B.Tech I Year (R13) Supplementary Examinations June 2017

MATHEMATICS - I

(Common to all branches)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Solve $(1-x^2)\frac{dy}{dx} - xy = 1$.

(b) Solve $\left(xy^2 - e^{1/x^2}\right)dx - x^2y dy = 0$.

(c) Show that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta), y = a(1-\cos\theta)\ is\ 4a\cos\theta/2$.

(d) Find the maximum and minimum values of $3x^4 - 2x^3 - 6x^2 + 6x + 1$ in the internal (0, 2).

(e) Evaluate $\iint_A xy\,dx\,dy$, where A is the domain bounded by x-axis, ordinate x = za and the curve x^2 = 4ay.

(f) Find by triple integration, the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

(g) Find the Laplace transform of the function

$f(t) = \sin\omega t, 0 < t < \pi/\omega$

$= 0, \quad \frac{\pi}{\omega} < t < 2\pi/\omega$

(h) Evaluate $L\left\{e^{-t}\int_0^t \frac{\sin\omega t}{t}dt\right\}$.

(i) If $u = x + y + z, C = x^2 + y^2 + z^2, w = yz + zx + xy$. Prove that grad u, grad v and grad w are coplanar.

(j) Prove that $div(r^nR) = (n+3)r^n$. Hence show that R/r^3 is solenoidal.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 (a) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = (1-e^x)^2$.

(b) A body originally at 80°c cools down to 60°c in 20 minutes, the temperature of the air being 40°c. What will be the temperature of the body after 40 minutes from the original?

OR

3 (a) Solve by the method of variation of parameters, $\frac{d^2y}{dx^2} - y = \frac{2}{(1+e^x)}$.

(b) An uncharged condenser of capacity C is charged by applying an e.m.f. $E\sin t/\sqrt{(LC)}$, through leads of self-inductance L and negligible resistance. Prove that at any time t, the charge on one of the plates is $\frac{EC}{2}\left\{\sin\frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}}\cos\frac{t}{\sqrt{LC}}\right\}$.

UNIT – II

4 (a) Using Maclaurin series, expand tan x up to the term containing x^5.

(b) Find the radius of curvature at the point (3a/2, 3a/2) of the Folium $x^3 + y^3$ = 3axy.

OR

5 (a) Find the volume of the largest possible right-circular cylinder that can be inscribed in sphere of radius a.

(b) If $y_1 = \frac{x_2x_3}{x_1}, y_2 = \frac{x_3x_1}{x_2}, y_3 = \frac{x_1x_2}{x_3}$, show that the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 is 4.

UNIT – III

6 (a) Trace the curve $y = x^3 - 12x - 16$.

(b) By changing the orders of integration of $\int_0^\infty\int_0^\infty e^{-xy}\sin px\,dx\,dy$ show that $\int_0^\infty \frac{\sin px}{x}dx = \frac{\pi}{2}$.

OR

7 (a) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

(b) Find the value of the portion of the sphere $x^2 + y^2 + z^2 = a^2$ using inside the cylinder $x^2 + y^2 = ay$.

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UNIT – IV

8 (a) If f(t) is a periodic function with period T, then prove that $L(f(t)) = \int_0^T \frac{e^{-st} f(t)dt}{1-e^{-sT}}$.

(b) Find the inverse Laplace transform of $\frac{5s+3}{(s-1)(s^2+2s+5)}$

OR

9 (a) Using convolution theorem, evaluate $L^{-1}\left\{\frac{1}{(s-2)(s+2)^2}\right\}$.

(b) Solve $\frac{d^2x}{dt^2} + 9x = \cos 2t, if\ x(0) = 1,\ x\left(\frac{\pi}{2}\right) = -1$

UNIT – V

10 (a) Show that $r^{\alpha}R$ is any irrotational vector for any value of α but is solenoidal if $\alpha + 3 = 0$, where R = xi + yj + 2k and r is the magnitude of R.

(b) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$.

OR

11 (a) Evaluate $\int_s F.N\,ds$, where $F = 2x^2yi - y^2j + 4xz^2k$ and S is the closed surface of the region in the first octant bounded by the cylinder $y^2+z^2=9$ and the planes x=0, x=2, y=0 and z=0.

(b) Verify divergence theorem for $F = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ taken over the rectangular parallelepiped, $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$.

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