Code: 9ABS302

R09

B.Tech II Year I Semester (R09) Supplementary Examinations December 2015

MATHEMATICS – III

(Common to EEE, ECE, EIE, E.Con.E & ECC)

Time: 3 hours Max. Marks: 70

Answer any FIVE questions
All questions carry equal marks
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1 (a) Prove that $\int_0^1 \frac{x^2dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \pi/_{4\sqrt{2}}$.
(b) Show that $(2n+1)x\,P_n(x) = (n+1)P_{n+1}(x) + n\,P_{n-1}(x)$.

2 (a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though C – R equations are satisfied there.
(b) Find the analytic function $z = u + iv$, if $2u + v = e^x(\cos y - \sin y)$.

3 (a) Prove that $\log\left(\frac{a+ib}{a-ib}\right) = 2i\ \tan^{-1}\left(b/_a\right)$. Hence evaluate $\cos\left[i\log\left(\frac{a+ib}{a-ib}\right)\right]$.
(b) Find the general value of $\log(-i)$.

4 (a) Evaluate $\int_{1-i}^{2+3i}(z^2+z)dz$ along the line joining the points (1, -1) and (2, 3).
(b) State and prove Cauchy's integral theorem.

5 (a) Find the first four terms of the Taylor's series expansion of the complex variable function $f(x) = \frac{z+1}{(z-3)(z-4)}$ about $z = 2$. Find the region of convergence.
(b) Find the Laurent's series expansion of $\frac{7z-2}{z(z+1)(z-2)}$ in annulus $1 < |z+1| < 3$.

6 (a) Use Cauchy's residue theorem to evaluate $\oint_c \frac{dz}{(z^2+4)^2}$ where C is the circle $|z-i| = 2$.
(b) Show that $\int_0^{2\pi} \frac{\cos 2\theta\, d\theta}{1-2a\cos\theta+a^2} = \frac{2\pi a^2}{1-a^2}, (a^2 < 1)$.

7 (a) If the real number $a > e$, prove by using Rouche's theorem that the equation $e^z = az^n$ has n roots inside the unit circle.
(b) Prove that all the zeros of $z^7 - 5z^3 + 12 = 0$ lie between the circles $C_1 : |z| = 1$ and $C_2 : |z| = 2$.

8 (a) Discuss the transformation of $\omega = z + \frac{1}{z}$ and also state one application of this transformation.
(b) Under the transformation $\omega = \frac{1}{z}$ find the image of the circle $|z-2i| = 2$.

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