## Code: 13A54102

## B.Tech I Year (R13) Supplementary Examinations June 2017 <br> MATHEMATICS - II

(Common to EEE, ECE, EIE, CSE \& IT)
Time: 3 hours
Max. Marks: 70

## PART - A

(Compulsory Question)
*****
1 Answer the following: (10 $\times 02=20$ Marks $)$
(a) Find the Eigen values of the matrix.

$$
A=\left(\begin{array}{lll}
3 & 1 & 4 \\
0 & 2 & 6 \\
0 & 0 & 5
\end{array}\right)
$$

(b) If $A=\left(\begin{array}{ccc}2+i & 3 & -1+3 i \\ -5 & i & 4-2 i\end{array}\right)$, show that $\mathrm{AA}^{*}$ is a Hermitian matrix, where $\mathrm{A}^{*}$ is the conjugate transpose of $A$.
(c) Find the real root of the equation $x^{3}-2 x+5 z=0$ by the method of false position correct to three decimal places.
(d) Find the positive root of $x^{2}-x=10$ correct to three decimal places, using Newton-Raphson method.
(e) Find the value of $y$ for $x=0.1$ by Picard's method, given that $\frac{d y}{d x}=\frac{y-x}{y+x} y(0)=1$.
(f) Use Runge-Kutta method of order 4, find $y(0.2)$ given that $\frac{d y}{d x}=3 x+\frac{1}{2} y, y(0)=1$ taking $h=0.1$.
(g) If $F(s)$ is the complex Fourier transform of $f(x)$ them prove that $F\left(f(a x)=\frac{1}{a} F\left(\frac{S}{a}\right), a \neq 0\right.$.
(h) State and prove convolution theorem for Fourier transforms.
(i) Form the partial differential equation (by eliminating the arbitrary constants) from

$$
Z=(x+y) \phi\left(x^{2}-y^{2}\right)
$$

(j) Using the method of separation of variables solve, $\frac{\partial u}{\partial x}=2 \frac{\partial u}{\partial y}+u$, where $u(x, 0)=6 e^{-3 x}$.

PART - B
(Answer all five units, $5 \times 10=50$ Marks)

## UNIT - I

2 (a) Verify Cayley-Hamilton theorem for the matrix $A=\left(\begin{array}{ll}1 & 4 \\ 2 & 3\end{array}\right)$ and find its inverse.
(b) Find the Rank of the matrix.

$$
A=\left(\begin{array}{lll}
1 & 2 & 3 \\
1 & 4 & 2 \\
2 & 6 & 5
\end{array}\right)
$$

## OR

3 (a) Find the Eigen values and Eigen vectors of the matrix $\left(\begin{array}{lll}1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1\end{array}\right)$.
(b) Find the matrix P which transforms the matrix $A=\left(\begin{array}{lll}1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1\end{array}\right)$ to the diagonal form. Hence calculate $\mathrm{A}^{4}$.

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## UNIT - II

4 (a) Using Newton's forward formula, find the value of $f(1.6)$, if

| $x$ | 1 | 1.4 | 1.8 | 2.2 |
| :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 3.49 | 4.82 | 5.96 | 6.5 |

(b) Compute the value of $\int_{0.2}^{1.4}\left(\sin x-\log x+e^{x}\right) d x$. Using Simpson's $3 / 8^{\text {th }}$ rule.

## OR

5 (a) Use Lagrange's interpolation formula to find the value of $y$ when $x=10$, if the following values of $x$ and y are given;

| $x$ | 5 | 6 | 9 | 11 |
| :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 12 | 13 | 14 | 16 |

(b) Evaluate $\int_{0}^{1} \frac{d x}{1+x^{2}}$ using trapezoidal rule taking $h=\frac{1}{4}$.

## UNIT - III

6 (a) Apply Milne's method, to find a solution of the differential equation $y^{\prime}=x-y^{2}$ in the range $0 \leq x \leq 1$ for the boundary conditions $y=0$ at $x=0$.
(b) Employ Taylor's method to obtain approximate value of y at $x=0.2$ for the differential equation $\frac{d y}{d x}=2 y+3 e^{x}, y(0)=0$.

## OR

If $f(x)=\left\{\begin{array}{cc}0, & -\pi \leq x \leq 0 \\ \sin x & 0 \leq x \leq \pi\end{array}\right.$
Prove that $f(x)=\frac{1}{\pi}+\frac{\sin x}{2}-\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\operatorname{cons} 2 n x}{4 n^{2}-1}$.
Hence show that $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-----\infty=\frac{1}{4}(\pi-2)$.

## UNIT - IV

(a) Find the Fourier integral representation for
$f(x)= \begin{cases}1-x^{2}, & \text { for }|x| \leq 1 \\ 0, & \text { for }|x| \leq 1\end{cases}$
(b) Find the Fourier sine and cosine transform of $x^{n-1}, n>0$.

## OR

9 (a) If $Z\left(u_{n}\right)=U(z)$ then prove that
$Z\left(u_{n-k}\right)=Z^{-k} U(z) \quad(k>0)$.
(b) If $U(z)=\frac{2 Z^{2}+5 Z+14}{(z-1)^{4}}$, evaluate $u_{2}$ and $u_{3}$.

## UNIT - V

(a) Solve by the method of separation of variables $\frac{\partial^{2} Z}{\partial x^{2}}-2 \frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$.
(b) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is vibrating by giving to each of its points a velocity $\lambda x(l-x)$, find the displacement of the string at any distance $x$ from one end at any time $t$.

## OR

Solve $\frac{\partial u}{\partial x}=2 \frac{\partial u}{\partial t}+u$ where $u(x, 0)=6 e^{-3 x}$.
A bar $A B$ of length 10 cm has its ends $A$ and $B$ kept at $30^{\circ}$ and $100^{\circ}$ temperatures respectively, until steady-state condition is reached. Then the temperature at $A$ is lowered to $20^{\circ}$ and that at $B$ to $40^{\circ}$ and these temperatures are maintained. Find the subsequent temperature distribution in the bar.

