

B.Tech II Year I Semester (R09) Supplementary Examinations June 2016

MATHEMATICS – III

(Common to EEE, EIE, E.Con.E, ECE & ECC)

Time: 3 hours

Max. Marks: 70

Answer any FIVE questions
All questions carry equal marks

- 1 (a) Show that $\Gamma(n) = \int_0^1 \left(\log \frac{1}{x}\right)^{n-1} dx$ ($n > 0$).
(b) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.
- 2 Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials.
- 3 (a) State and prove C – R equations in polar form.
(b) Find the general value of $\log(-i)$.
- 4 (a) Find the analytic function whose imaginary part is $e^x(x \sin y + y \cos y)$.
(b) Show that the function $u(x, y) = e^x \cos y$ is harmonic. Determine its harmonic conjugate $v(x, y)$ and $f(z) = u + i v$.
- 5 (a) State and prove Cauchy's theorem.
(b) Evaluate $\int_C \frac{\log z}{(z-1)^3} dz$ where $C: |z-1| = \frac{1}{2}$, using Cauchy's integral formula.
- 6 (a) Expand $\log(1-z)$ where $|z| < 1$ using Taylor series.
(b) Find Laurent series expansion of the function $f(z) = \frac{z^2 - 6z - 1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z+2| < 5$.
- 7 Prove that $\int_0^\infty \frac{dx}{x^6+1} = \frac{\pi}{3}$.
- 8 (a) State and prove Rouché's theorem.
(b) Find the bilinear transformation that maps the points $(\infty, i, 0)$ into the points $(0, i, \infty)$.
