B.Tech II Year I Semester (R13) Regular & Supplementary Examinations December 2015

MATHEMATICS – III

(Common to EEE, ECE and EIE)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Define Gamma function and evaluate $\int_0^\infty e^{-x^2}\,dx$.

(b) Express $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ in terms of Beta function.

(c) Express $Cos\,\theta$ and $Sin\,\theta$ interms of Bessel function.

(d) Prove that $P_n^1(1) = \frac{1}{2}\,n(n+1)$

(e) Write the C – R equations in both Cartesian and Polar co-ordinates.

(f) Find the fixed points of the transformation: $\omega = \frac{2i-6z}{iz-3}$.

(g) State Cauchy's integral theorem.

(h) Define pole of a complex function with example.

(i) State Cauchy's residue theorem.

(j) Evaluate $\int_C \frac{5z-2}{z(z-1)}\,dz$ where $C:\ |z| = 2$.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 Show that $\beta(m.n) = 2\int_0^{\pi/2} Sin^{2m-1}\theta\ Cos^{2n-1}\theta$ and deduce that $\int_0^{\pi/2} Sin^n\,\theta\,d\theta = \int_0^{\pi/2} Cos^n\,\theta\,d\theta = \frac{\Gamma\left[\frac{(n+1)}{2}\right]\sqrt{\pi}}{2\Gamma\left(\frac{n+2}{2}\right)}$.

OR

3 Find the power series solution of the equation $y'' + xy' + y = 0$ in powers of x.

UNIT – II

4 Prove that $P_n(x) = \frac{1}{2^n.n!}\frac{d^n}{dx^n}(x^2-1)^n$ and hence express $2x^2 - 4x + 2$ as Legendre polynomial.

OR

5 (a) Prove that $\frac{d}{dx}[xJ_n(x)J_{n+1}(x)] = x[J_n^2(x) - J_{n+1}^2(x)]$.

(b) Prove that $J_{n+1}(x) = \frac{n}{x}J_n(x) - J_n^1(x)$.

UNIT – III

6 (a) Determine P such that the function: $f(z) = \frac{1}{2}\log(x^2+y^2) + i\ \tan^{-1}\left(\frac{px}{y}\right)$ be an analytic function.

(b) Find the analytic function, whose real part is $u = e^x[(x^2-y^2)\cos y - 2xy\sin y]$.

OR

7 (a) Show that the function $\omega = \frac{4}{z}$ transform the straight line $x = c$ in the z-plane into a circle in the ω – plane.

(b) Find the bilinear transformation that maps the points $(0,\ i,\ 1)$ into the points $(-1,\ 0,\ 1)$.

UNIT – IV

8 Generate z^2 along the straight line OM and along the path OLM where 'O' is the origin, L is the point z = 3 and M is $z = 3 + i$ and hence establish the Cauchy's integral theorem.

OR

9 (a) Obtain the Taylor's series to represent the function $\frac{z^2-1}{(z+2)(z+3)}$, in the region $|z| < 2$.

(b) Expand $f(z) = \frac{e^{2z}}{(z-1)^3}$ about z = 1 as Laurent's series. Also find the region of convergence.

UNIT – V

10 Show that $\int_0^{2\pi}\frac{d\theta}{a+b\sin\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$, $a > b > 0$ using Residue theorem.

OR

11 Prove that $\int_{-\infty}^{\infty}\frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{a+b}$ $(a > 0,\ b > 0,\ a \neq b)$

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