# I B.Tech Examinations,December 2010 <br> MATHEMATICS-I 

Common to CE, ME, CHEM, BME, IT, MECT, MEP, AE, BT, AME, ICE, E.COMP.E, MMT, ETM, EIE, CSE, ECE, EEE

Time: 3 hours
Max Marks: 75

## Answer any FIVE Questions

All Questions carry equal marks

1. (a) Find $L\left[\frac{\cos 4 t \sin 2 t}{t}\right]$
(b) Find the Laplace inverse transform of $\log \left(\frac{s^{2}+4}{s^{2}+9}\right)$
2. (a) Test the convergence of the series $1+\frac{3 x}{7}+\frac{3.6}{7.10} x^{2}+\frac{3.6 .9}{7.10 .13} x^{3}+\frac{3.6 .9 .12}{1.10 .13 .16} x^{4}+\ldots .$.
(b) Find the interval of convergence for the series $\sum_{n=1}^{\infty}(-1)^{n} \frac{n(x+1)^{n}}{2^{n}}$
[7+8]
3. (a) Find the length of an arc of the curve $x=e^{\theta} \sin \theta, y=e^{\theta} \cos \theta$ from $\theta=$ 0 to $\frac{\pi}{2}$
(b) Evaluate $\iint_{R} y^{2} d x d y$ were R is the region bounded by the parabolas $y^{2}=4 x$ and $x^{2}=4 y$
4. (a) Find the differential equation of all circles whose radius is $r$
(b) Solve the differential equation $(x+1) \frac{d y}{d x}-y=e^{3 x}(x+1)^{2}$
(c) Find the equation of the curve, in which the length of the subnormal is proportional to the square of the ordinate.
$[4+6+5]$
5. (a) Solve the differential equation $\left(D^{4}+2 D^{2}+1\right) y=x^{2} \cos ^{2} x$
(b) Solve the differential equation $\left(D^{2}+2 D+1\right) y=e^{-x}$
6. (a) Expand $e^{x \sin x}$ in powers of x .
(b) Find the volume of the greatest rectangular parallelopiped that can be 'inscribed in the ellipsoid $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}+\frac{z^{2}}{c^{2}}=1$.
$[8+7]$
7. (a) Show that the evolute of the ellipse $x=a \cos \theta, y=b \sin \theta$ is $(a x)^{\frac{2}{3}}+(b y)^{\frac{2}{3}}=$ $\left(a^{2}-b^{2}\right)^{\frac{2}{3}}$
(b) Show that the envelope of the lines whose equations are $x \sec ^{2} \theta+y \operatorname{cosec}^{2} \theta=c$ is a parabola which touches the axes of coordinates. $\quad[8+7]$
8. (a) Find the work done by the force $\bar{F}=(2 y+3) i+x z j+(y z-x) k$ when it moves a particle from the point $(0,0,0)$ to $(2,1,1)$ along the curve $x=2 t^{2}$, $y=t$ and $z=t^{3}$
(b) Use divergence theorem to Evaluate $\iint_{S}\left(y^{2} z^{2} i+z^{2} x^{2} j+z^{2} y^{2} k\right) . \bar{n}$ ds where S is the part of the unit sphere above the x y plane.

# I B.Tech Examinations,December 2010 <br> MATHEMATICS-I 

Common to CE, ME, CHEM, BME, IT, MECT, MEP, AE, BT, AME, ICE, E.COMP.E, MMT, ETM, EIE, CSE, ECE, EEE

Time: 3 hours
Max Marks: 75

## Answer any FIVE Questions

All Questions carry equal marks

1. (a) Find $\mathrm{L}\left[\left(t^{2}+1\right)^{2}\right]$
(b) Find Inverse Laplace transform of $\frac{3 s+7}{\left(s^{2}-2 s-3\right)}$
2. (a) Find the radius of curvature at any point on $y^{2}=4 a x$ and hence show that the radius of curvature at the vertex is equal to the semi latys rectum.
(b) Trace the curve $r=a(1+\cos \theta)$
3. (a) Test the convergence of the series $\frac{3^{2}}{6^{2}}+\frac{3^{2} \cdot 5^{2}}{6^{2} .8^{2}}-\frac{3^{2} .55^{2} \cdot 7^{2}}{6^{2} \cdot 8^{2} \cdot 10^{2}}+\ldots .$.
(b) Test whether the following series is absolutely convergent or conditionally convergent $\frac{1}{5 \sqrt{2}}-\frac{1}{5 \sqrt{3}}+\frac{1}{5 \sqrt{4}} \ldots . .(-1)^{n} \frac{1}{5 \sqrt{n}}$
4. (a) Solve the differential equation $\left(D^{2}+2\right) y=e^{x} \cos x$
(b) Solve the differential equation $\left(D^{3}+2 D^{2}+D\right) y=x^{3}$
5. (a) If $u=x^{2}-2 y, v=x+y+z, w=x-2 y+3 z$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$
(b) Find the maximum and minimum values of $f(x)=x^{3}+3 x y^{2}-3 x^{2}-3 y^{2}+4$

$$
[8+7]
$$

6. (a) The curve $y^{2}(a+x)=x^{2}(3 a-x)$ revolved about the $x$-axis. Find the volume of the solid generated.
(b) Evaluate $\int_{0}^{a} \int_{0}^{\sqrt{a^{2}-y^{2}}}\left(x^{2}+y^{2}\right) d x d y$ by changing into polar coordinates $\quad[8+7]$
7. (a) Form the differential equation by eliminating arbitrary constants $\mathrm{y}=\mathrm{A} e^{x}+\mathrm{B} e^{-x}$
(b) Solve the differential equation $\left(e^{y}+1\right) \cos x d x+e^{y} \sin x d y=0$
(c) Find the curve in which the perpendicular upon the tangent from the foot of the ordinate of the point of contact is constant and equal to a.
$[4+6+5]$
8. (a) If $\bar{F}$ and G are two vectors, then prove that $\operatorname{div}(\bar{F} \times \bar{G})=\bar{F} \operatorname{curl} \bar{G}-\bar{G}$ .curl $\bar{F}$
(b) Evaluate $\oint_{c} x d y+y d x$ where c is the loop of the Folium of $\mathrm{D}^{\prime}$ cartes $x=\frac{3 a t}{1+t^{3}}$, $y=\frac{3 a t^{2}}{1+t^{3}}$

# I B．Tech Examinations，December 2010 <br> MATHEMATICS－I 

Common to CE，ME，CHEM，BME，IT，MECT，MEP，AE，BT，AME，ICE， E．COMP．E，MMT，ETM，EIE，CSE，ECE，EEE
Time： 3 hours
Max Marks： 75

## Answer any FIVE Questions

All Questions carry equal marks

1．（a）Find $L[3 \cos .3 t \cos 4 t]$
（b）Find the inverse Laplace transform of $\log \left(1+\frac{16}{s^{2}}\right)$
$[7+8]$
2．（a）Solve the differential equation $\left(D^{4}-2 D^{3}+2 D^{2}-2 D+1\right) y=\cos x$
（b）Solve the differential equation $\left(D^{3}-3 D-2\right) y=x^{2} \quad[7+8]$
3．（a）Form the differential equation by eliminating arbitrary constants $\operatorname{Sin}^{-1} x+\operatorname{Sin}^{-1} y=C$ ．
（b）Solve the differential equation $\frac{y}{x} \frac{d y}{d x}=\sqrt{1+x^{2}+y^{2} x^{2}+y^{2}}$ ．
（c）Prove that the system of Parabolas $y^{2}=4 a(x+a)$ is self orthogonal．$[3+6+6]$
4．（a）Apply Rolle＇s theorem for $\sin n \sqrt{\cos 2 n}$ in $\left[0, \frac{\pi}{4}\right]$ and find x such that $0<x<\frac{\pi}{4}$
（b）Expand $e^{x} \cdot \cos y$ near the point $\left[1, \frac{\pi}{4}\right]$ by Taylor＇s theorem．
$[7+8]$
5．（a）Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1.3 .5 \cdots(2 n+1)}{2.5 .8 \cdots(3 n+2)}$
（b）Test the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{n^{2}}{2^{n}}+\frac{1}{n^{2}}\right)$
6．（a）Find the radius of curvature at the point $\theta$ on $x=a \log (\sec \theta+\tan \theta)$ and $y=a \sec \theta$
（b）Trace the curve $x^{3}+y^{3}=3 a x y$
7．（a）Prove that the surface area of the solid generated when the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ is revolved about its major axis is $2 \pi a b\left[\sqrt{1-e^{2}}+\frac{\sin ^{-1} e}{e}\right]$ where e is the eccentricity of the ellipse．
（b）Evaluate $\iiint(x y+y z+z x) d x d y d z$ ，where V is the region of space founded by $x=0, x=1, y=0, y=2$ and $z=0, z=3$

8．Verify stoke＇s theorem for $F=(2 x-y) i-y z^{2} j-y^{2} z k$ over upper half surface of $x^{2}+y^{2}+z^{2}=1$ bounded by its projection on the xy plane．

# I B．Tech Examinations，December 2010 <br> MATHEMATICS－I 

Common to CE，ME，CHEM，BME，IT，MECT，MEP，AE，BT，AME，ICE， E．COMP．E，MMT，ETM，EIE，CSE，ECE，EEE
Time： 3 hours
Max Marks： 75

## Answer any FIVE Questions

All Questions carry equal marks

1．（a）Find $L\left[t e^{2 t} \sin 3 t\right]$
（b）Find $L^{-1}\left[\frac{1}{s^{3}\left(s^{3}+1\right)}\right]$
2．（a）By considering the function $(x-2) \log x$ show that the equation $x \log x=$ $2-x$ is satisfied by at least one value of x lying between 1 and 2 ．
（b）Find the minimum of $x^{2}+y^{2}+z^{2}$ subject $x+y z=3 a \quad[7+8]$
3．（a）Find the volume of the solid obtained byrevolving one arch of the cycloid $x=a$ $(\theta+\sin \theta) y=,\mathrm{a}(1+\cos \theta)$ about its base.
（b）Calculate $\int_{R} \int r^{3} d r d \theta$ over the area included between the circles $r=2 \sin \theta$ and $r=4 \sin \theta$

4．（a）If $\bar{F}$ and $G$ are two vectors，then $\operatorname{div}(\bar{F} \times \bar{G})=\bar{F} \operatorname{curl} \bar{G}-\bar{G} . \operatorname{curl} \bar{F}$
（b）Evaluate by Greens theorem $\int_{C}\left(x^{2}-\operatorname{Cosh} y\right) d x+(y+\sin x) d y$ where C is the rectangle with vertices $(0,0),(\pi, 0),(\pi, 1),(0,1)$

5．（a）Test the convergence of the series $\sum_{n=1}^{\alpha} \frac{2 n!}{n!(n)}$
（b）Test the convergence of the series $=\frac{2.5 .8 . \ldots .3 n-1}{1.5 .9 \ldots .4 n-3}$
（c）Find the interval of convergence for the following series $\sum \frac{\left(n^{2}-1\right)}{n^{2}+1} x^{n}$ ．$[5+5+5]$
6．（a）If CP and CD are a pair of conjugate diameters of an ellipse prove that the radius of curvature at P is $\frac{(C D)^{3}}{a b}$ a and b being the lengths of the semiarcs of the ellipse．
（b）Trace the curve $y^{2}=x^{2} \frac{(3 a-x)}{(a+x)}$
7．（a）Find the differential equation of all circles which pass through the origin and whose centers are on x －axis．
（b）Solve the differential equation $\frac{d y}{d x}+x \sin 2 y=x^{3} \cos ^{2} y$
（c）The rate at which bacteria multiply is proportional to the instantaneous N numbers present．If the original number doubles in 2 hrs？When it will be trebled？

$$
[4+6+5]
$$

8．（a）Solve the differential equation $\left(D^{2}-1\right) y=x \sin x+\left(1+x^{2}\right) e^{x}$
(b) Solve the differential equation $\left(D^{2}+4\right) y=\operatorname{Tan} 2 x$


