Code No: R05221801 **R05** **Set No. 2**

II B.Tech II Semester Examinations,November 2010
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Evaluate $\int_0^{2\pi} \frac{Cos2\theta}{5+4Cos\theta} d\theta$ using residue theorem.
   (b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ using residue theorem. [8+8]
2. (a) Find the image of 1< |z| <2 under the transformation w= 2iz+1.
   (b) Find the bilinear transformation which maps the points (–1, i, 1+i) onto the points (0, 2i, 1-i). [8+8]
3. (a) State and derive Laurent's series for an analytic function f (z).
   (b) Expand $\frac{1}{(z^2-3z+2)}$ in the region.
      i. $0 < |z - 1| < 1$
      ii. $1 < |z| < 2$. [8+8]
4. (a) Show that $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$.
   (b) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.
   (c) Show that $P_n(1) = 1 \text{ and } P_n(-x) = (-1)^n P_n(x)$. [5+5+6]
5. Evaluate the following using $\beta - \Gamma$ functions.
   (a) $\int_0^1 x^7 (1-x)^5 dx$.
   (b) $\int_0^{\pi/2} \sin^5\theta \cos^{\frac{7}{2}}\theta \, d\theta$. [5+5+6]
6. (a) Evaluate $\int_{(0,0)}^{(1,1)} (z^2 + 2z) dz$ along the closed path bounded by the curves $y^2=x$, $y=x^2$.
   (b) Evaluate $\int_C \frac{(4-3z)dz}{z(z-1)(z-2)}$ where C is the circle |z| = 3/2. using Cauchy's integral formula. [8+8]
7. (a) Find the poles and residues at each pole $\frac{\cot z \ coth z}{z^3}$.
   (b) Evaluate $\int_C \frac{3\sin z.dz}{(z^2-\frac{\pi^2}{4})}$ where C is |z| = π by residue theorem. [8+8]
8. (a) Derive Cauchy Riemann equations in polar coordinates.
   (b) Prove that the function f(z) = $\bar{z}$ is not analytic at any point.

(c) Find the general and the principal values of (i) $\log_e(1+\sqrt{3}i)$ (ii) $\log_e(-1)$. [5+5+6]

⋆ ⋆ ⋆ ⋆ ⋆

Code No: R05221801 **R05** **Set No. 4**

II B.Tech II Semester Examinations,November 2010
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Find the image of 1< |z| <2 under the transformation w= 2iz+1.
   (b) Find the bilinear transformation which maps the points (–1, i, 1+i) onto the points (0, 2i, 1-i). [8+8]

2. (a) Show that J_{n-1} (x) + J_{n+1}(x) = $\frac{2n}{x}J_n(x)$.
   (b) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x$.
   (c) Show that $P_n(1) = 1 \text{ and } P_n(-x) = (-1)^n P_n(x)$. [5+5+6]

3. (a) Find the poles and residues at each pole $\frac{\cot z \ coth z}{z^3}$.
   (b) Evaluate $\int_C \frac{3\sin z.dz}{(z^2-\frac{\pi^2}{4})}$ where C is $|z| = \pi$ by residue theorem. [8+8]

4. (a) Derive Cauchy Riemann equations in polar coordinates.
   (b) Prove that the function f(z) = $\bar{z}$ is not analytic at any point.
   (c) Find the general and the principal values of (i) log $_e(1+\sqrt{3}i)$ (ii) log $_e(-1)$. [5+5+6]

5. (a) State and derive Laurent's series for an analytic function f (z).
   (b) Expand $\frac{1}{(z^2-3z+2)}$ in the region.
      i. $0 < |\, z - 1 \,| < 1$
      ii. $1 < |\, z \,| < 2$. [8+8]

6. Evaluate the following using $\beta - \Gamma$ functions.
   (a) $\int_0^1 x^7(1-x)^5 dx$.
   (b) $\int_0^{\pi/2} \sin^5\theta \cos^{\frac{7}{2}}\theta \, d\theta$. [5+5+6]

7. (a) Evaluate $\int_0^{2\pi} \frac{Cos2\theta}{5+4Cos\theta} d\theta$ using residue theorem.
   (b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ using residue theorem. [8+8]

8. (a) Evaluate $\int_{(0,0)}^{(1,1)} (z^2+2z)dz$ along the closed path bounded by the curves y^2=x, y=x^2.

(b) Evaluate $\int_C \frac{(4-3z)dz}{z(z-1)(z-2)}$ where C is the circle |z| = 3/2. using Cauchy's integral formula. [8+8]

⋆ ⋆ ⋆ ⋆ ⋆

Code No: R05221801 **R05** **Set No. 1**

II B.Tech II Semester Examinations,November 2010
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Show that J_{n-1} (x) + J_{n+1}(x) = $\frac{2n}{x}J_n(x)$.
   (b) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x$.
   (c) Show that $P_n(1) = 1 \text{ and } P_n(-x) = (-1)^n P_n(x)$. [5+5+6]

2. (a) Evaluate $\int_0^{2\pi} \frac{Cos2\theta}{5+4Cos\theta} d\theta$ using residue theorem.
   (b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ using residue theorem. [8+8]

3. (a) Derive Cauchy Riemann equations in polar coordinates.
   (b) Prove that the function f(z) = $\bar{z}$ is not analytic at any point.
   (c) Find the general and the principal values of (i) log $_e(1+\sqrt{3}i)$ (ii) log $_e(-1)$. [5+5+6]

4. (a) Find the poles and residues at each pole $\frac{\cot z \ coth z}{z^3}$.
   (b) Evaluate $\int_C \frac{3\sin z\, dz}{(z^2-\frac{\pi^2}{4})}$ where C is |z| = π by residue theorem. [8+8]

5. (a) Find the image of 1< |z| <2 under the transformation w= 2iz+1.
   (b) Find the bilinear transformation which maps the points (–1, i, 1+i) onto the points (0, 2i, 1-i). [8+8]

6. (a) State and derive Laurent's series for an analytic function f (z).
   (b) Expand $\frac{1}{(z^2-3z+2)}$ in the region.
      i. $0 < |\, z - 1 \,| < 1$
      ii. $1 < |\, z \,| < 2$. [8+8]

7. Evaluate the following using $\beta - \Gamma$ functions.
   (a) $\int_0^1 x^7(1-x)^5 dx$.
   (b) $\int_0^{\pi/2} \sin^5\theta \cos^{\frac{7}{2}}\theta \, d\theta$. [5+5+6]

8. (a) Evaluate $\int_{(0,0)}^{(1,1)} (z^2+2z)dz$ along the closed path bounded by the curves y^2=x, $y=x^2$.

(b) Evaluate $\int_C \frac{(4-3z)dz}{z(z-1)(z-2)}$ where C is the circle |z| = 3/2. using Cauchy's integral formula. [8+8]

⋆ ⋆ ⋆ ⋆ ⋆

Code No: R05221801 **R05** **Set No. 3**

II B.Tech II Semester Examinations,November 2010
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

⋆ ⋆ ⋆ ⋆ ⋆

1. Evaluate the following using $\beta - \Gamma$ functions.
   (a) $\int_0^1 x^7(1-x)^5 dx$.
   (b) $\int_0^{\pi/2} \sin^5\theta \cos^{\frac{7}{2}}\theta \, d\theta$. [5+5+6]

2. (a) State and derive Laurent's series for an analytic function f (z).
   (b) Expand $\frac{1}{(z^2-3z+2)}$ in the region.
      i. $0 < |\, z - 1 \,| < 1$
      ii. $1 < |\, z \,| < 2$. [8+8]

3. (a) Evaluate $\int_0^{2\pi} \frac{Cos2\theta}{5+4Cos\theta} d\theta$ using residue theorem.
   (b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ using residue theorem. [8+8]

4. (a) Show that $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$.
   (b) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.
   (c) Show that $P_n(1) = 1 and\ P_n(-x) = (-1)^n P_n(x)$. [5+5+6]

5. (a) Derive Cauchy Riemann equations in polar coordinates.
   (b) Prove that the function f(z) = $\bar{z}$ is not analytic at any point.
   (c) Find the general and the principal values of (i) $\log_e(1+\sqrt{3}i)$ (ii) $\log_e(-1)$. [5+5+6]

6. (a) Find the poles and residues at each pole $\frac{\cot z \ coth z}{z^3}$.
   (b) Evaluate $\int_C \frac{3\sin z.dz}{(z^2-\frac{\pi^2}{4})}$ where C is $|z| = \pi$ by residue theorem. [8+8]

7. (a) Evaluate $\int_{(0,0)}^{(1,1)} (z^2 + 2z)dz$ along the closed path bounded by the curves y^2=x, y=x^2.
   (b) Evaluate $\int_C \frac{(4-3z)dz}{z(z-1)(z-2)}$ where C is the circle $|z| = 3/2$. using Cauchy's integral formula. [8+8]

8. (a) Find the image of $1 < |z| < 2$ under the transformation w= 2iz+1.

(b) Find the bilinear transformation which maps the points (–1, i, 1+i) onto the points (0, 2i, 1-i). [8+8]

⋆ ⋆ ⋆ ⋆ ⋆