R15

Code No: 123AH

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech II Year I Semester Examinations, March - 2017
MATHEMATICS-III
(Common to EEE, ECE, EIE, ETM)

Time: 3 Hours   Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit.
Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

                                                                (25 Marks)

1.a)   Find the particular integral of $x^2 \frac{d^2y}{dx^2} - 6x\frac{dy}{dx} + 10y = x^2$.   [2]

  b)   Find the indicial equation of $x^2 y'' - 2xy' - (x^2 - 2)y = 0$.   [3]

  c)   Prove that $\int_{-1}^{1} P_2^2(x)dx = \frac{2}{5}$.   [2]

  d)   Prove that $J_1(0) = 0$.   [3]

  e)   Find the value of 'a' if cosax sinhy is harmonic.   [2]

  f)   Find the analytic function whose real part is xy.   [3]

  g)   Find the residue of $\frac{2z+3}{z^2-z-2}$ at z = -1.   [2]

  h)   Expand $\frac{1}{3-z}$ when $|z| > 3$ as Laurent series.   [3]

  i)   Prove that $w = C + z$ where C is a complex constant is conformal at all points.

                                                                [2]

  j)   Find the fixed points of $\frac{z+i}{1+iz}$.   [3]

PART-B

                                                                (50 Marks)

2.   Solve the differential equation $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x\log x$.   [10]

OR

3.   Solve the differential equation in series $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ around x = 0.

                                                                [10]

4.a)   Express $x^2 + x + 1$ in terms of Legendre Polynomials.

  b)   Prove that $\frac{d}{dx}\left(x^n J_n(x)\right) = x^n J_{n-1}(x)$.   [5+5]

OR

5.a)   Prove that $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + (n)P_{n-1}(x)$.

  b)   Prove that $J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right)J_1(x) + \left(1 - \frac{24}{x^2}\right)J_0(x)$.   [5+5]

6.a) Find the analytic function whose real part is $e^{-x}(x \sin y - y \cos y)$.

b) Evaluate $\int\limits_{C} \dfrac{dz}{(z-2)(z-4)}$ where C is $|z-3|=1/2$. [5+5]

OR

7.a) If f(z) is an analytic function then show that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} \right)|f(z)|^2 = 4|f'(z)|^2$.

b) Evaluate $\int\limits_{C} \dfrac{dz}{(z^2-4)(z+1)}$ where C is $|z| = 3$. [5+5]

8. Evaluate $\int_0^{2\pi} \dfrac{d\theta}{2+\cos\theta}$ using residue theorem. [10]

OR

9. Evaluate using residue theorem $\int_{-\infty}^{\infty} \dfrac{x^2 dx}{(x^2+1)(x^2+4)}$. [10]

10.a) Under the transformation $w = \dfrac{z-i}{1-iz}$ find the image of the circle $|z| = 1$.

b) Find the image of $|z - 3i| = 3$ under the mapping $w = \dfrac{1}{z}$. [5+5]

OR

11.a) Find the image of the region bounded by the lines $x = 1$, $y = 1$, $x + y = 1$ under the transformation $w = z^2$.

b) Find the bilinear mapping which maps the points $z = \infty$, i, 0 into 0, i, ∞. [5+5]

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