

B.Tech II Year I Semester (R13) Supplementary Examinations June 2016

MATHEMATICS – III

(Common to EEE, ECE and EIE)

Time: 3 hours

Max. Marks: 70

PART – A
(Compulsory Question)

1 Answer the following: (10 X 02 = 20 Marks)

- Evaluate $\int_0^1 x^4 (1-x)^2 dx$
- Compute $\beta\left(\frac{9}{2}, \frac{7}{2}\right)$
- Use Rodrigues formula, show that $P_0(x) = 1$.
- Prove that $x^2 = \frac{1}{3}P_0(x) + \frac{2}{3}P_2(x)$
- If $f(z)$ is an analytic function with constant modulus, show that $f(z)$ is constant.
- Define Bilinear transformation and write cross ratio of four points.
- Evaluate $\int_0^{2+i} z^2 dz$, along the line $y = x/2$.
- Write the statement of Cauchy's integral formula.
- Define isolated singular point of an analytic function.
- Evaluate $\int_C \frac{z-3}{z^2+2z+5} dz$, where C is circle $|z| = 1$.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

- Show that $\int_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1} \beta(m+1, n+1)$
 - Show that $\int_0^\infty x^m e^{-x^n} dx = \frac{\Gamma(m)}{n^m}$, ($m, n > 0$).

OR

- Obtain the series solution of the equation $x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + xy = 0$.

UNIT – II

- Show that $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x \right)$
 - Show that $x^4 = \frac{1}{35} [8P_4(x) + 20P_2(x) + 7P_0(x)]$

OR

- Prove that $J_{-\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{3}{x} \sin x + \frac{3-x^2}{x^2} \cos x \right)$
 - Use Rodrigues formula, show that: (i) $P_1(x) = x$. (ii) $P_2(x) = \frac{3x^2-1}{2}$.

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UNIT – III

6 (a) If $f(z)$ is an analytic function of z prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \log |f(z)| = 0$

(b) Find the analytic function $f(z) = u + i v$ if $u + v = \frac{\sin 2x}{(\cosh 2y - \cos 2x)}$

OR

7 (a) Find the analytic function whose real part $u = \frac{\sin 2x}{(\cosh 2y - \cos 2x)}$.

(b) Find the bilinear transformation which maps the points $(\infty, i, 0)$ into the points $(-1, -1, 1)$ in w - plane.

UNIT – IV

8 (a) Evaluate $\int_C (y - x - 3x^2 i) dz$, where C is the straight line from $Z = 0$ to $Z = 1 + i$.

(b) Find Taylor's expansion of $f(z) = \frac{2z^3 + 1}{z^2 + z}$ about the point $z = i$.

OR

9 (a) Evaluate:

$$\int_C \frac{\cos z - \sin z}{(z + i)^3} dz \text{ with } C : |z| = 2 \text{ using Cauchy's integral formula.}$$

(b) Expand $\frac{\tan z}{z}$ by Laurent's series and then find nature of singularity.

UNIT – V

10 (a) Determine the poles of the function $\frac{z^2 + 1}{z^2 - 2z}$ and the residue at each pole.

(b) Evaluate $\int_{-\infty}^{\infty} \frac{1}{x^4 + 1} dx$.

OR

11 Apply the calculus of residues, to prove that:

$$\int_{-\infty}^{\infty} \frac{\cos x}{(x^2 + a^2)(x^2 + b^2)} dx = \frac{\pi}{a^2 - b^2} \left(\frac{e^{-b}}{b} - \frac{e^{-a}}{a} \right) \quad (a > b > 0)$$
