Code No: A109210101 **R09** **Set No. 2**

II B.TECH – I SEM EXAMINATIONS, NOVEMBER - 2010

MATHEMATICS-II

Common to CE, CHEM, AE, BT, MMT

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions

All Questions carry equal marks

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1. A long rectangular plate of width 9 cms with insulated surface has its temperature equal to zero on both the long edges and one of the shorter sides so that u(0, y) = u(a, y) = u(x, ∞) = 0 and u(x, 0) = Kx . Find the steady state temperature with in the plate. [15]

2. (a) Solve completely the system of equations:
   x+2y+3z=0; 3x+4y+4z =0; 7x+10y+12z =0.
   (b) Solve the system of equations:
   x+3y-2z =0; 2x-y+4z =0; x-11y+14z =0. [15]

3. (a) Find the Hermitian form H for A = $\begin{bmatrix} 0 & i & 0 \\ -i & 1 & -2i \\ 0 & 2i & 2 \end{bmatrix}$ with $X = \begin{bmatrix} i \\ 1 \\ -i \end{bmatrix}$.
   (b) Determine the skew-Hermitian form S for A = $\begin{bmatrix} 2i & 3i \\ 3i & 0 \end{bmatrix}$ with X= $\begin{bmatrix} 4i \\ -5 \end{bmatrix}$. [15]

4. (a) Using Fourier integral show that $\int_0^\infty \frac{w \sin xw}{1+w^2}\, dw = \frac{\pi}{2} e^{-x} (x > 0)$
   (b) Find the Fourier Transform of f(x) = x e^{-x}, $0 \leq x < \infty$ [15]

5. (a) Obtain the Fourier series for the function $f(x) = e^{-x}\ in\ -1 < x < 1$
   (b) Find the half range cosine series for $f(x) = x^2\ in\ [0, \pi]$ and find the sum of the series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ...$ [15]

6. (a) Form the partial differential equation from
   $z = a(x + \log y) - \frac{x^2}{2} - b$
   (b) Solve the partial differential equation
   $x^4 p^2 - yzp = z^2$ [15]

7. Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_3x_2 + x_3x_1$ to the canonical form. Find rank, index and signature of the quadratic form. [15]

8. Verify Cayley-Hamilton theorem for the matrix A = $\begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$. [15]

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Code No: A109210101 **R09** **Set No. 4**

II B.TECH – I SEM EXAMINATIONS, NOVEMBER - 2010

MATHEMATICS-II

Common to CE, CHEM, AE, BT, MMT

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions

All Questions carry equal marks

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1. (a) Solve the equations completely:
   $x_1 + 2x_2 - x_3 = 0; 3x_1 + x_2 - x_3 = 0; 2x_1 - x_2 = 0.$

   (b) Solve the equations completely
   4x+2y+z+3w =0; 6x+3y+4z+7w=0; 2x+y+w=0. [15]

2. (a) Obtain the Fourier series for the function = = x -1 in (0, 1)
   = 1 - x in (1, 2)

   (b) Find the half range Sine series for f(x) = x^3 in $[0, 2\pi]$ [15]

3. (a) Find the rank, index, and signature of the sylvester's canonical form $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$.

   (b) Determine the nature, index and signature of the Quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_3x_1$. [15]

4. (a) Prove that the determinant of a unitary matrix is of unit modulus.

   (b) Show that the matrix $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ is skew-Hermitian and hence find eigen values and eigen vectors. [15]

5. Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ using cayley-Hamilton theorem. [15]

6. Solve the partial differential equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ with the conditions

   (a) $u \to 0, ast \to \infty$

   (b) $\frac{\partial u}{\partial x} = 0$ when x = 0 and L, t > 0

   (c) u = x(L - x), when t = 0 and x = 0 and L [15]

7. Find the Fourier sine Transform of $\frac{x}{a^2+x^3}$ and Find the Fourier cosine Transform of $\frac{1}{a^2+x^2}$ [15]

8. (a) Form the partial differential equation from
   $z = \frac{1}{z}\left[\sqrt{x+a} + \sqrt{y-a}\right] + b$

   (b) Solve the partial differential equation
   $(x + y)(p + q)^2 + (x - y)(p - q)^2 = 1$ [15]

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Code No: A109210101 **R09** **Set No. 1**

II B.TECH – I SEM EXAMINATIONS, NOVEMBER - 2010

MATHEMATICS-II
Common to CE, CHEM, AE, BT, MMT

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Obtain the Fourier series for the function f(x) = $|\cos x|$ in (-π, π)
   (b) Find the half range Sine series for f(x) = x in [0, $\frac{\pi}{2}$]
   = π - x in [$\frac{\pi}{2}$, π] [15]

2. (a) Reduce the quadratic form $2(x_1^2 + x_1x_2 + x_2^2)$ to canonical form.
   (b) Determine the nature index and signature of the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$. [15]

3. (a) Find the Fourier cosine transform of $\frac{e^{-ax}}{x}$
   (b) Find the finite Fourier Cosine transforms of f(x) = 1 if $0 < x < \frac{\pi}{2}$
   = -1 if $\frac{\pi}{2} < x < \pi$ [15]

4. (a) Determine the value of b such that the rank of A = $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$ is 3.
   (b) Find the rank of the matrix A = $\begin{bmatrix} 3 & -2 & 0 & -1 & -7 \\ 0 & 2 & 2 & 1 & -5 \\ 1 & -2 & -3 & -2 & 1 \\ 0 & 1 & 2 & 1 & -6 \end{bmatrix}$. [15]

5. (a) Find the eigen values and eigen vectors of matrix A = $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$
   (b) If $\lambda_1, \lambda_2, ,$ ... λ_n are the eigen values of A, then prove that the eigen values of (A-kI) are $\lambda_1 - k_1, \lambda_2 - k, ,$ ..... $\lambda_n - k$. [15]

6. (a) Form the partial differential equation from
   Z = xy + f(x^2 + y^2)
   (b) Solve the partial differential equation
   $x^2p^2 + y^2q^2 = z^2$ [15]

7. Solve the boundary value problem $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, with u(0, y) = u($\pi$, y) = u(x, π) = 0 and u (x, 0) = Sin^{2}x, $0 < x < \pi$ [15]

8. (a) Prove that the matrix $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary and determine the eigen values and eigen vectors.

(b) Prove that the characteristic roots of a Hermitian matrix are real. [15]

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Code No: A109210101 **R09** **Set No. 3**

II B.TECH – I SEM EXAMINATIONS, NOVEMBER - 2010

MATHEMATICS-II
Common to CE, CHEM, AE, BT, MMT

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Show that the matrix A $= \begin{bmatrix} a+ic & -b+id \\ b+id & a-ic \end{bmatrix}$ is unitary if and only if $a^2 + b^2 + c^2 + d^2 = 1$.
   (b) i. If A and B are Hermitian matrices, prove that AB-BA is skew Hermitian.
   ii. Find the Hermitian form of A $= \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ with X$=\begin{bmatrix} 1 \\ i \end{bmatrix}$. [15]

2. (a) Find the nature of the quadratic form $6x^2 + 35y^2 + 11z^2 - 4zx$.
   (b) By Lagrange's reduction transform X^{1}AX to sum of squares form for A $= \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & -2 \\ 4 & -2 & 18 \end{bmatrix}$.
   [15]

3. (a) Find the Fourier sine transform of $e^{-|x|}$ and have evaluate $\int\limits_0^\infty \frac{x \sin mx}{1+x^2} dx$
   (b) Find the finite Fourier cosine transform of $f(x) = \begin{cases} x \; if \; 0 < x < \frac{\pi}{2} \\ \pi - x \; if \; \frac{\pi}{2} < x < \pi \end{cases}$ [15]

4. A square plate is bounded by the lines x=0, y=0 , x=20 and y=20. Its faces are insualted. The temperature along the upper horizontal edge is given by u(x, 20) = x (20-x) when $0 < x < 20$. While the other three edges are kept at 0^0 C. Find the steady state temperature in the plate. [15]

5. Diagonalize the matrix A $= \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and hence find A^4. [15]

6. (a) Solve the partial differential equation q(p - Sinx) = Cos y
   (b) Solve the partial differential equation
   $x^2p^2 + xpq = z^2$ [15]

7. (a) Obtain the Fourier series for the function $f(x) = x\,Sin\,x \; in \; [0, 2\pi]$
   (b) Find the half range cosine series for $\begin{aligned} f(x) &= 1 \; in \; [0,1] \\ &= x \; in \; [1,2] \end{aligned}$. [15]

8. (a) Show that the system of equations $2x_1 - 2x_2 + x_3 = \lambda x_1; 2x_1 - 3x_2 + 2x_3 = -\lambda x_2; -x_1 + 2x_2 = \lambda x_3$ can possess a non-trivial solutions only if λ =1, λ = -3. Obtain the general solution in each case.

(b) Solve completely the system of equations:
2x-2y+5z+3w=0; 4x-y+z+w = 0;
3x-2y+3z+4w =0; x-3y+7z+6w = 0. [15]

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