Code No: 07A3BS02 **R07** **Set No. 2**

II B.Tech I Semester Examinations,November 2010
MATHEMATICS - III
Common to ICE, E.COMP.E, ETM, E.CONT.E, EIE, ECE, EEE

Time: 3 hours Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Expand f(z) = $\frac{e^{2z}}{(z-1)^3}$ about z=1 as a Laurent series. Also find the region of convergence.
   (b) Find the Taylor series for $\frac{z}{z+2}$ about z=1, also find the region of convergence. [8+8]

2. (a) Use method of contour integration to prove that $\int_0^{2\pi} \frac{d\theta}{1+a^2-2a\cos\theta} = \frac{2\pi}{1-a^2}$, 0< a<1.
   (b) Evaluate $\int_0^{\infty} \frac{dx}{(x^2+9)(x^2+4)^2}$ using residue theorem. [8+8]

3. Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & if\ m \neq n \\ \frac{2}{2n+1} & if\ m = n \end{cases}$. [16]

4. (a) Find the poles and residues at each pole of the function $\frac{(2z+1)}{(z^2-z-2)}$.
   (b) Evaluate $\int_C \frac{(3z-4)dz}{z(z-1)(z-2)}$ by residue theorem.where C:$|z| = 3$. [8+8]

5. (a) Evaluate $\int_{1-i}^{2+3i} (z^2 + z)dz$ along x=t and y=t^2 using Cauchy's integral formula.
   (b) Evaluate $\int_C \frac{\sin^6 z\ dz}{(z-\frac{\pi}{2})^3}$ where C is $|z|$ = 1 using Cauchy's integral formula.
   (c) Evaluate $\int_C \frac{\cos \pi z^2\ dz}{(z-1)(z-2)}$ where C is $|z|$ = 3 using Cauchy's integral formula. [5+5+6]

6. (a) State necessary condition for f (z) to be analytic and derive C-R equations in Cartesian coordinates.
   (b) If u and v are functions of x and y satisfying Laplace's equations show that (s+it) is analytic where s = $\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$ and $t = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$. [8+8]

7. Evaluate using $\beta - \Gamma$ functions.
   (a) $\int_0^1 x^2(\log \frac{1}{x})^3 dx$
   (b) $\int_0^{\pi/2} \sin^{7/2}\theta \cos^{3/2}\theta\, d\theta$

(c) Show that $\int\limits_{-1}^{1}(1+x)^{m-1}(1-x)^{n-1}dx = 2^{m+n+1}\beta\,(m,n)$. [5+5+6]

8. (a) Show that the transformation w=z+1/z maps the circle |z| =c into the ellipse u=(c+1/c) cos θ, v =(c–1/c)sinθ. Also discuss the case when c=1 in detail.

   (b) Find the bilinear transformation which maps the points (2, i, –2) into the points (l, i, –l). [8+8]

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Code No: 07A3BS02 **R07** **Set No. 4**

II B.Tech I Semester Examinations,November 2010
MATHEMATICS - III
Common to ICE, E.COMP.E, ETM, E.CONT.E, EIE, ECE, EEE
Time: 3 hours **Max Marks: 80**
Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Use method of contour integration to prove that $\int\limits_{0}^{2\pi} \frac{d\theta}{1+a^2-2acos\theta} = \frac{2\pi}{1-a^2}$, $0< a<1$.
   (b) Evaluate $\int\limits_{0}^{\infty} \frac{dx}{(x^2+9)(x^2+4)^2}$ using residue theorem. [8+8]

2. Prove that $\int\limits_{-1}^{1} P_m(x)P_n(x)dx = \left\{ \begin{array}{ll} 0 & if\ m \neq n \\ \frac{2}{2n+1} & if\ m = n \end{array} \right.$. [16]

3. (a) Show that the transformation w=z+1/z maps the circle |z| =c into the ellipse u=(c+1/c) cos θ, v =(c–1/c)sinθ. Also discuss the case when c=1 in detail.
   (b) Find the bilinear transformation which maps the points (2, i, –2) into the points (l, i, –l). [8+8]

4. (a) State necessary condition for f (z) to be analytic and derive C-R equations in Cartesian coordinates.
   (b) If u and v are functions of x and y satisfying Laplace's equations show that (s+it) is analytic where s = $\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$ and $t = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$. [8+8]

5. (a) Expand f(z) = $\frac{e^{2z}}{(z-1)^3}$ about z=1 as a Laurent series. Also find the region of convergence.
   (b) Find the Taylor series for $\frac{z}{z+2}$ about z=1, also find the region of convergence. [8+8]

6. (a) Evaluate $\int\limits_{1-i}^{2+3i} (z^2 + z)dz$ along x=t and y=t^2 using Cauchy's integral formula.
   (b) Evaluate $\int\limits_{C} \frac{\sin^6 z\ dz}{(z-\frac{\pi}{2})^3}$ where C is $|z| = 1$ using Cauchy's integral formula.
   (c) Evaluate $\int\limits_{C} \frac{\cos \pi z^2\ dz}{(z-1)(z-2)}$ where C is $|z| = 3$ using Cauchy's integral formula. [5+5+6]

7. Evaluate using $\beta - \Gamma$ functions.
   (a) $\int\limits_{0}^{1} x^2(\log \frac{1}{x})^3 dx$
   (b) $\int\limits_{0}^{\pi/2} \sin^{7/2}\theta \cos^{3/2}\theta\, d\theta$

(c) Show that $\int\limits_{-1}^{1} (1+x)^{m-1}(1-x)^{n-1}dx = 2^{m+n+1}\beta\,(m,n)$. [5+5+6]

8. (a) Find the poles and residues at each pole of the function $\frac{(2z+1)}{(z^2-z-2)}$.

(b) Evaluate $\int\limits_C \frac{(3z-4)dz}{z(z-1)(z-2)}$ by residue theorem.where C:$|z|=3$. [8+8]

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Code No: 07A3BS02 **R07** **Set No. 1**

II B.Tech I Semester Examinations,November 2010
MATHEMATICS - III
Common to ICE, E.COMP.E, ETM, E.CONT.E, EIE, ECE, EEE

Time: 3 hours Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Show that the transformation w=z+1/z maps the circle |z| =c into the ellipse u=(c+1/c) cos θ, v =(c–1/c)sinθ. Also discuss the case when c=1 in detail.
   (b) Find the bilinear transformation which maps the points (2, i, –2) into the points (l, i, –l). [8+8]
2. (a) Use method of contour integration to prove that $\int\limits_0^{2\pi} \frac{d\theta}{1+a^2-2a\cos\theta} = \frac{2\pi}{1-a^2}$, 0< a<1.
   (b) Evaluate $\int\limits_0^{\infty} \frac{dx}{(x^2+9)(x^2+4)^2}$ using residue theorem. [8+8]
3. Prove that $\int\limits_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & if\ m \neq n \\ \frac{2}{2n+1} & if\ m = n \end{cases}$. [16]
4. (a) Find the poles and residues at each pole of the function $\frac{(2z+1)}{(z^2-z-2)}$.
   (b) Evaluate $\int\limits_C \frac{(3z-4)dz}{z(z-1)(z-2)}$ by residue theorem.where C:$|z| = 3$. [8+8]
5. (a) Evaluate $\int\limits_{1-i}^{2+3i} (z^2+z)dz$ along x=t and y=t^2 using Cauchy's integral formula.
   (b) Evaluate $\int\limits_C \frac{\sin^6 z\ dz}{(z-\frac{\pi}{2})^3}$ where C is $|z|$ = 1 using Cauchy's integral formula.
   (c) Evaluate $\int\limits_C \frac{\cos \pi z^2\ dz}{(z-1)(z-2)}$ where C is $|z|$ = 3 using Cauchy's integral formula. [5+5+6]
6. (a) State necessary condition for f (z) to be analytic and derive C-R equations in Cartesian coordinates.
   (b) If u and v are functions of x and y satisfying Laplace's equations show that (s+it) is analytic where s = $\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$ *and* $t = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$. [8+8]
7. Evaluate using $\beta - \Gamma$ functions.
   (a) $\int\limits_0^1 x^2(\log \frac{1}{x})^3 dx$
   (b) $\int\limits_0^{\pi/2} \sin^{7/2}\theta \cos^{3/2}\theta\, d\theta$

(c) Show that $\int_{-1}^{1} (1+x)^{m-1}(1-x)^{n-1}dx = 2^{m+n+1}\beta\,(m,n)$. [5+5+6]

8. (a) Expand f(z) = $\frac{e^{2z}}{(z-1)^3}$ about z=1 as a Laurent series. Also find the region of convergence.

   (b) Find the Taylor series for $\frac{z}{z+2}$ about z=1, also find the region of convergence. [8+8]

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Code No: 07A3BS02 **R07** **Set No. 3**

II B.Tech I Semester Examinations,November 2010
MATHEMATICS - III
Common to ICE, E.COMP.E, ETM, E.CONT.E, EIE, ECE, EEE

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Use method of contour integration to prove that $\int\limits_0^{2\pi} \frac{d\theta}{1+a^2-2a\cos\theta} = \frac{2\pi}{1-a^2}$, 0< a<1.
   (b) Evaluate $\int\limits_0^{\infty} \frac{dx}{(x^2+9)(x^2+4)^2}$ using residue theorem. [8+8]

2. (a) Show that the transformation w=z+1/z maps the circle |z| =c into the ellipse u=(c+1/c) cos θ, v =(c–1/c)sinθ. Also discuss the case when c=1 in detail.
   (b) Find the bilinear transformation which maps the points (2, i, –2) into the points (l, i, –l). [8+8]

3. (a) Find the poles and residues at each pole of the function $\frac{(2z+1)}{(z^2-z-2)}$.
   (b) Evaluate $\int\limits_C \frac{(3z-4)dz}{z(z-1)(z-2)}$ by residue theorem.where C:$|z| = 3$. [8+8]

4. Evaluate using $\beta - \Gamma$ functions.
   (a) $\int\limits_0^1 x^2(\log \frac{1}{x})^3 dx$
   (b) $\int\limits_0^{\pi/2} \sin^{7/2}\theta \cos^{3/2}\theta \, d\theta$
   (c) Show that $\int\limits_{-1}^1 (1+x)^{m-1}(1-x)^{n-1}dx = 2^{m+n+1}\beta\,(m,n)$. [5+5+6]

5. (a) Evaluate $\int\limits_{1-i}^{2+3i} (z^2+z)dz$ along x=t and y=t^2 using Cauchy's integral formula.
   (b) Evaluate $\int\limits_C \frac{\sin^6 z \, dz}{(z-\frac{\pi}{2})^3}$ where C is $|z| = 1$ using Cauchy's integral formula.
   (c) Evaluate $\int\limits_C \frac{\cos \pi z^2 \, dz}{(z-1)(z-2)}$ where C is $|z| = 3$ using Cauchy's integral formula. [5+5+6]

6. (a) State necessary condition for f (z) to be analytic and derive C-R equations in Cartesian coordinates.
   (b) If u and v are functions of x and y satisfying Laplace's equations show that (s+it) is analytic where $s = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$ *and* $t = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$. [8+8]

7. Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & if\ m \neq n \\ \frac{2}{2n+1} & if\ m = n \end{cases}$. [16]

8. (a) Expand f(z) = $\frac{e^{2z}}{(z-1)^3}$ about z=1 as a Laurent series. Also find the region of convergence.

   (b) Find the Taylor series for $\frac{z}{z+2}$ about z=1, also find the region of convergence. [8+8]

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