Code No: A109210201 **R09** **Set No. 2**

II B.Tech I Semester Examinations,November 2010
MATHEMATICS-III
Common to ICE, E.COMP.E, ETM, EIE, ECE, EEE

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Evaluate $\int\limits_c (x+y)dx + x^2y$ dy along y=3x between (0,0) and (3,a)
   (b) Evaluate $\int\limits_c e^z dz$ where C: $|z| = 1$ [15]
2. (a) Find the Residues of f (z) = $\frac{z^2}{z(z+2)^3}$ at z = -2.
   (b) Find the Residues of f (z) = $\frac{z^3}{(z-1)^4(z-2)(z-3)}$ at $|z| = 1$. [8+7]
3. (a) Define conformal Transformation. Show that a bilinear transformation is conformal.
   (b) Show that circles are invariant under linear transformation $w = az + c$. [8+7]
4. (a) P.T. $xJ_n^1(x)$ = nJ$_n$(x) - x J$_{n+1}$(x).
   (b) Evaluate $\int_0^\alpha \frac{x}{1+x^6}dx$ Using $\beta - \Gamma$ functions. [15]
5. (a) S.T. the real & imaginary parts of the function w = log z satisfy the C-R equations when z is not zero.
   (b) S.T. f(z) = z+2$\bar{z}$ is not analytic anywhere in the complex plane. [15]
6. (a) Prove that, for all x, $x^7 = \frac{16}{429}P_7(x) + \frac{8}{39}P_5(x) + \frac{14}{33}P_3(x) + \frac{1}{3}P_1(x)$
   (b) Show that $\int_{-1}^1 x^k P_n(x)dx = 0$ for k=0,1,2,n-1 [15]
7. (a) Represent the function f (z) = $\frac{1}{z(z+2)^3(z+1)^2}$ in Laurent series with in $\frac{5}{4} \le |z| \le \frac{7}{4}$
   (b) Define for a complex function (z)
      i. Isolated Singularity
      ii. Removable Singularity
      iii. Essential singularity [15]
8. (a) How many vertices do the following graphs have if they contain (explain your answer)
      i. 16 edges and all vertices of degree 2
      ii. 21 edges, 3 vertices of degree 4 and others each of degree 3
   (b) Determine whether the following graphs (Figures 1) are isomorphic or not. [8+7]

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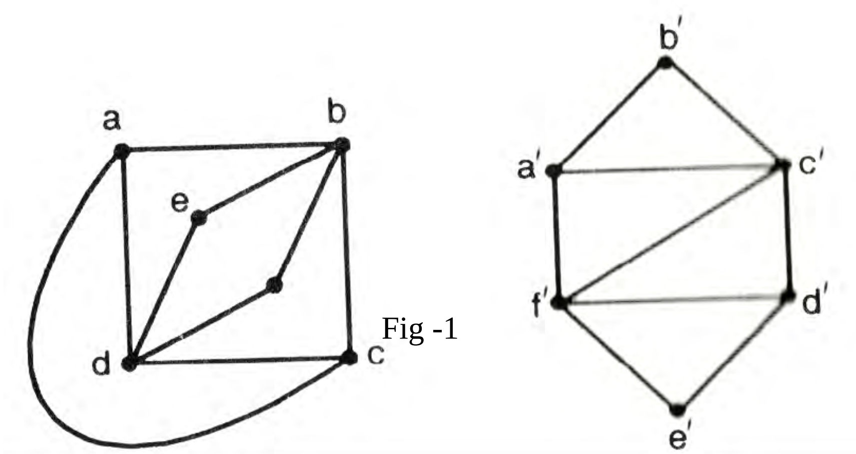

Fig -1

Code No: A109210201 **R09** **Set No. 4**

II B.Tech I Semester Examinations,November 2010
MATHEMATICS-III
Common to ICE, E.COMP.E, ETM, EIE, ECE, EEE

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
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1. (a) Evaluate $\int\limits_{-1}^{1} x^4(1-x^2)^{-\frac{1}{2}}T_2(x)dx.$

   (b) Prove that $\int\limits_{-1}^{1} x^2P_{n-1}(x)P_{n+1}(x)dx = 0$ [15]

2. (a) Evaluate $\int \frac{\sin^2 z}{(z-\pi/6)^3}dz$ where C: $|z| = 1$

   (b) Evaluate $\int\limits_{c} \frac{\log z}{(z-1)^3}dz$ where C: $|z-1| = 1/2$ [15]

3. (a) S.T. $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\{1/x\sin x - \cos x\}$

   (b) S.T. $\int_0^1 x^3(1-\sqrt{x})^5dx = 2\beta(8,6)$ [15]

4. Expand $\frac{7z-2}{(z+1)z(z-2)}$ about the point z = -1 in the region $1 < |z+1| < 3$ as Laurent's series. [15]

5. (a) Using the method of contour integration prove that $\int\limits_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2}\,dx$

   (b) Determine the poles and the residues at each pole of the function f (z) = Cot z. [8+7]

6. (a) Determine 'P' such that the function f(z) = $\frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\left(\frac{px}{y}\right)$ be an analytic function.

   (b) Find the analytic function whose real part is $\frac{y}{x^2+y^2}$. [15]

7. (a) Find the image of $|z| = 2$ under the transformation w = 3z.

   (b) Show that the transformation $w = z^2$ maps the circle $|z - 1| = 1$ in to the cardiod $r = 2(1 + \cos\theta)$ [8+7]

8. (a) Draw a corresponding binary tree for the following algebraic expression.

   (b) Show that a tree is a bipartite graph. [8+7]

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Code No: A109210201 **R09** **Set No. 1**

II B.Tech I Semester Examinations,November 2010
MATHEMATICS-III
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Time: 3 hours Max Marks: 75

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Find the residue of $\frac{1}{(z-Sinz)}$ at z = 0.
   (b) Evaluate $\int \frac{(z-3)dz}{(z^2+2z+5)}$ where c is $|z+1-i| = 2$. [7+8]
2. (a) P.T. J_n(x)=0 has no repeated roots except at x=0
   (b) P.T. $\frac{d}{dx}\{xJ_1(x)\} = xJ_0(x)$ [15]
3. (a) Evaluate $\int\limits_{1-i}^{z+i} (2x + iy + 1)dz$ along the straight line joining (1,-i) & (2,i)
   (b) If C is the boundary of the square with vertices at the points z = 0, z = 1, z = 1+i and z = i show that $\int\limits_{c} (3z + 1)dz = 0$ [15]
4. (a) P.T. $\int_{-1}^{1} (P_n^1)^2 dx = n(n + 1)$
   (b) P.T. $P_n^1 - P_{n-2}^1 = (2n - 1)P_{n-1}$ [15]
5. (a) Find the map of the circle $|z| = c$ under the transformation $w = z - 2 + 4i$
   b)Show that both the transformations $w = \frac{z-i}{z+i}$ andw=$\frac{i-z}{i+z}$ transform$|w| \leq 1$ into upper half plane I(z)>0. [7+8]
6. (a) S.T. $\tan^{-1} z = \frac{1}{2}\log\frac{i+z}{i-z}$
   (b) Find the roots of sinz = cosh4 [15]
7. (a) Write the Kruskal's algorithm for minimal spanning tree.
   (b) Consider the following graph.Determine which of the following sequenc s are paths; simple paths; cycle and simple cycle.
      i. $v_1\, e_1\, v_2\, e_6\, v_4\, e_3\, v_3\, e_2\, v_2$
      ii. $v_5\, e_5\, v_1\, e_8\, v_4\, e_3\, v_3\, e_2\, v_2\, e_6\, v_4\, e_4\, v_5$
      iii. $v_1\, e_8\, v_4\, e_3\, v_3\, e_7\, v_1\, e_8\, v_4$
      iv. $v_5\, e_5\, v_1\, e_8\, v_4\, e_3\, v_3\, e_2\, v_2\, e_6\, v_4\, e_4\, v_5$ [7+8]
8. Expand f(z)= $\frac{z+3}{z(z^2-z-2)}$ in powers of z.
   (a) With in the unit circle about the origin
   (b) With in the annular region between the concentric circles about the origin having radii 1 and 2 respectively.
   (c) The exterior to the circle of radius 2. [15]

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Code No: A109210201 **R09** **Set No. 3**

II B.Tech I Semester Examinations,November 2010
MATHEMATICS-III
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Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Evaluate $\int_0^a x^4\sqrt{a^2-x^2}dx$
   (b) P.T. $\int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}}\times\int_0^{\pi/2}\sqrt{\sin\theta}d\theta=\pi$ [15]

2. For the function f(z) = $\frac{2z^3+1}{z^2+z}$ find
   (a) A tailors expansion valid in the neighborhood of the point 'i'.
   (b) A Laurent's series valid within the annulus of which centre is origin. [15]

3. (a) Find the image of the triangle with vertices at i,1+i,1-i in the z-plane, under the transformation $e^{\frac{5\Pi i}{3}}.(z-2+4i)$
   (b) Find the image of the infinite strip, $0<y<\frac{1}{2}$ under the mapping function w $=\frac{1}{z}$. [7+8]

4. (a) Find the incidence matrix to represent the following graph (Figure 2).

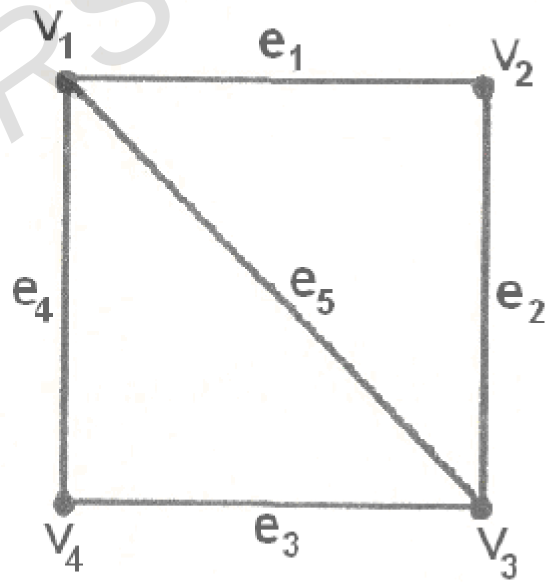

Figure 2:

   (b) Draw the graph of the expression $((x+y)\uparrow 2)+((x-4)/3)$ and write its postfix notation by Traversing the tree in post order. [8+7]

5. (a) Evaluate $\int\limits_0^\infty \frac{1}{1+x^6}\,dx$

(b) Evaluate by Residue theorem $\int\limits_{c} \frac{4z^2-4z+1}{(z-2)(z^2+4)}\ dz$ where C: $|z| = 1$. [8+7]

6. (a) Evaluate $\int\limits_{c} \frac{z^3+z^2+2z-1}{(z-1)^3}$dz where C: $|z| = 3$

(b) Evaluate $\int\limits_{c} \frac{z^4}{(z+1)(z-i)^2}dz$ where 'C' is the ellipse $9x^2+4y^2=36$ [15]

7. (a) Prove that $\text{Tan}^{-1}z = \left[\log \frac{(i+z)}{(i-z)}\right]$.

(b) Find the analytic function whose real part is $r^{-4} \cos 4\theta$. [15]

8. If f(x) = $\begin{cases} 0 & ; \quad -1 < x \leq 0 \\ x & ; \quad 0 < x < 1 \end{cases}$ then
S.T. f(x) = $\frac{1}{4}P_0(x) + \frac{1}{2}P_1(x) + \frac{5}{16}P_2(x) - \frac{3}{32}P_4(x) + --$ [15]

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