

Code No: RR10102

RR

Set No. 2

I B.Tech Examinations, December 2010

MATHEMATICS - I

Common to CE, ME, CHEM, BME, IT, MECT, MEP, AE, BT, AME, ICE,
E.COMP.E, MMT, ETM, E.CONT.E, EIE, CSE, ECE, CSSE, EEE

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

- Prove that $\text{curl}(\mathbf{A} \times \mathbf{B}) = \mathbf{A} \text{div} \mathbf{B} - \mathbf{B} \text{div} \mathbf{A} + (\mathbf{B} \cdot \nabla) \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{B}$.
 - Find the directional derivative of $\phi(x, y, z) = x^2 yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the normal to the surface $f(x, y, z) = x \log z - y^2$ at $(-1, 2, -1)$. [8+8]
- Test the convergence of the series $\frac{\sqrt{2}-1}{3^2-1} + \frac{\sqrt{3}-1}{4^2-1} + \frac{\sqrt{4}-1}{5^2-1} + \dots$ [5]
 - Test whether the following series converges absolutely or conditionally.
 $1 \cdot \frac{2}{3} - \frac{1}{2} \cdot \frac{3}{4} + \frac{1}{3} \cdot \frac{4}{5} - \frac{1}{4} \cdot \frac{5}{6} + \dots$ [5]
 - Verify Rolle's theorem for $f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$ [6]
- If $u = \sin^{-1} \left[\frac{x^{1/3} + y^{1/3}}{\sqrt{x} + \sqrt{y}} \right]^{1/2}$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{12} \tan u$
and $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144} (13 + \tan^2 u)$
 - Find the evolute of the curve $x = a \cos^3 \theta, y = a \sin^3 \theta$. [8+8]
- Form the differential equation by eliminating the arbitrary constant
 $x \tan(y/x) = c$. [3]
 - Solve the differential equation: $\frac{dy}{dx} + y \cos x = y^3 \sin 2x$. [7]
 - Find the orthogonal trajectories of the family of circles $x^2 + y^2 = ax$. [6]
- Trace the curve: $x^4 + y^4 = 2a^2 xy$.
 - Find the surface area got by rotating one loop of the curve $r^2 = a^2 \cos 2\theta$ about the initial line. [8+8]
- Solve the differential equation: $(D^2 + 1)y = \sin x \sin 2x$.
 - Solve the differential equation: $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4$ [8+8]
- Verify divergence theorem for $\mathbf{F} = 4xzi - y^2 \mathbf{j} + yzk$, where S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$. [16]
- Find $L \left[\frac{\sin^2 t}{t} \right]$ [5]
 - Find $L^{-1} \left[\frac{s+2}{s^2-4s+13} \right]$ [6]
 - Evaluate the triple integral $\int_0^{\pi/2} \int_0^a \int_1^{\sin \theta} r \, dz \, dr \, d\theta$ [5]

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Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) If $u = \sin^{-1} \left[\frac{x^{1/3} + y^{1/3}}{\sqrt{x} + \sqrt{y}} \right]^{1/2}$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{12} \tan u$
and $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144} (13 + \tan^2 u)$
- (b) Find the evolute of the curve $x = a \cos^3 \theta$, $y = a \sin^3 \theta$. [8+8]
2. (a) Test the convergence of the series $\frac{\sqrt{2}-1}{3^2-1} + \frac{\sqrt{3}-1}{4^2-1} + \frac{\sqrt{4}-1}{5^2-1} + \dots$ [5]
- (b) Test whether the following series converges absolutely or conditionally.
 $1 \cdot \frac{2}{3} - \frac{1}{2} \cdot \frac{3}{4} + \frac{1}{3} \cdot \frac{4}{5} - \frac{1}{4} \cdot \frac{5}{6} + \dots$ [5]
- (c) Verify Rolle's theorem for $f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$ [6]
3. (a) Trace the curve : $x^4 + y^4 = 2a^2 xy$.
- (b) Find the surface area got by rotating one loop of the curve $r^2 = a^2 \cos 2\theta$ about the initial line. [8+8]
4. (a) Solve the differential equation: $(D^2 + 1)y = \sin x \sin 2x$.
- (b) Solve the differential equation: $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4$ [8+8]
5. (a) Form the differential equation by eliminating the arbitrary constant
 $x \tan(y/x) = c$. [3]
- (b) Solve the differential equation: $\frac{dy}{dx} + y \cos x = y^3 \sin 2x$. [7]
- (c) Find the orthogonal trajectories of the family of circles $x^2 + y^2 = ax$. [6]
6. Verify divergence theorem for $F = 4xz\mathbf{i} - y^2\mathbf{j} + yz\mathbf{k}$, where S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$. [16]
7. (a) Prove that $\text{curl}(\mathbf{A} \times \mathbf{B}) = \mathbf{A} \text{div} \mathbf{B} - \mathbf{B} \text{div} \mathbf{A} + (\mathbf{B} \cdot \nabla) \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{B}$.
- (b) Find the directional derivative of $\phi(x, y, z) = x^2 yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the normal to the surface $f(x, y, z) = x \log z - y^2$ at $(-1, 2, -1)$. [8+8]
8. (a) Find $L \left[\frac{\sin^2 t}{t} \right]$ [5]
- (b) Find $L^{-1} \left[\frac{s+2}{s^2-4s+13} \right]$ [6]
- (c) Evaluate the triple integral $\int_0^{\pi/2} \int_0^a \int_0^{\sin \theta} r \, dz \, dr \, d\theta$ [5]

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Set No. 1

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Time: 3 hours

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Answer any FIVE Questions
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1. (a) Solve the differential equation: $(D^2 + 1)y = \sin x \sin 2x$.
(b) Solve the differential equation: $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4$ [8+8]
2. (a) If $u = \sin^{-1} \left[\frac{x^{1/3} + y^{1/3}}{\sqrt{x+y}} \right]^{1/2}$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{12} \tan u$
and $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144} (13 + \tan^2 u)$
(b) Find the evolute of the curve $x = a \cos^3 \theta$, $y = a \sin^3 \theta$. [8+8]
3. Verify divergence theorem for $F = 4xz\mathbf{i} - y^2\mathbf{j} + yz\mathbf{k}$, where S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$. [16]
4. (a) Find $L \left[\frac{\sin^2 t}{t} \right]$ [5]
(b) Find $L^{-1} \left[\frac{s+2}{s^2-4s+13} \right]$ [6]
- (c) Evaluate the triple integral $\int_0^{\pi/2} \int_0^a \int_1^{\sin \theta} r \, dz \, dr \, d\theta$ [5]
5. (a) Prove that $\text{curl}(\mathbf{A} \times \mathbf{B}) = \mathbf{A} \text{div} \mathbf{B} - \mathbf{B} \text{div} \mathbf{A} + (\mathbf{B} \cdot \nabla) \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{B}$.
(b) Find the directional derivative of $\phi(x, y, z) = x^2 yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the normal to the surface $f(x, y, z) = x \log z - y^2$ at $(-1, 2, -1)$. [8+8]
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(c) Verify Rolle's theorem for $f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$ [6]
7. (a) Trace the curve: $x^4 + y^4 = 2a^2 xy$.
(b) Find the surface area got by rotating one loop of the curve $r^2 = a^2 \cos 2\theta$ about the initial line. [8+8]
8. (a) Form the differential equation by eliminating the arbitrary constant $x \tan(y/x) = c$. [3]
(b) Solve the differential equation: $\frac{dy}{dx} + y \cos x = y^3 \sin 2x$. [7]
(c) Find the orthogonal trajectories of the family of circles $x^2 + y^2 = ax$. [6]

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1. (a) Test the convergence of the series $\frac{\sqrt{2}-1}{3^2-1} + \frac{\sqrt{3}-1}{4^2-1} + \frac{\sqrt{4}-1}{5^2-1} + \dots$ [5]
 (b) Test whether the following series converges absolutely or conditionally.
 $1.\frac{2}{3} - \frac{1}{2}.\frac{3}{4} + \frac{1}{3}.\frac{4}{5} - \frac{1}{4}.\frac{5}{6} + \dots$ [5]
 (c) Verify Rolle's theorem for $f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$ [6]
2. (a) Trace the curve : $x^4 + y^4 = 2a^2 xy$.
 (b) Find the surface area got by rotating one loop of the curve $r^2 = a^2 \cos 2\theta$ about the initial line. [8+8]
3. Verify divergence theorem for $F = 4xzi - y^2j + yzk$, where S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$. [16]
4. (a) Prove that $\text{curl}(\mathbf{A} \times \mathbf{B}) = \mathbf{A} \text{div} \mathbf{B} - \mathbf{B} \text{div} \mathbf{A} + (\mathbf{B} \cdot \nabla) \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{B}$.
 (b) Find the directional derivative of $\phi(x, y, z) = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the normal to the surface $f(x, y, z) = x \log z - y^2$ at $(-1, 2, -1)$. [8+8]
5. (a) Solve the differential equation: $(D^2 + 1)y = \sin x \sin 2x$.
 (b) Solve the differential equation: $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4$ [8+8]
6. (a) Form the differential equation by eliminating the arbitrary constant $x \tan(y/x) = c$. [3]
 (b) Solve the differential equation: $\frac{dy}{dx} + y \cos x = y^3 \sin 2x$. [7]
 (c) Find the orthogonal trajectories of the family of circles $x^2 + y^2 = ax$. [6]
7. (a) Find $L \left[\frac{\sin^2 t}{t} \right]$ [5]
 (b) Find $L^{-1} \left[\frac{s+2}{s^2-4s+13} \right]$ [6]
 (c) Evaluate the triple integral $\int_0^{\pi/2} \int_0^a \int_0^{\sin \theta} r \, dz \, dr \, d\theta$ [5]
8. (a) If $u = \sin^{-1} \left[\frac{x^{1/3} + y^{1/3}}{\sqrt{x} + \sqrt{y}} \right]^{1/2}$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{12} \tan u$
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 (b) Find the evolute of the curve $x = a \cos^3 \theta, y = a \sin^3 \theta$. [8+8]
