Code No: 07A4BS02 **R07** **Set No. 2**

II B.Tech II Semester Examinations,November 2010
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Verify Cauchy's integral theorem for f(z)=z^3 taken over the boundary of the rectangle with vertices at -1, 1, 1+i, -1+i.
   (b) Use Cauchy's integral formula to evaluate $\oint\limits_c \frac{\cos z}{(z-\pi i)^2} dz$ where c is the circle and $|z| = 5$ [8+8]

2. (a) Prove that $\int\limits_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(b+cx)^{m+n}} dx = \frac{\beta(m,n)}{(b+c)^m b^n}$.
   (b) Show that $\int\limits_0^\infty x^m e^{-x^n} dx = \frac{\Gamma(m)}{n^m}$, (m, n> 0). [8+8]

3. (a) Represent each of the following in the exponential form $r\, e^{i\theta}$
      i. 3 + 4i
      ii. -4i
      iii. -2
   (b) Separate the real and imaginary parts of exp (i z^2) [10+6]

4. (a) Find the Laurent series of $\frac{7z-2}{(z+1)z(z-2)}$ in the annulas $1 < |z+1| < 3$.
   (b) Expand the Laurent series of $\frac{z^2-1}{(z+2)(z+3)}, for\, |z| > 3$. [8 + 8]

5. (a) Find all values of k, such that f(z) = e^x [cos ky + i sin ky] is analytic.
   (b) If f(z) is an analytic function of z prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \log |f(z)| = 0$ [8+8]

6. Evaluate $\int\limits_C \frac{f'(z)}{f(z)} dz$ by using argument principle where C is a simple closed curve C and f(z) = z^5 - $8z^2$i + 2z - 3 + 5i. [16]

7. Using the method of contour integration, prove that $\int\limits_0^\infty \frac{dx}{x^6+1} = \frac{\pi}{3}$ [16]

8. Discuss about the conformal mapping of w = tan z. [16]

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Code No: 07A4BS02 **R07** **Set No. 4**

II B.Tech II Semester Examinations,November 2010
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
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1. Determine and classify all singularities of the given functions.
   (a) $\frac{1}{z-z^3}$
   (b) $\frac{z^4}{1+z^4}$
   (c) $\cot\frac{1}{z} - \frac{1}{z}$
   (d) $\frac{1-e^{2z}}{z^4}$
   (e) $\frac{1-\cos z}{z}$
   (f) $e^{z/(z-2)}$ [16]

2. (a) If w = u + iv = z^3, prove that u = c_1 and v = c_2 cut each other orthogonally where C_1 and C_2 are constants.
   (b) Find the analytic function whose imaginary part is e^{-x}[x cos y + y sin y]. [8+8]

3. Prove that the all roots of z^5+z-16 lie between the circles $|z|$=1 and $|z|$=2. [16]

4. (a) Determine all values and the principal value of
      i. log 3i
      ii. $\log(\sqrt{3} - i)$
   (b) Find the real and imaginary parts of log cos (x + iy) [8+8]

5. (a) Integrate f(z) = x^2+ixy from A(1,1) to B(2,4) along the curve x =t , y = t^2.
   (b) If f(z) is analytic in a region R,P and Q are two points in R, then prove that $\int_P^Q f(z)dz$ is independent of the path joining P and Q. [8+8]

6. (a) Find the image of the infinite strip $0 < y < \frac{1}{2}$ under the transformation w = $\frac{1}{z}$
   (b) Show that the image of the hyperbola x^2 - y^2 = 1 under the transformation w = $\frac{1}{z}$ is the lemniscate p^2 = cos 2 ϕ. [8+8]

7. (a) Prove that Rodrigue's formula $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$
   (b) Express f(x) = x^3 - 5x^2 + x + 2 in terms of Legender's polynomials. [10+6]

8. Evaluate $\int_0^\infty \frac{dx}{x^4+a^4}$ [16]

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Code No: 07A4BS02 **R07** **Set No. 1**

II B.Tech II Semester Examinations,November 2010
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
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1. (a) Determine the residues of the function
$f(z) = \frac{e^z}{z^2+\pi^2}$ at the poles.

   (b) Find the residues of the function $f(z) = \frac{1-e^{2z}}{z^4}$ at the poles. [8+8]

2. (a) If tan(x+iy) = A+iB show that $A^2+B^2+2A\cot 2x = 1$.

   (b) Find the principal value of $(2i)^{2i}$ [8+8]

3. State Rouche's theorem and use it to find the number of zeros of the polynomial $z^8 - 4z^5 + z^2 - 1$ that lie inside the circle $|z| = 1$. [16]

4. (a) Find where the function w = $\frac{z-2}{(z+1)(z^2+1)}$ fails to be analytic.

   (b) If w = φ + iψ represents the complex potential for an electric field and $= x^2 - y^2 + \frac{x}{x^2+y^2}$ find the function φ. [6+10]

5. (a) Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the path

      i. y = x
      ii. y = x^2

   (b) Use Cauchy's integral formula to evaluate $\oint_c \frac{\sin^2 z}{\left(z-\frac{\pi}{6}\right)^3}dz$ where c is the circle $|z| = 1$ [8+8]

6. (a) Prove that the transformation w = $\frac{1}{z}$ maps every straight line or circle on to a circle or straight line.

   (b) Define bilinear transformation. Prove that the bilinear transformation is conformal. [8+8]

7. (a) Determine the poles and the corresponding residues of $f(z) = \frac{1}{(z^2+4)^2}$ and f(z) $= \frac{z+1}{z^2(z-2)}$

   (b) Expand $\frac{e^{2z}}{(z-1)^4}$ in powers of (z-1). [8+8]

8. (a) Prove that (2n+1) $P_n(x) = P^1_{n+1}(x) + P^1_{n-1}(x)$

   (b) Prove that $\int_{-1}^{1} x^2 P_{n+1}(x)P_{n-1}(x)dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$ [8+8]

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Code No: 07A4BS02 **R07** **Set No. 3**

II B.Tech II Semester Examinations,November 2010
MATHEMATICS - III
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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
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1. (a) Separate the real and imaginary parts of
   i. cosec z
   ii. exp (z^2)

   (b) Find all solutions of : exp (2z - 1) = 1 + i . [10+6]

2. (a) Find whether the function f(z) is analytic or not .
   i. f(z) = cosh z ,
   ii. f(z) = 2xy + i(x^2 - y^2)

   (b) Show that the function u = $\frac{1}{2}$ log (x^2 + y^2) is harmonic and find its harmonic conjugate function. [8+8]

3. (a) Determine the bilinear transformation that maps the points (1 - 2i, 2 + i, 2 + 3i) into the points (2 + i, 1 + 3i, 4).

   (b) Find the bilinear transformation which maps the points (-i, 0, i) into the points (-1, i, 1) respectively. [8+8]

4. Evaluate $\oint\limits_C \frac{f'(z)}{f(z)}dz$ where C is simple closed curve, where f(z) = cos πz, $c : |z| = \pi$ [16]

5. Evaluate $\int\limits_0^\infty \frac{\cos x}{(1+x^2)^2}dx$ [16]

6. (a) Show that when $|z + 1| < 1, z^{-2} = 1 + \sum\limits_{n=1}^{\infty}(n + 1)z^n$

   (b) Find the poles and the corresponding residues of the function f(z) =$\frac{1}{(z+1)(z+3)}$ [8+8]

7. (a) Determine F(2), F(4), F(-3i), F$'$ (i), F$''$ (-2i), if F(α) = $\int\limits_c \frac{5z^2-4z+3}{z-\alpha}dz$ where c is the ellipse 16x^2+9y^2=144.

   (b) Use Cauchy's integral formula to evaluate $\oint\limits_c \frac{z+4}{z^2+2z+5}dz$ where C is the circle |z + 1| = 1 [10+6]

8. (a) Show that P_n(x) is the co-efficient h^n in the expansion in ascending powers of $(1 - 2xh + h^2)^{-1/2}$

   (b) Show that

i. $P_n(1) = 1$

ii. $P_n(-x) = (-1)^n \, P_n(x)$
Hence deduce that $P_n(-1)=(-1)^n$. [8+8]

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