## B.Tech II Year I Semester (R13) Supplementary Examinations June 2017

## PROBABILITY THEORY \& STOCHASTIC PROCESSES

(Electronics \& Communication Engineering)
Time: 3 hours
Max. Marks: 70

## PART - A

(Compulsory Question)

1 Answer the following: ( $10 \times 02=20$ Marks $)$
(a) Define probability of the event with an example.
(b) Determine the value of K , such that the given density function is valid
$f_{X}(x)=K, a<x<b$
0 , elsewhere
(c) State the central limit theorem.
(d) Derive the expression of 'constant $\mathrm{a}^{\prime}$ in terms of moments of X \& Y , if V \& W are orthogonal, where $\mathrm{V}=\mathrm{X}+\mathrm{aY}$ \& $\mathrm{W}=\mathrm{X}-\mathrm{aY}$.
(e) List the various classifications of random processes.
(f) Prove the statement $R_{X X}(-\tau)=R_{X X}(\tau)$.
(g) List any two properties of cross PSD.
(h) What is the average power in $\mathrm{X}(\mathrm{t})$, if the $R_{X X}(\tau)=3+2 \exp \left(-4 \tau^{2}\right)$ ?
(i) Write the relation between $S_{X Y}(\omega), H(\omega) \& S_{X X}(\omega)$.
(j) Describe the condition for stable system.

PART - B
(Answer all five units, $5 \times 10=50$ Marks)

## UNIT-1

2 (a) State and prove the Bayes theorem.
(b) A random variable X has the density function $f_{X}(x)=\frac{1}{2} u(x) \exp \left(-\frac{x}{2}\right)$, evaluate the probabilities of events $A=\{1<X \leq 3\}, B=\{X \leq 2.5\}$.

OR
3 (a) Compute the joint and conditional probabilities based on the given data. In a box there are 100 resistors having resistances and tolerance as shown in below table. Define the three events, A as "draw a $470 \Omega$ resistor", B as "draw a 5\% tolerance resistor", and C as "draw a $100 \Omega$ resistor".

| Resistance $(\Omega)$ | Tolerance |  | Total |
| :---: | :---: | :---: | :---: |
|  | $5 \%$ | $10 \%$ |  |
| 22 | 10 | 14 | 24 |
| 47 | 28 | 16 | 44 |
| 100 | 24 | 08 | 32 |
| Total | 62 | 38 | 400 |

(b) Define and explain the following distribution and densities with an application.
(i) Exponential
(ii) Uniform.

Contd. in page 2

## UNIT - II

4 (a) State and prove the joint density function properties.
(b) Identify the value of moment $\mu_{22}$, if statistically independent random variables $X$ and $Y$ have moments $m_{10}=2, m_{20}=14, m_{02}=12$ and $m_{11}=-6$. OR
5 (a) If statistically independent random variables X and Y having respective densities $f_{X}(x)=5 u(x) e^{-5 x}$, $f_{Y}(y)=2 u(y) e^{-2 y}$ then derive the density function of $\mathrm{W}=\mathrm{X}+\mathrm{Y}$.
(b) Two random variables $X$ and $Y$ have means $\bar{X}=1$ and $\bar{Y}=2$ variances $\sigma_{X}^{2}=4$ and $\sigma_{Y}^{2}=1$ and a correlation coefficient $\rho_{X Y}=0.4$. New random variables W and V are defined by $\mathrm{V}=-\mathrm{X}+2 \mathrm{Y}, \mathrm{W}=\mathrm{X}+3 \mathrm{Y}$. Find (i) The means (ii) The variances (iii) The correlations (iv) The correlation coefficient $\rho_{V W}$ of V and W .

UNIT - III
6 (a) If a random process $X(t)=A \operatorname{Cos}(w t)+B \operatorname{Sin}(w t)$ is given, where $A \& B$ are uncorrelated, zero mean random variables having same variance $\sigma^{2}$, then check $X(t)$ is WSS or not.
(b) Evaluate the mean, average power and variance of random process having $R_{X X}(\tau)=36+25 \exp (-|\tau|)$.

OR
7 (a) Define the terms:
(i) Random process.
(ii) Stationary random process.
(iii) Wide sense stationary random process.
(iv) Ergodic random process.
(b) Let $\mathrm{X}(\mathrm{t})$ be a stationary continuous random process that is differentiable.

Denoted by $\dot{X}(t)=\frac{d}{d t}(X(t))$
Determine (i) $\dot{X}(t)$ (ii) Express auto correlation function $R_{X X X}(\tau)$ in terms of $R_{X X}(\tau)$.

## UNIT - IV

8 (a) Interpret the Wiener Khintchine relation for auto power spectral density and autocorrelation of a random process.
(b) Find spectrum of random process whose auto correlation function $R_{X X}(\tau)=\frac{A_{0}{ }^{2}}{2} \operatorname{Cos}\left(\omega_{0} \tau\right)$, plot correlation and its spectrum.

## OR

Discuss the properties of auto power spectral density in detail.

## UNIT - V

10 (a) Derive the expression for mean and mean squared value of system response.
(b) A stationary random process $\mathrm{X}(\mathrm{t})$ with zero mean and autocorrelation $R_{X X}(\tau)=e^{-2|\tau|}$ is applied to a system of function $H(\omega)=\frac{1}{2+j \omega}$ develop the PSD of output.

OR
11 (a) Write a short note on band limited, band pass and narrow band process.
(b) Consider a linear system as shown in figure:

$\mathrm{X}(\mathrm{t})$ is the input and $\mathrm{Y}(\mathrm{t})$ is the output of the system. The autocorrelation of $\mathrm{X}(\mathrm{t})$ is $R_{X X}(\tau)=5 \delta(\tau)$ determine the PSD and autocorrelation.

