

R15

Code No: 123AN
JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech II Year I Semester Examinations, March - 2017
PROBABILITY AND STATISTICS
(Common to ME, CSE, IT, MCT, AME, MIE, MSNT)
Time: 3 Hours
Max. Marks: 75
Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

PART- A
(25 Marks)

- 1.a) The distribution function of random variable X is given by

$$F(X) = \begin{cases} 0, & x < 2 \\ k(x-2), & 2 \leq x \leq 6 \\ 1, & x > 6 \end{cases} \quad \text{find the value of } k. \quad [2]$$

- b) Find moment generating function. Deduce moment generating function of Poisson distribution. [3]

- c) The Probability density function of two-dimensional random variable is

$$f(x, y) = \begin{cases} \frac{8}{9}xy, & 1 < x < y < 2 \\ 0, & \text{Other wise} \end{cases}$$

Compute marginal density function of X. [2]

- d) If
- $\sigma_x = \sigma_y = \sigma$
- and angle between two regression lines is
- $\tan^{-1}\left(\frac{4}{3}\right)$
- , compute
- r
- . [3]

- e) If we can assert with 99% confidence that the maximum error is 0.05 and
- $P = 0.2$
- . deduce the size of the sample. [2]

- f) State the properties of F-distribution. [3]

- g) Define transient and study states in queueing model. [2]

- h) Explain Customer behaviour in the queue. [3]

- i) Differentiate random variable and random process. [2]

- j) Compose steady-state distribution of the Markov chain
- $\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$
- . [3]

PART-B
(50 Marks)

- 2.a) A coin is tossed until a head appears. Expect the number of tosses required?

- b) If the random variable X takes the values 1,2,3 and 4 such that
- $2P(X=1) = 3P(X=2) = p(X=3) = 5P(X=4)$
- , derive the probability distribution function and cumulative distribution function of X. [5+5]

OR

- 3.a) A machine manufacturing bolts is known to produce 5% defective. In a random sample of 10 bolts, compute the probability that there are (i) exactly 3 defective bolts (ii) not more than 3 defective bolts.

- b) In Normal distribution, 7% of items under 35 and 89% under 63. Compute mean and variance of the distribution. [5+5]

- 4.a) The joint probability mass function of X and Y is given by

$$f(x, y) = \begin{cases} \frac{x+y}{21}, & x = 1, 2, 3; y = 1, 2 \\ 0, & \text{Other wise} \end{cases}$$

Compute covariance of (x, y) .

- b) Using the formula $r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x\sigma_y}$, compute r from the following data. [5+5]

X	92	89	87	86	83	77	71	63	53	50
Y	86	88	91	77	68	85	52	82	37	57

OR

- 5.a) In a partially destroyed laboratory record of an analysis of correlation data, the following results only are legible: variance of X is 9, regression equations are $8x - 10y + 66 = 0$, $40x - 18y = 214$

Compute (i) the mean values of X and Y .

(ii) coefficient of correlation between X and Y .

(iii) the standard deviation of Y .

- b) From the data relating to the yield of dry bark (X_1), height (X_2) and girth (X_3) for 18 cinchona plants, the following correlations were obtained:

$$r_{12} = 0.77, r_{13} = 0.72 \text{ and } r_{23} = 0.52.$$

Compute (i) $r_{12.3}$ (ii) $R_{1.23}$.

[5+5]

- 6.a) A coin was tossed 400 times and head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance.

- b) A sample of 100 electric bulbs produced by manufacturer A showed a mean life time of 1190 hours and a standard deviation of 90 hours. A sample of 75 bulbs produced by manufacturer by B showed a mean life time of 1230 hours, with a standard deviation of 120 hours. Is there a difference between the mean life time of two brands at a significance level of (i) 0.05 (ii) 0.01. [5+5]

OR

- 7.a) Eleven school boys were given a test in drawing. They were given a month's further tuition and a second test of equal difficulty was held at the end of it. Do the marks given evidence that the students have benefitted by extra coaching?

Boys	1	2	3	4	5	6	7	8	9	10	11
Marks I test	23	20	19	21	18	20	18	17	23	16	19
Marks II test	24	19	22	18	20	22	20	20	23	20	17

- b) A set of 5 similar coins is tossed 320 times and the result is

No. of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution.

[5+5]

8. Customers arrive at a one-man barber shop according to a Poisson process with mean inter-arrival time of 12 minutes. Customers spend an average of 10 minutes in barber chair.
- compute the expected number of customers in the barber shop?
 - compute the percentage of time an arrival can walk straight into the barber's chair without having to wait.
 - compute the average time customers spend in the queue?
 - compute the probability that more than 3 customers in the system? [10]

OR

9. In a single-server queuing system with Poisson input and exponential service times, if the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hour, and the maximum possible number of calling units in the system is 2, compute
- $P_n (n \geq 0)$.
 - the average number of calling units in the system.
 - the average waiting in the queue. [10]

- 10.a) A stochastic(random) process is described by $X(t) = A \sin t + B \cos t$ where A and B are independent random variables with zero means and equal standard deviation. Show that the process is stationary of second order.
- b) Three advocates A,B,C have 400,500 and 600 clients respectively at $t=0$. During one year though no new client has been added, migration from one to the other have taken places as given below:
- From A 50 have gone to B and 25 to C
 From B 50 have gone to A and 100 to C
 From C 25 have gone to A
- Prepare the transition probability matrix and estimate the number of clients associated with A, B, C after one year. [5+5]

OR

11. The transition probability matrix of a Markov chain $\{X_n\}; n=1,2,3,\dots$ having 3 states 1,2,and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)} = (0.7, 0.2, 0.1)$
- Compute a) $P(X_2 = 3)$
 b) $P(X_3 = 2, X_1 = 3, X_0 = 2)$. [10]

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