Code No: 07A62203 **R07** **Set No. 2**

III B.Tech II Semester Examinations,December 2010
DIGITAL AND OPTIMAL CONTROL SYSTEMS
Instrumentation And Control Engineering

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Derive the state space model of discrete control system using partial fraction expansion programming method and draw its block diagram.
   (b) Derive the general equation for phase variable form of state model and canonical form of state model. [8+8]

2. (a) Define shanon's sampling theorem. Analyse the sampling process in frequency domain and explain how the original signal is reconstructed from the sampled signal. Also explain the aliasing effect.
   (b) What are the advantages of sampled data control system? [12+4]

3. Find the curve with the minimum arc length joining the point (0, 0) and the time θ(t) = (1 - t). [16]

4. (a) Consider the system described by
   y(k) - 0.6 y(k-1) - 0.81 y(k-2) + 0.67 y(k-3) - 0.12 y(k-4) = x(k)
   where x(k) is the input and y(k) is the output of the system. Determine the stability of the system.
   (b) Consider the following characteristic equation
   $z^3 + 2.1 z^2 + 1.44 z + 0.32 = 0$
   Determine whether or not any of the roots of the characteristic equation lie outside the unit circle centered at the origin of the z - plane. [8+8]

5. Consider an n^{th} order single input system $x(k+1) = Ax(k) + bu(k)$ and feed back control of $u(k) = -KX(k) + r(k)$ where 'r' is the reference input signal. Show that the zeros of the system are invariant under the state feed back. [16]

6. (a) A state feed back control system has following system equations
   X(k+1) = GX(k) + HU(k)
   Y(k) = CX(k)
   U(k) = -KX(k)
   where K is state feed back gain matrix
   Draw the necessary block diagram for the control system and derive the observer error equation.
   (b) Briefly explain design of digital control systems that must follow changing reference inputs, applying observed-state feed back method. Draw necessary block diagram.

   [8+8]

7. (a) What are the major theoretical approaches for optimal control design
   (b) Explain the same in detail. [8+8]

8. Consider the completely observable system
   X(k+1) = G X(k)
   y(k)=C X(k)
   Define the observability matrix as N:
   $N=\left[C* \vdots G* C* \vdots \text{...........} \vdots (G*)^{n-1} C*\right]$

   Show that

$$N^*G\,(N^*)^{-1} = \begin{bmatrix} 0 & 1 & 0 & . & . & 0 \\ 0 & 0 & 1 & . & . & 0 \\ \vdots & \vdots & \vdots & & & \vdots \\ 0 & 0 & 0 & . & . & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & & & -a_1 \end{bmatrix}$$

   where, a_1, a_2, ... a_n are the coefficients of the characteristic polynomial $|zI - G| = z^n + a_1 z^{n-1} + \ldots\ldots\ldots\ldots + a_{n-1} z + a_n$ [16]

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Code No: 07A62203 **R07** **Set No. 4**

III B.Tech II Semester Examinations,December 2010
DIGITAL AND OPTIMAL CONTROL SYSTEMS
Instrumentation And Control Engineering

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
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1. For the control system defined by
$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u\ (k)$$
$$y(k) = [1 \quad 0] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$
Compare the controllability and observability. [16]

2. (a) Give an example of minimum-fuel problem. Explain its performance expression.
   (b) What is the objective of a state regulator problem. Write down the equation for a reasonable measure of the system transient response in terms of state variables and time. [8+8]

3. With suitable diagram, illustrate with the one point fixed end, terminal time t_1 free and derive the necessary conditions of variational calculus. [16]

4. (a) Consider the system described by the equation
   $\dot{x}_1 = x_2$
   $\dot{x}_2 = = -x_1 - x_2 + x_2^2$

   Investigate the stability of the equilibrium state. Use the direct method of Liapunov.
   (b) State and explain the stability and instability theorems. [8+8]

5. Design a dead beat-response controller for a plant having transfer function as $G_p(z) = (z + 0.5)/(z^2 - z - 1)$ [16]

6. (a) Given the Z-transforms
   $X\ (z) = \frac{z^{-1}}{(1-z^{-1})\ (1+1.3z^{-1}+0.4z^{-2})}$
   Determine the initial and final values of x (k). Also find x (k), in a closed form.
   (b) Solve the following difference equation
   (k + 1) x (k + 1) -x (k) = k + 1
   Where, x (k) = 0 for k < 0 and x (0) = 1. [8+8]

7. If a system is given by X(k+1) = GX(k) + HU(k); Y(k) = CX(k)
   Where $G = \begin{pmatrix} 0 & -0.2 \\ 1 & -1 \end{pmatrix}; H = \begin{pmatrix} 0 \\ 1 \end{pmatrix}; C = \begin{pmatrix} 0 & 1 \end{pmatrix}$

Design a Full-order state observer with desired eigen values of the observer matrix as z = 0.3±j0.3.
Assume e(k+1) = (G-KeC)e(k)
where e(k) = X(k)- $\widetilde{X}$(k) and Ke is feedback gain matrix [16]

8. Consider the following system given in the controllable canonical form.

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{u (k).}$$

$$\text{y (k)} = [\, b_3 - a_3 b_0 \,;\, b_2 - a_2 b_0 \,;\, b_1 - a_1 b_0 \,] \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + b_0 \text{ u (k).}$$

It is desired to transform the system equation into the observable canonical form by means of the transformation of the state vector.
X=Q$\overline{X}$

Determine a transformation matrix ?Q? that will give the desired observable canonical form. [16]

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1. With suitable diagram explain both terminal time (t_1) and x(t_1) free problem and derive the necessary conditions of variational calculus. [16]

2. Given the discrete time system
G(z)= $\frac{2+2.2z^{-1}+0.2z^{-2}}{1+0.4z^{-1}-0.12^{-2}}$ Realize the system in
   (a) The parallel scheme and
   (b) The ladder scheme using pure delay element z^{-1}. [8+8]

3. (a) Find the state controllability of the system given below.

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} u(k)$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

   (b) Examine the controllability & observability of the systems given below

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 11 \\ 1 \\ -14 \end{bmatrix} u(k)$$

$$Y(k) = \begin{bmatrix} -3 & 5 & -2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$ [8+8]

4. Explain the different methods to determine state feed back gain matrix for the system with state equation as X(k+1) = GX(k)+ HU(k). Assume that characteristic equation is selected and the system is completely state controllable. [16]

5. (a) A sampled data system with dead time is shown in figure 5a determine the condition for system stability, if $\delta < T$

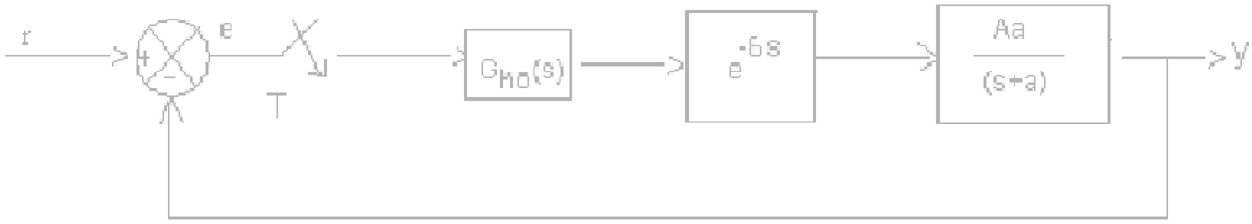

Figure 5a

(b) Consider the system described by the equations,
$x_1(k+1) = 2\,x_1(k) + 0.5\,x_2(k) - 5$
$x_2(k+1) = 0.8\,x_2(k) + 2.$
Investigate the stability of the equilibrium state. Using the direct method of Lyapunov. [8+8]

6. A discrete time system is described by the following difference equation
$x(k+2) + 5x(k+1) + 6x(k) = u(k)$
$x(0)=x(1)=0$, T=1 sec

   (a) Determine the state model in canonical form
   (b) Find the state transition matrix
   (c) For input $u(k)=1$, $k \geq 1$, find the output $x(k)$. [5+6+5]

7. Consider the system $X(k+1) = GX(k) + HU(k)$, $Y(k) = CX(k)$
where $G = \begin{pmatrix} 0 & -0.2 \\ 1 & -1 \end{pmatrix}; H = \begin{pmatrix} 0 \\ 1 \end{pmatrix}; C = \begin{pmatrix} 0 & 1 \end{pmatrix}$
Design a full order state observer. The desired eigen values of the observer matrix are $z = 0.5 \pm j0.5$
Assume $e(k+1) = (G-K_eC)e(k)$
where $e(k) = X(k) - \widetilde{X}(k)$ and K_e is feed back gain matrix. [16]

8. (a) Write an explicit statement of the optimal control problem and explain various terms contained in the equation.
   (b) What are the various mathematical procedures for the optimal control system design, Briefly explain them. [8+8]

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Code No: 07A62203 **R07** **Set No. 3**

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1. (a) State the advantages and disadvantages in controllability and observability.
   (b) What is the need for
      i. Controllability test
      ii. Observability test. [8+8]

2. Find the response of the system
$x(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)\ ;\ x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
if u (k) = 1 , for k = 0, 1 , 2. [16]

3. A discrete-time regulator system has the plant equation
$X(k+1) = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} X(k) + \begin{bmatrix} 4 \\ 3 \end{bmatrix} u(k)$ and
$y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} X(k) + 7u(k)$. Design a state feed back control system with $u(k) = -KX(k)$ to place the closed loop poles at $\pm j0.5$. [16]

4. (a) What are the aspects, to be considered while introducing optimal control theory in an industry?
   (b) What are the characteristics of a plant to be considered while planning optimal control? [8+8]

5. Show that the extremal for the functional $J(x) = \int_0^{\pi/2} (\dot{x}^2 - x^2)dt$, which satisfies the boundary conditions $x(0) = 0; x(\pi/2) = 1$ is $x^*(t) = \sin t$ [16]

6. Derive the necessary and sufficient condition for state observation for a system having following state and output equations.
$X(k+1) = GX(k) + Hu(k)$
$y(k) = CX(k)$
Where G is a 'n×n' non singular matrix. [16]

7. (a) Consider the discrete - time unity feedback control system (with sampling period T=1 sec) whose open loop pulse transfer function is given by
$G(z) = \frac{K(0.3679z+0.2642)}{(z-0.3679z)(z-1)}$
Determine the range of gain K for stability by using the Jury stability test.
   (b) What is bilinear transformation? Explain briefly the stability analysis using bilinear transformation and Routh stability? [10+6]

8. (a) Obtain C (z, m), the modified Z-transform of the output of the system shown in figure 8a : [16]

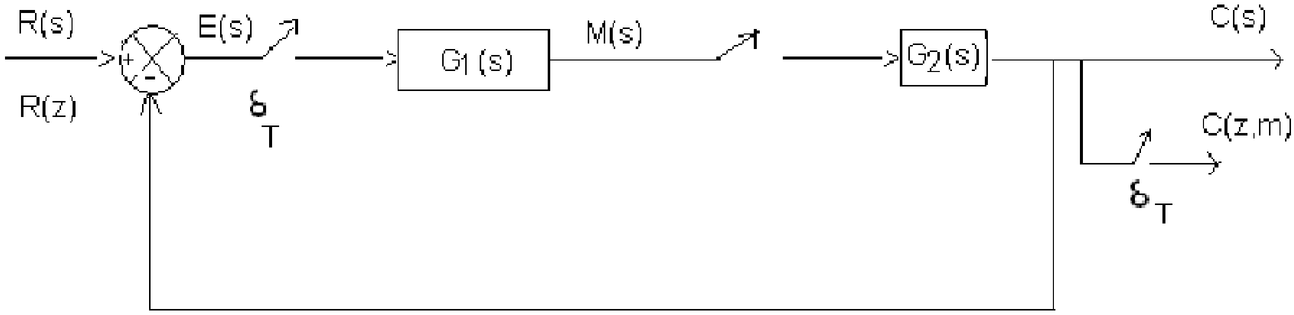

Figure 8a

(b) Find y(z) for the sampled data closed loop system. Shown in figure. 8b [8+8]

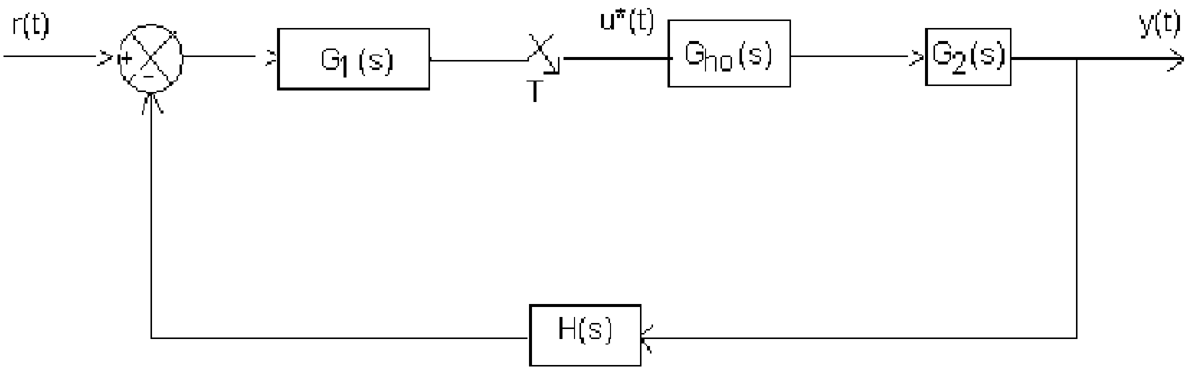

Figure 8b

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