Code No: 07A4BS04 **R07** **Set No. 2**

**II B.Tech II Semester Examinations,December 2010**
**MATHEMATICS FOR AEROSPACE ENGINEERS**
**Aeronautical Engineering**

**Time: 3 hours** **Max Marks: 80**

**Answer any FIVE Questions**
**All Questions carry equal marks**

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1. (a) Prove that $\Gamma(m) = \int_0^1 \left[\left(\log\left(\frac{1}{x}\right)\right)\right]^{m-1} dx$
   (b) Evaluate $\int_0^1 \frac{x^4}{\sqrt{1-x^2}} dx$
   (c) Prove that $\int_0^1 x^n [\log x]^m \, dx = \frac{(-1)^m \, m}{n+1^{m+1}}$ [5+6+5]

2. (a) Find the analytic function f(z) = u + iv If u - v = $e^x(\cos y - \sin y)$
   (b) Find the general and Principal values of
      i. $\log_e\left(1 + i\sqrt{3}\right)$
      ii. $\log_e(-1)$
      iii. $\log_e(-i)$ [8+8]

3. (a) If a random variable has a probability density function
   $f(x) = K(x^2 - 1), \quad for -1 \leq x \leq 3$
   $= 0 \quad otherwise$
   Find the value of K and P (1/ 2≤ x ≤ 5 / 2)
   (b) A book of 500 pages contains 500 print mistakes. If a distributor receives1000 books, find how many books may contain 4 print mistakes in a randomly selected page. [8+8]

4. (a) Find the expansion of by Taylor's series about z=1.
   (b) Expand f(z) = $\frac{z}{(z-1)(2-z)}$ in a Laurent's series for $1 < |z| < 2$
   (c) Expand $f(z) = z\, e^{2z}$ in a Taylor's series about z=-1. [5+6+5]

5. (a) Determine the region of the w- plane in to which the region bounded by x = 2, y = 2 , x + y = 2 in z- plane is mapped under the transformation w = $z^2$ .
   (b) Find the bilinear transformation mapping the points z = -i ,0, i into w = -1 ,i,1 Also find the image |z| < 1 under such transformation. [8+8]

6. (a) If the components of two tensors are equal in one coordinate system, show that they are equal in all coordinate systems.
   (b) Define Christoffel symbol of first kind. Prove that $\partial g_{ij}/\partial x^k$= [ik, j] + [jk, i] [8+8]

7. (a) Using Cauchy's integral formula Evaluate $\int_c \frac{dz}{(2-zi)^2(z+2i)^2}$, c being the circumference of the Ellipse $x^2 + 4(y-2)^2 = 4$
   (b) Evaluate $\int_{1+i}^{2+4i} z^2 dz$

i. Along the parabola x=t, y=$t^2$ where $1 \leq t \leq 2$ and

ii. Along the straight line joining 1+i, and 2+4i [8+8]

8. (a) The chances that a doctor will diagnose a disease X correctly is 70%.The chances that a patient will die by his treatment after correct diagnosis is 30% and the chance of death by wrong diagnosis is 70%. A patient of doctor A who has a disease X, died. What is the chance that his disease was diagnosed correctly?

   (b) Four students A, B, C, D try independently to solve a problem , with respective probabilities to solve the problem as 1/2, 1/3, 1/5 and 1/5. What is the probability that the problem remains unsolved? [8+8]

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Code No: 07A4BS04 **R07** **Set No. 4**

**II B.Tech II Semester Examinations,December 2010**
**MATHEMATICS FOR AEROSPACE ENGINEERS**
**Aeronautical Engineering**

**Time: 3 hours** **Max Marks: 80**

**Answer any FIVE Questions**
**All Questions carry equal marks**
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1. (a) Show that an analytic function of constant absolute value is constant.
   (b) Prove that $\omega = z^n$ where n is a positive integer is analytic and find its derivative
   (c) Prove that the f(z)= $\overline{z}$ is not analytic at any point. [6+7+3]

2. (a) Find the poles and residues at each pole of the function $\frac{z^2}{z^3-1}$
   (b) Evaluate $\int_c \frac{ze^z}{z^2+9}dz$ where c is $|z| = 5$ by residue theorem. [8+8]

3. (a) What is summation convention in tensor analysis? Write out all the tensor in $x^{pq}$ $x_{qr}$ if n = 2
   (b) Define Christoffel symbol of first and second kind. If $(ds)^2 = (dr)^2 + r^2 (d\theta)^2 + r^2\sin^2\theta\,(d\varphi)^2$, then find the value of [22,1 ]and $\left[{1 \atop 22}\right]$ [8+8]

4. (a) Prove that $\int\limits_c \frac{dz}{z-a} = 2\pi i$ where c is given by the equation $|z - a| = r$
   (b) Evaluate $\int\limits_c \frac{ze^z dz}{(z+2)^3}$ where c is $|z| = 3$ using Cauchy's integral formula
   (c) Evaluate $\int\limits_c \left(\frac{e^z}{z^3} + \frac{z^4}{(z+i)^2}\right)dz$ where c is $|z| = 2$ using Cauchy's integral theorem. [5+5+6]

5. (a) Find the image of $1 \le |z| \le 2$ under the mapping w =2 iz + 1.
   (b) Determine bilinear transformation which map the points z = -1, 1,i into w= -2, 2, -i Find the critical and fixed points of the transformation. [8+8]

6. (a) A person seeks advice regarding one of the two possible courses of action from three experts who act independently. The person follows the majority advice. The probability that the individual advisors are wrong is 0.1, 0.05 and 0.05 respectively. What is the probability that the person takes incorrect advice?
   (b) Two ordinary six faced dice are thrown. Given the sum of two numbers are 8,find the conditional probability that the number noted on the first dice is larger than the number noted on the second dice. [8+8]

7. (a) The owner of lodging house with 5 cabins is considering installing air conditioners(A.C) to those cabins. He expects that about half of his customers would be willing to stay in A.C cabins and finally he buys 3 A.Cs. Assuming 100% occupancy for all times, find the probability that
      i. there will be no more requests for A.C cabins

ii. a customer who requires an A.C cabin will get it.

(b) The weekly wages of 1000 workers are normally distributed with a mean of Rs.70 /- and standard deviation of Rs. 5 /- Estimate the number of of workers whose weekly wages will be

i. more than Rs.80.

ii. less than Rs. 50

iii. Between Rs 69 and Rs 72. [8+8]

8. (a) Prove that $\int_{-1}^{1} x\, P_n(x) P_{n-1}(x)\, dx = \frac{2n}{4n^2-1}$

(b) Prove that $\cos(x \sin\theta) = J_o + (2\cos 2\theta)\, J_2 + (2\cos 4\theta) J_4 + - - - - - -$
And $\sin(x \sin\theta) = (2\sin\theta)\, J_1 + (2\sin 3\theta)\, J_3 + (2\sin 5\theta)\, J_5 + - - - - -$ [8+8]

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Code No: 07A4BS04 **R07** **Set No. 1**

**II B.Tech II Semester Examinations,December 2010**
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**Aeronautical Engineering**

**Time: 3 hours** **Max Marks: 80**

**Answer any FIVE Questions**
**All Questions carry equal marks**
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1. (a) Define Knonecker delta. Show that $a_{ij}\ A^{kj} = \Delta\delta^k_i$, where $\Delta$ is a determinant of order three and $A^{ij}$ are cofactors of $a^{ij}$.
   (b) Define Christoffel symbol of first and second kind. If $(ds)^2 = r^2(d\theta)^2 + r^2\sin^2\theta\ (d\varphi)^2$ , then find the value of [12 ,2 ] , [2 ,12 ] [8+8]

2. (a) State and prove the addition theorem of probabilities for any two events A and B. State the extension of the theorem for any n events $A_1$, $A^2$, $A^3$, .... $A_n$
   (b) Define independent events. Prove that, if A and B are independent events then $A^c$ and $B^c$ are also independent. [8+8]

3. (a) State and prove Laurent's theorem
   (b) Expand $\frac{1}{(z^2+1)(z^2+2)}$ in positive and negative powers of z if $1 < |z| < \sqrt{2}$ [8+8]

4. (a) Evaluate $\int_{1-i}^{2+i} (2x + iy + 1)\, dz$ along the straight line joining (1,-1) and (2,1)
   (b) Evaluate $\int\limits_c \frac{\cosh \pi z}{z(z^2+1)} dz$ using Cauchy's integral formula where c is $|z| = 2$. [8+8]

5. (a) The diameter of an electric cable is a continuous random variable X with p.d.f $f(x) = 6x(1 - x)$ , $0 \le x \le 1$.
      i. Check whether f (x) is a p.d.f.
      ii. Determine b such that P(X<b) = P(X >b)
   (b) Fit a Poisson distribution to the following data.

| x: | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| f: | 46 | 38 | 22 | 9 | 1 |

[8+8]

6. (a) State and prove Rodrigue's Formula
   (b) Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$ [10+6]

7. (a) If w=f(z)=u+iv is an analytic function and $\phi$ is any function of x and y with partial derivatives of first and second orders, than prove that $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = \left\{\frac{\partial^2\phi}{\partial u^2} + \frac{\partial^2\phi}{\partial v^2}\right\} |f'(z)|^2$
   (b) If f(z)=u+iv is an analytic function of z and if u-v=$(x - y)(x^2 + 4xy + y^2)$ , find f(z) in terms of z. [8+8]

8. (a) Find the image and draw a rough sketch of the mapping of the region $1 \le x \le 2$, $2 \le y \le 3$ under the transformation $w = e^z$

(b) Find the bilinear transformation which maps the points -2, 1+2i, 0 of the z-plane into 0, 1, $\infty$ of the w-plane. Find the invariant points of the transformation. [8+8]

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Code No: 07A4BS04 **R07** **Set No. 3**

**II B.Tech II Semester Examinations,December 2010**
**MATHEMATICS FOR AEROSPACE ENGINEERS**
**Aeronautical Engineering**

**Time: 3 hours** **Max Marks: 80**

**Answer any FIVE Questions**
**All Questions carry equal marks**

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1. (a) Determine the poles of the function f(z) and residue at each pole where $f(z) = \frac{(z^2-2z)}{(z+1)^2(z^2+1)}$

   (b) Evaluate $\int_c \frac{(12z-7)}{(z-1)^2(2z+3)}dz$ by Cauchy's residue theorem where C is the circle $|z| = 2$ [8+8]

2. (a) Find the image of

      i. the infinite strip 1 < x < 2

      ii. |z + 1| = 1 under the transformation w =1 /z.

   (b) Find the bilinear transformation which maps the points z = 1, i, - 1 into the points w = i,0, -i respectively. Hence find the image of $|z| < 1$ [8+8]

3. (a) prove that $\int_0^\infty \cos(x^2)dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$

   (b) Prove that $\int_0^{\frac{\pi}{2}} \sin^m(x)\cos^n(x)\,dx = \frac{\Gamma\frac{m+1}{2}\Gamma\frac{n+1}{2}}{2\Gamma\left(\frac{m+n+2}{2}\right)}$

   (c) Show that $\int_0^\infty x^2e^{-x^4}dx$ X $\int_0^\infty e^{-x^4}\ dx = \frac{\pi\sqrt{2}}{16}$. [5+5+6]

4. (a) Prove that f $(x, y) = \frac{x^2y(y-x)}{(x^6+y^2)(x+y)}, (x, y) \neq (0, 0) = 0$ if (x,y)=(0,0) Is discontinuous at (0,0).

   (b) Find f(z)=u+iv given that
   $u + v = \frac{\sin 2x}{\cosh 2y-2\cos 2x}$ [8+8]

5. Define fundamental tensor. Determine the components of the fundamental tensor in cylindrical coordinates. [16]

6. (a) Evaluate $\int_c \frac{e^z}{z(1-z)^3}dz$ if

      i. z=1 lies inside c and z=0 lies outside and

      ii. z=0 and z=1 both lie inside c.

   (b) Using Cauchy's integral formula, evaluate $\int\limits_c \frac{z^3-2z+1}{z^2(z-i)^2}dz$ where c is the circle $|z| = 2$ [8+8]

7. (a) The mean yield for one acre plot is 662 kgs with standard deviation 32 .Assuming normal distribution, how many one acre plots in a batch of 1000 plots are expected to yield

      i. over 700 kgs.

ii. below 650 kgs.

iii. what is the lowest yield of the best 100 plots.

(b) Show that if X (t) and Y(t) are two random processes and $R_{XY}(\tau)$ and $R_{YX}(\tau)$ are their respective auto correlation functions, then, $R_{XY}(\tau)| \leq \sqrt{[R_{XX}(0) + R_{YY}(0)]}$ holds. [8+8]

8. (a) A German language class has only three students A,B,C and they independently attend classes. The probabilities of attendance of A, B, C on any given day are respectively 1/3, 2/3, and 3/4 respectively. Find the probability that the total number of attendances in two consecutive days is exactly three.

(b) The chances of three candidates A, B, C to become the excise ministers of state are in the ratio 30%, 45% and 25%. The probabilities of introducing prohibition scheme by them if selected as excise minister are 0.6, 0.4 and 0.5 respectively. If the prohibition is indeed is introduced, what is the probability that B has become the excise minister? [8+8]

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