B.Tech II Year II Semester (R15) Regular Examinations May/June 2017

MATHEMATICS – IV

(Common to EEE, ECE and EIE)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Compute $\Gamma(3.5)$.

(b) $\beta\left(\frac{9}{2},\frac{7}{2}\right)$.

(c) Compute $J_0(2)$

(d) $J_0(x)=\frac{9}{2}[J_2(x)-J_0(x)]$ use recurrence relation.

(e) Find the critical points of the transformation $w^2=(z-a)(z-b)$.

(f) Compute $V(r,\theta)$ when $f(z)=u(r,\theta)+iv(r,\theta)$. Here $u(r,\theta)=\left(r+\frac{1}{r}\right)\cos\theta$.

(g) Expand Taylor's series $\cos z$ about the point $z=\pi/2$.

(h) Write the formula of pole of order n at $z=z_0$.

(i) Evaluate $\oint_C e^{1/z^2}\,dz$ where C is $|z|=2$ traversed counterclockwise.

(j) Evaluate $\oint_C \frac{dz}{z^2(z+4)}dz$ where C is $|z|=2$.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 State and prove relation between Beta and Gamma function.

OR

3 (a) Find the value of $\Gamma\left(\frac{1}{2}\right)$.

(b) Derive $\int_0^{\frac{\pi}{2}}\sin^n\theta\,d\theta$.

UNIT – II

4 Show that $J_0(x)=\frac{1}{\pi}\int_0^{\pi}\cos(x\cos\phi)d\phi$.

OR

5 Find the value of $J_{\frac{1}{2}}(x)$.

UNIT – III

6 Find the analytical function of the complex potential for an electric field $w=\varphi+i\Psi$, given that $\Psi=x^2-y^2+\frac{x}{x^2+y^2}$. Use Milne Thomson method.

OR

7 (a) Find the bilinear transformation that maps the points $z_1=-i,\ z_2=0, z_3=i$ into the points $w_1=0$, $w_2=-1\ and\ w_3=\infty$.

(b) Find the fixed points of the bilinear transformation $w=(z-1)/(z+1)$.

UNIT – IV

8 Evaluate $\int_C \frac{e^{2z}}{(z+1)^4}\,dz$ where C is the circle $|z|=2$, using complex integration formula.

OR

9 Represent the function $f(z)=\frac{4z+3}{z(z-3)(z-2)}$ as Laurent series:

(i) With in $|z|=1$. (ii) In the annulus region $|z|=2\ and\ |z|=3$. (iii) Exterior to $|z|=3$.

UNIT – V

10 Apply the calculus of residues to evaluate $\int_0^{2\pi}\frac{d\theta}{2-\sin\theta}$.

OR

11 (a) Evaluate $\int_{|z|=\frac{1}{2}}\frac{dz}{(z-1)(z+2)^2}=0$ using Residue theorem.

(b) Evaluate $\oint_C \frac{z^2}{z^2-jz+2}dz$ where C is $|z|=3/2$, traversed counterclockwise.