Answer any FIVE questions
All questions carry equal marks
1 (a) Three boxes of identical appearance contain two coins each. In one box are gold; in the second box are silver and in the third box one is silver and the other is gold. Suppose that a box is selected random and further that a coin in that box is selected at random. If this coin is gold what is the probability that the other coin is also gold.
(b) In a school $60 \%$ are boys and $40 \%$ are girls. Suppose that $20 \%$ and $25 \%$ of the girls and boys respectively play tennis. What is the probability that at randomly selected student is: (i) A girl who plays tennis? (ii) A boy who plays tennis? (iii) A tennis player?

2 (a) The distribution function of a random variable X is given by:

$$
\begin{aligned}
F_{x}(x) & =1-(1+x) e^{-x}, \text { for } x \geq 0 ; \\
& =0 ; \text { otherwwise } .
\end{aligned}
$$

Find the probability density function.
(b) State and prove any four properties of variance of random variable.

3 (a) A random variable defined by the density function

$$
\begin{aligned}
F_{x}(x) & =(\pi / 6) \cos (\pi x / 8) ; \text { for }-4 \leq x \leq 4 ; \\
& =0 ; \text { elsewhere }
\end{aligned}
$$

Find $\mathrm{E}[3 \mathrm{X}]$ and $\mathrm{E}\left[\mathrm{X}^{2}\right]$.
(b) List the properties of characteristic function.

A random process $X(t)$ is defined as $X(t)=(A+2) \operatorname{Cos}(t)+B \operatorname{Sin}(t)$, where $A$ and $B$ are independent random variables with zero mean and same mean square value of 1 . Verify that $X(t)$ is stationary or not. Find its covariance.

8 (a) Find the power spectral density of a random process $Z(t)=X(t)+Y(t)$, where $X(t)$ and $Y(t)$ are zero mean independent random process.
(b) Find the average power of a random process with power spectral density given as $S_{x x}(\omega)=6 \omega^{2} /\left(1+\omega^{2}\right)$.

