## B.Tech II Year I Semester (R09) Supplementary Examinations June 2017 <br> MATHEMATICS - III

(Common to EEE, EIE, E.Con.E, ECE \& ECC)
Time: 3 hours
Max. Marks: 70

> Answer any FIVE questions
> All questions carry equal marks
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1 (a) Show that $\sqrt{\pi} \Gamma(2 m)=2^{2 m-1} \Gamma(m) \Gamma\left(m+\frac{1}{2}\right)$.
(b) Show that $\beta(m, n+1)+\beta(m+1, n)=\beta(m, n)$.

2 (a) Show that an analytic function with constant modulus is constant.
(b) If the potential function is $\log \left(x^{2}+y^{2}\right)$, find the flux function and the complex potential function.

3 (a) Prove that $(i)^{i}=e^{-(4 n+1) \pi / 2}$.
(b) Separate $\sin ^{-1}(\cos \theta+i \sin \theta)$ into real and imaginary parts, where $\theta$ is a positive acute angle.

4 (a) Evaluate $\int_{1-i}^{2+3 i}\left(z^{2}+z\right) d z$ along the line joining the points $(1,-1)$ and $(2,3)$.
(b) If $f(\xi)=\oint_{c} \frac{3 z^{2}+7 z+1}{z-\xi} d z$, where C is the circle $\mathrm{x}^{2}+\mathrm{y}^{2}=4$, find the values of $\mathrm{f}(3), f^{\prime}(1-\mathrm{i}), f^{\prime \prime}(1-\mathrm{i})$.

5 (a) Expand $e^{z}$ as Taylor's series about $z=1$.
(b) Obtain Laurent's series for $f(z)=e^{2 z /(z-1)^{3}}$ about $\mathrm{z}=1$.

6 (a) Use Cauchy's residue theorem to evaluate $\oint_{c} \frac{d z}{\left(z^{2}+4\right)^{2}}$ where c is the circle $|z-i|=2$.
(b) Evaluate $\int_{0}^{\infty} \frac{d x}{\left(1+x^{2}\right)^{2}}$.

7 Prove that one root of the equation $z^{4}+z^{3}+1=0$ lies in the first quadrant.
8 Find the bilinear transformation which maps the points $\mathrm{z}=1, \mathrm{i},-1$ on to the points $\mathrm{w}=\mathrm{i}, 0,-\mathrm{i}$. Hence find: (i) The image of $|z|<1$. (ii) The invariant points of this transformation.

