Code: R7100102

R07

B.Tech I Year (R07) Supplementary Examinations, June 2013

MATHEMATICS - I

(Common to all branches)

Time: 3 hours

Max. Marks: 80

Answer any FIVE questions

All questions carry equal marks

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1 (a) Solve: $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$.

(b) Find particular member or orthogonal trajectories of $x^2 + cy^2 = 1$ passing through the point $(2, 1)$.

2 (a) Solve: $y'' + 4y' + 20y = 23\sin t - 15\cos t$, $y(0) = 0$, $y'(0) = -1$.

(b) Solve: $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x\,e^x \sin x$

3 (a) Show that $h < \sin^{-1}h < \frac{h}{\sqrt{(1-h^2)}}$ for $0 < h < 1$.

(b) Verify Lagrange's mean value theorem for $f(x) = \begin{cases} x\sin{}^1\!/_x\ (x \neq 0) \\ 0\ (x = 0) \end{cases}$ in$[-1, 1]$.

4 (a) Show that the radius of curvature at any point of the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ is equal to three times the length of the perpendicular from the origin to the tangent at that point.

(b) If $\sqrt{r} = \sqrt{a}\cos(\theta/2)$, prove that $\rho = \frac{2}{3}\sqrt{ar}$.

5 (a) Find the length of the arc of the parabola $y^2 = 4ax$ cutoff by the line $3y = 8x$.

(b) Evaluate $\iint \frac{r\,dr\,d\theta}{\sqrt{a^2+r^2}}$ over one loop of the lemniscates $r^2 = a^2\cos 2\theta$.

6 (a) Test the convergence of the following series: $\sum_{n=1}^{\infty} \frac{x^{2n}}{(n+1)\sqrt{n}}$.

(b) Show that the given exponential series $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ converges absolutely for all x.

7 (a) By transforming into triple integral, evaluate $\iint_s x^3dy\,dz + x^2y\,dz\,dx + x^2z\,dx\,dy$ where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$ and the circular discs $z = 0$, $z = b$.

(b) Verify Green's theorem for $\int_c [(xy + y^2)\,dx + x^2dy]$, where C is bounded by $y = x$ and $y = x^2$.

8 (a) Find: $L\left[\frac{e^{-3t}\sin 2t}{t}\right]$.

(b) Evaluate: $L^{-1}\left[\frac{(S+1)e^{-\pi s}}{s^2+s+1}\right]$.

(c) Using convolution theorem, find $L^{-1}\left\{\frac{1}{(s+a)(s+b)}\right\}$.

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