

R16

Code No: 131AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year I Semester Examinations, May/June - 2017

MATHEMATICS-II

(Common to CE, ME, MCT, MMT, MIE, CEE, MSNT)

Time: 3 hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

Part- A (25 Marks)

- 1.a) Find $\lim_{t \rightarrow 0} f(t)$, if $L(f(t)) = \frac{s}{s^2 + w^2}$. [2]
- b) Find the inverse Laplace Transform of $\frac{s+5}{(s+1)(s+3)}$. [3]
- c) Find the value of $\int_0^\infty \frac{dx}{1+x^4}$. [2]
- d) Evaluate $\int_0^1 x^{11}(1-x)^{16} dx$. [3]
- e) Find the area enclosed between the parabola $y = x^2$ and the line $y = x$. [2]
- f) Evaluate $\int_0^{\frac{\pi}{4}} \cos^2 x dx$. [3]
- g) Find the magnitude of the gradient of the function $f(x, y, z) = xyz^3$ at $(1, 0, 2)$. [2]
- h) The velocity vector in 2-dimensional field is $\vec{V} = 2xy\vec{i} + (2y^2 - x^2)\vec{j}$. Find the $\text{curl } \vec{V}$. [3]
- i) Find the Curl of the gradient of the scalar field $V = 2x^2y + 3y^2z + 4z^2x$. [2]
- j) Find the divergence of the vector field \vec{A} at $(1, -1, 1)$ $\vec{A} = x^2z\vec{i} + xy\vec{j} - yz^2\vec{k}$. [3]

Part-B (50 Marks)

- 2.a) State and prove the second shifting theorem of Laplace Transform.

$$\text{b) Find } L(F(t)) \text{ if } F(t) = \begin{cases} \sin\left(t - \frac{\pi}{3}\right), & t > \frac{\pi}{3} \\ 0, & t < \frac{\pi}{3} \end{cases} \quad [5+5]$$

OR

- 3.a) Find the Laplace Transform of $F(t) = a + bt + \frac{c}{\sqrt{t}}$.
- b) Solve using Laplace Transform $\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = 3\cos 3t - 11\sin 3t$, $y(0) = 0$, $y'(0) = 16$. [5+5]
- 4.a) Show that $\int_0^{\frac{\pi}{2}} \sqrt{\tan \theta} d\theta = \int_0^{\frac{\pi}{2}} \sqrt{\cot \theta} d\theta = \frac{\pi}{\sqrt{2}}$.
- b) Evaluate $\frac{\beta(m+1, n)}{\beta(m, n)}$. [5+5]
- OR**
5. Show that $\Gamma(m) \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma(2m)$, $m > 0$. [10]
6. Find the mass, center of gravity and moment of inertia relative to the x-axis, y-axis and origin of a rectangle $0 \leq x \leq 4$, $0 \leq y \leq 2$ having the mass density function $f(x, y) = xy$. [10]
- OR**
7. Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy dy dx$ and integrate it. [10]
- 8.a) Show that $\text{div}(r^n \bar{r}) = (n+3)r^{-n}$.
- b) If $\bar{u} = \frac{1}{r} \bar{r}$, find $\text{grad}(\text{div} \bar{u})$. [5+5]
- OR**
- 9.a) Show that $\text{div}(\bar{A} \times \bar{B}) = \bar{B} \cdot \text{curl} \bar{A} - \bar{A} \cdot \text{curl} \bar{B}$.
- b) Find the gradient of the Scalar function $f(x, y, z) = x^2y^2 + xy^2 - z^2$ at $(3, 1, 1)$. [5+5]
10. Verify the Gauss's divergence theorem for $\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ over the rectangular parallelepiped $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq c$. [10]
- OR**
11. Evaluate $\oint_C x^2 dx + 2y dy - dz$ by Stoke's theorem where C is the curve $x^2 + y^2 = 4$, $z = 2$. [10]

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