II B.Tech I Semester Examinations,November 2010
MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE
Common to Information Technology, Computer Science And Engineering
Time: 3 hours
Max Marks: 75

## Answer any FIVE Questions

All Questions carry equal marks

1. (a) Prove or disprove the validity of the following arguments using the rules of inference.
All men are fallible
All kings are men
Therefore, all kings are fallible
(b) Show that $(\exists \mathrm{x})(\mathrm{p}(\mathrm{x}) \Lambda \mathrm{Q}(\mathrm{x})) \Rightarrow(\exists \mathrm{x})(\mathrm{p}(\mathrm{x}) \Lambda \exists(\mathrm{x}) \mathrm{Q}(\mathrm{x}))$.
2. (a) Draw a planar representation of the following graph. Figure 1.
(b) What do you mean by a spanning tree? ExplainDFS method for finding a spanning tree for the graph.

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Figure 1:
3. (a) Show that the following statements are logically equivalent without using truth table.
$\neg(\mathrm{PV}(\neg \mathrm{P} \Lambda \mathrm{Q})) \Leftrightarrow \neg \mathrm{P} \Lambda \neg \mathrm{Q}$
(b) Show that the following statements is a tautology.
$(\neg \mathrm{P} \Lambda(\mathrm{P} \rightarrow \mathrm{Q})) \rightarrow \neg \mathrm{Q}$
4. Find the general solution for the recurrence relation $a_{n}-a_{n-1}=4\left(n+n^{3}\right)$, where $\mathrm{n} \geq 1$, and $a_{0}=5$
5. (a) Using the binomial theorem to prove that

$$
3^{n}=\sum_{r=0}^{n} c(n, r) 2^{r} .
$$

(b) If $\mathrm{x}>2, \mathrm{y}>0, \mathrm{z}>0$ then find the number of solutions of $\mathrm{x}+\mathrm{y}+\mathrm{z}+\mathrm{w}=21$.
6. (a) Write an algorithm to determine if a connected graph is Eulerian,using its adjacency list representation.
(b) Write an algorithm to determine if a connected graph contains an Eulerian path, using its adjacency matrix.
7. Prove the theorem: Every equivalence relation R on a set generates a unique partition of the set. The blocks of this partition correspond to the R equivalence classes.
8. If $\left(\mathrm{G},{ }^{*}\right)$ and $(\mathrm{H}, \Delta)$ are two groups and $\mathrm{f}: \mathrm{G} \rightarrow \mathrm{H}$ is homorphism, then prove that the kernel of ' $f$ ' is a normal subgroup.
[15]

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1. (a) State the binomial theorem.
(b) Show that the number of r-permutations of a set of n (distinct) etements is given by $\mathrm{P}(\mathrm{n}, \mathrm{r})=\mathrm{n}!/(\mathrm{n}-\mathrm{r})$ !
2. (a) Prove that $\mathrm{H}=\{0,2,4$,$\left.\} forms a sub group of <\mathrm{Z}_{6},+_{6}\right\rangle$.
(b) Consider the group $\mathrm{G}=\{1,2,4,7,8,11,13,14\}$ under multiplication modulo 15. Construct the multiplication table of G and verify whether G is cycle or not.

$$
[7+8]
$$

3. (a) Construct the truth table for the following statement
$(\neg \mathrm{P} \leftrightarrow \neg \mathrm{Q}) \leftrightarrow(\mathrm{Q} \leftrightarrow \mathrm{R})$
(b) Show that the following statements are logically equivalent without using truth table.
$(\mathrm{P} \rightarrow \mathrm{Q}) \Lambda(\mathrm{P} \rightarrow \mathrm{R}) \Leftrightarrow \mathrm{P} \rightarrow(\mathrm{Q} \Lambda \mathrm{R})$
4. (a) Prove that if $G$ is a plane graph, then the sum of the degrees of the regions determined by $G$ is $2|E|$, where $|E|$ is the number of edges of $G$.
(b) Determine if bipartite graph $K_{2,2}$ is planar or not.
5. (a) Give an example to show that $(\mathrm{x})(\mathrm{A}(\mathrm{x}) \Lambda \mathrm{B}(\mathrm{x}))$ need not be a conclusion form $(\exists \mathrm{x}) \mathrm{A}(\mathrm{x})$ and $(\exists \mathrm{x}) \mathrm{B}(\mathrm{x})$.
(b) FShow that ( $\exists \mathrm{x}) \mathrm{M}(\mathrm{x})$ follows logically from the premises (x) $H(x) \rightarrow M(x)$ and ( $\exists x) H(x)$.
6. (a) A function $f(Z \times Z) \rightarrow Z$ is defined by $f(x, y)=4 x=5 y$. Prove that $f$ is not one-to-one, but onto
(b) If $\mathrm{A}, \mathrm{B}, \mathrm{C}$ are three sets such that $\mathrm{A} \subseteq \mathrm{B}$. Show that $(A \times C) \subseteq(B \times C)$
(c) If $\mathrm{A}=\{1,2,3\}, \mathrm{B}=\{4,5\}$. Find
i. $\mathrm{A} \times \mathrm{B}$
ii. $\mathrm{B} \times \mathrm{A}$
7. (a) State and prove Five Colour theorem.
(b) Find a subgraph of G which is isomorphic to $\mathrm{K}_{33}$.
8. (a) Solve the recurrence relation $u_{n+2}+4 u_{n+1}+3 u_{n}=5(-2)^{n}, u_{0}=1, u_{1}=0$ using generating function.
(b) Solve the recurrence relation $\mathrm{u}_{n+2}-\mathrm{u}_{n+1}-12 \mathrm{u}_{n}=10, \mathrm{u}_{1}=\frac{1}{3}, \mathrm{u}_{0}=0$.


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1. (a) Determine the truth value of each of the following statements
i. $6+2=7$ and $4+4=8$.
ii. four is even.
iii. $4+3=7$ and $6+2=8$.
(b) Write each of the following statements in symbolic form
i. Anil \& Sunil are rich.
ii. Neither Ramu nor Raju is poor.
iii. It is not true that Ravi \& Raju are both rich.
(c) Write a short note on normal forms
2. (a) Give an example of a connected graph $\mathbb{C}$ where removing any edge of G results in a disconnected graph.
(b) Give an example for a bipartite graph with examples.
(c) Discuss graph coloring problem with required examples.
3. (a) Find the number of positive integers less than are equal to 2076 and divisible by 3 or 4 .
(b) Find the coefficient of $x^{4} y^{7}$ in the expansion of $(x-y)^{11}$.
4. (a) Find the generating function of $n^{2}-2$.
(b) Solve $a_{n}=a_{n-1}+n$, where $a_{0}=2$ by substitution
5. (a) Let $f(x): x^{2}-3 x+2$. Find
i. $f\left(x^{2}\right)$
ii. $f(x+3)$
(b) Prove that $A-(B \cap C)=(A-B) \cup(A-C)$
(c) Define equivalence relation
6. (a) Show that if a plane graph is self-dual, then $|E|=2|V|-2$
(b) Give the adjacency matrix of the digraph $\mathrm{G}=(\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\}, \mathrm{R})$, where $\mathrm{R}=$ $\{(\mathrm{a}, \mathrm{b}),(\mathrm{b}, \mathrm{c}),(\mathrm{d}, \mathrm{c}),(\mathrm{d}, \mathrm{a})\}$.
7. In a symmetric group S3 find those elements of a and b, such that
(a) $\mathrm{a}^{2}=e$

> (b) $\mathrm{a}^{3}=\mathrm{e}$
> (c) $(\mathrm{a}+\mathrm{b})^{2} \neq \mathrm{a}^{2} * \mathrm{~b}^{2}$.
8. (a) Is the following conclusion validly derivable from the premises given? Verify. If $(\mathrm{x})(\mathrm{P}(\mathrm{x}) \rightarrow \mathrm{Q}(\mathrm{x}))$, (exists y$) \mathrm{P}(\mathrm{y})$ then (exists z$) \mathrm{Q}(\mathrm{z})$.
(b) Prove that $(x)(H(x) \rightarrow A(x)) \Rightarrow(x)((\exists y)(H(y) \Lambda N(x, y)) \rightarrow(\exists y)(A(y) \Lambda$ $\mathrm{N}(\mathrm{x}, \mathrm{y}))$.

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1. (a) Let $\mathrm{f}: \mathrm{Z} \rightarrow \mathrm{N}$ be defined by
$F(x)=\{2 x-1$ if $x>0$
$\{-2 \mathrm{x}$ if $\mathrm{x} \leq 0$
(b) Let $\mathrm{A}=\{0,1,2,3,4\}$. Show that the relation $R=\{(0,0),(0,4),(1,1),(1,3)$, $(2,2),(3,1),(3,3),(4,0),(4,4)$,$\} is an equivalence relation. Find the distinct$ equivalence classes of $R$.
[15]
2. (a) Using Krushall's algorithm, find a minimal spanning tree for the graph given in the following table.

| Weight | 7 | 10 | 10 | 11 | 12 | 12 | 13 | 13 | 13 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Edges | $(\mathrm{a}, \mathrm{b})$ | $(\mathrm{a}, \mathrm{d})$ | $(\mathrm{b}, \mathrm{d})$ | $(\mathrm{b}, \mathrm{e})$ | $(\mathrm{a}, \mathrm{e})$ | $(\mathrm{c}, \mathrm{e})$ | $(\mathrm{b}, \mathrm{c})$ | $(\mathrm{c}, \mathrm{d})$ | $(\mathrm{d}, \mathrm{e})$ | $(\mathrm{a}, \mathrm{c})$ |

(b) Prove that a connected plane graph with 7 vertices and degree $(\mathrm{V})=4$ for each vertex V of G must have 8 regions of degree 3 and one region of degree 4. [15]
3. Obtain the PCNF of the following formula $(\neg \mathrm{P} \rightarrow \mathrm{R}) \Lambda(\mathrm{Q} \leftrightarrow \mathrm{P})$
(a) Using Truth Table.
(b) Without using Truth Table.
4. (a) Show that the set $G=\{1,2,3,4,5\}$ is not a group under addition and multiplication modulo 6 (i.e. $\mathrm{X}_{6}$ and $+{ }_{6}$ ).
(b) If G is the set of all matrices of the type $\left(\begin{array}{cc}a & 0 \\ 0 & a^{-1}\end{array}\right)$ where $\mathrm{a} \neq \mathrm{e}$. Prove that G is an abelian group under matrix multiplication.
5. (a) Applying the multiplication principle show that a set S with n elements has $2^{n}$ subsets.
(b) One type of automobile license plate number in Masachusetts consists of one letter and five digits. Compute the number of such license plate numbers possible.
6. (a) Prove or disprove the conclusion given below from the following axioms. If Socrates is a man, then Socrates is mortal. Socrates is a man. Therefore, Socrates is mortal.
(b) Using proof by contradiction show that $\sqrt{2}$ is not a rational number.
7. (a) Find the generating function of $(n-1)^{2}$.
(b) Solve the difference equation $\mathrm{u}_{n}-2 \mathrm{u}_{n-1}=5 .(2)^{n}$ using generating function. [15]
8. (a) Find whether the following (figure 2) is Hamiltonian or Eulerian. If so find the cycle otherwise write the reason.


Figure 2:
(b) Find the chromatic number of the following graph (figure 3).


Figure 3:

