

PROBABILITY THEORY & STOCHASTIC PROCESSES

(Electrical & Electronics Engineering)

Time: 3 hours

Max. Marks: 70

Answer any FIVE questions
All questions carry equal marks

- 1 (a) A pair of fair dice are thrown. Person A wins if the sum of numbers showing up is six or less and one of the dice shows four. Person B wins if the sum is five or more and one of the dice shows a four. Find: (i) The probability that A wins. (ii) The probability that both A and B wins.
- (b) A manufacturing plant makes radios that each contain an integrated circuit (IC) supplied by three sources A, B, C. The probability that the IC in a radio came from one of the sources is $1/3$, the same for all sources. ICs are known to be defective with probabilities 0.001, 0.003 and 0.002 for sources A, B, C respectively.
 - (i) What is the probability any given radio will contain a defective IC?
 - (ii) What is the probability that the defective IC came from source A?
- 2 (a) List the properties of probability density function.
- (b) A random variable 'X' is known to be Poisson with $b = 4$.
 - (i) Plot the density and distribution functions for this random variable.
 - (ii) What is the probability of the event $(0 \leq X \leq 5)$?
- 3 (a) A Gaussian voltage random variable X has mean value zero and variance 9. The voltage X is applied to a square law full wave diode detector with a transfer characteristic $Y = 5X^2$. Find the mean value of the output voltage.
- (b) Find the characteristic function of a Poisson random variable.
- 4 Two random variable X and Y have a joint density:

$$f_{X,Y}(x,y) = \frac{10}{4} [u(x) - u(x-4)]u(y)y^3 \exp[-(x+1)y^2]$$
 Find marginal distributions of X,Y.
- 5 Two statistically independent random variables X and Y have mean values 2 and 4, second moments 8 and 25 respectively. Find the mean and variance of the random variable $W = 3X - Y$.
- 6 A random process is defined by $X(t) = A$, where A is a continuous random variable uniformly distributed on (0,1).
 - (a) Determine the form of the sample functions.
 - (b) Classify the process.
 - (c) Is it deterministic?
 - (d) Find the first-order density function of X(t) at any time t.
- 7 (a) List the properties of auto correlation function properties.
- (b) Two random processes X(t) and Y(t) are defined as:

$$X(t) = A \cos(\omega_0 t) + B \sin(\omega_0 t); Y(t) = B \cos(\omega_0 t) - A \sin(\omega_0 t)$$
 Find the cross-correlation function $R_{XY}(t, t + \tau)$. Show that X(t) and Y(t) are jointly wide sense stationary. Given A and B are uncorrelated zero mean random variables.
- 8 (a) List the properties of cross power density spectrum.
- (b) Random process X(t) has the autocorrelation function as follows $R_{XX}(\tau) = Pe^{-\tau^2/2a^2}$ where $p > 0$ and $a > 0$ are constants. Find the power density spectrum of X(t).