Code No: RR220202

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Set No. 2

II B.Tech II Semester Examinations,December 2010 MATHEMATICS - III Common to AE, ICE, MMT, ETM, E.CONT.E, EIE, ECE, EEE Time: 3 hours Answer any FIVE Questions All Questions carry equal marks

(a) Find the poles and residue at each pole of the function cosec²z
(b) Evaluate \$\int_C \frac{ze^{iz} dz}{(z^2+9)^2}\$ where C is the circle \$|z| = 4\$ by residue theorem. [8+8]

2. (a) If f(z) is an analytic function, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$.

- (b) If $\tan \log (x+iy) = a + i b$ where $a^2 + b^2 \neq 1$ prove that $\tan \log (x^2 + y^2) = \frac{2a}{1-a^2-b^2}$ [8+8]
- 3. (a) State and prove Laurent's theorem.
 - (b) Obtain the Laurent series of the function $\frac{7z-2}{(z+1)(z)(z-2)}$ about z = -1. [8+8]
- 4. (a) Evaluate $\int_C \frac{z^2 z 1}{z(z i)^2} dz$ with C: $|z \frac{1}{2}| = 1$, using Caucy's integral theorem.
 - (b) Evaluate $\int_{1+i}^{2+4i} z^2 dz$ along
 - i. the parabola x=t, $y=t^2$ where $1 \le t \le 2$ ii. the stright line joining (1, 1) and (2, 4) iii. the stright lines from (1, -1) to (2, 1) and then to (2, 4) What is the inference and justify.
- 5. (a) Prove that $1 + \frac{1}{2} P_1(\cos \theta) + \frac{1}{3} P_2(\cos \theta) + ... = \log \left(\frac{1 + \sin \frac{\theta}{2}}{\sin \frac{\theta}{2}}\right)$ (b) Prove that $\frac{1}{2} x J_n = (n+1) J_{n+1} - (n+3) J_{n+3} + (n+5) J_{n+5} - \frac{x}{2} J_{n+6}[8+8]$
- 6. (a) In the transformation z=(i-w)/(i+w), show that the positive half of
 - the w-plane given by $w \ge 0$ corresponds to the circle $|z| \le 1$ in the z- plane.
 - (b) Show that the transformation w=(z+a)/z transforms circles with origin at the centre in the z -plane in the co-axial concentric confocal ellipses in the w-plane.

[8+8]

- 7. (a) Show that $\int_{0}^{1} x^{m} (\log x)^{n} dx = \frac{(-1)^{n} n!}{(m+1)^{n+1}}$ where n is a positive interger and m>-1
 - (b) Show that $\beta(\mathbf{m},\mathbf{n}) = \int_{0}^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}} dy$

\mathbf{RR} Set No. 2 Code No: RR220202 (c) Show that $\int_{0}^{\infty} x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$ [6+5+5]8. (a) Evaluate $\int_{0}^{2\pi} \frac{\sin^2 \theta \, d\theta}{a^+ b \cos \theta}$ using residue theorem.

(b) Evaluate
$$\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$$
 using residue theorem. [8+8]

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Set No. 4

II B.Tech II Semester Examinations, December 2010 MATHEMATICS - III Common to AE, ICE, MMT, ETM, E.CONT.E, EIE, ECE, EEE Time: 3 hours Max Marks: 80 Answer any FIVE Questions All Questions carry equal marks **** (a) Prove that $1 + \frac{1}{2} P_1(\cos\theta) + \frac{1}{3} P_2(\cos\theta) + \dots = \log\left(\frac{1 + \sin\frac{\theta}{2}}{\sin\frac{\theta}{2}}\right)$ 1. (b) Prove that $\frac{1}{2}xJ_n = (n+1)J_{n+1} - (n+3)J_{n+3} + (n+5)J_{n+5} - \frac{x}{2}J_{n+6}[8+8]$ (a) Evaluate $\int_{0}^{2\pi} \frac{\sin^2 \theta \, d\theta}{a^+ b \cos \theta}$ using residue theorem. 2. (b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ using residue theorem. [8+8]3. (a) Show that $\int_{0}^{1} x^{m} (\log x)^{n} dx = \frac{(-1)^{n} n!}{(m+1)^{m+1}}$ where n is a positive interger and m>-1 (b) Show that $\beta(\mathbf{m},\mathbf{n}) = \int_{0}^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}} dy$ (c) Show that $\int_{0}^{\infty} x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$ [6+5+5](a) State and prove Laurent's theorem. 4. (b) Obtain the Laurent series of the function $\frac{7z-2}{(z+1)(z)(z-2)}$ about z = -1. [8+8]5.(a) Find the poles and residue at each pole of the function $\csc^2 z$ (b) Evaluate $\int_{C} \frac{ze^{iz} dz}{(z^2+9)^2}$ where C is the circle |z| = 4 by residue theorem. [8+8](a) In the transformation z=(i-w)/(i+w), show that the positive half of 6. the w-plane given by $w \ge 0$ corresponds to the circle $|z| \le 1$ in the z- plane. (b) Show that the transformation $w=(z+a^{-2})/z$ transforms circles with origin at the centre in the z -plane in the co-axial concentric confocal ellipses in the wplane. [8+8](a) Evaluate $\int_C \frac{z^2 - z - 1}{z(z-i)^2} dz$ with C: $|z - \frac{1}{2}| = 1$, using Caucy's integral theorem. 7. (b) Evaluate $\int_{1+i}^{2+4i} z^2 dz$ along

- i. the parabola x=t, y=t² where $1 \leq t \leq 2$
- ii. the stright line joining (1, 1) and (2, 4)
- iii. the stright lines from (1, 1) to (2, 1) and then to (2, 4)

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What is the inference and justify.

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[8+8]

- 8. (a) If f(z) is an analytic function, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$.
 - (b) If $\tan \log (x+iy) = a + i b$ where $a^2 + b^2 \neq 1$ prove that $\tan \log (x^2 + y^2) = \frac{2a}{1-a^2-b^2}$ [8+8]

FIRST

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 \mathbf{RR}

Set No. 1

II B.Tech II Semester Examinations, December 2010 MATHEMATICS - III Common to AE, ICE, MMT, ETM, E.CONT.E, EIE, ECE, EEE Max Marks: 80 Time: 3 hours Answer any FIVE Questions All Questions carry equal marks **** 1. (a) If f(z) is an analytic function, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$. (b) If tan log (x+iy) = a + i b where $a^2 + b^2 \neq 1$ prove that tan log (x²+ y²) = $\frac{2a}{1-a^2-b^2}$ [8+8]2. (a) Prove that $1 + \frac{1}{2} P_1(\cos\theta) + \frac{1}{3} P_2(\cos\theta) + \dots = \log \left(\frac{1 + \sin\frac{\theta}{2}}{\sin\frac{\theta}{2}}\right)$ (b) Prove that $\frac{1}{2} x J_n = (n+1) J_{n+1} - (n+3) J_{n+3} + (n+5) J_{n+5} - \frac{x}{2} J_{n+6}[8+8]$ 3. (a) Evaluate $\int_C \frac{z^2 - z - 1}{z(z-i)^2} dz$ with C: $|z - \frac{1}{2}| = 1$, using Caucy's integral theorem. (b) Evaluate $\int_{1+i}^{2+4i} z^2 dz$ along (b) Evaluate $\int_{1+i}^{2+4i} z^2 dz$ along i. the parabola x=t, y=t² where $1 \leq t \leq 2$ ii. the stright line joining (1, 1) and (2, 4)iii. the stright lines from (1, 1) to (2, 1) and then to (2, 4)What is the inference and justify. [8+8]4. (a) Find the poles and residue at each pole of the function $\csc^2 z$ (b) Evaluate $\int_{C} \frac{ze^{iz} dz}{(z^2+9)^2}$ where C is the circle |z| = 4 by residue theorem. [8+8](a) In the transformation z=(i-w)/(i+w), show that the positive half of 5. the w-plane given by $w \ge 0$ corresponds to the circle $|z| \le 1$ in the z- plane. (b) Show that the transformation $w=(z+a^2)/z$ transforms circles with origin at the centre in the z –plane in the co-axial concentric confocal ellipses in the wplane. [8+8]6. (a) Evaluate $\int_{0}^{2\pi} \frac{\sin^2 \theta \, d\theta}{a^+ b \cos \theta}$ using residue theorem.

(b) Evaluate
$$\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$$
 using residue theorem. [8+8]

7. (a) Show that $\int_{0}^{1} x^{m} (\log x)^{n} dx = \frac{(-1)^{n} n!}{(m+1)^{n+1}}$ where n is a positive interger and m>-1

(b) Show that
$$\beta(\mathbf{m},\mathbf{n}) = \int_{0}^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}} dy$$

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(c) Show that
$$\int_{0}^{\infty} x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$$
 [6+5+5]

- 8. (a) State and prove Laurent's theorem.
 - (b) Obtain the Laurent series of the function $\frac{7z-2}{(z+1)(z)(z-2)}$ about z = -1. [8+8]

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II B.Tech II Semester Examinations, December 2010

Set No. 3

MATHEMATICS - III Common to AE, ICE, MMT, ETM, E.CONT.E, EIE, ECE, EEE Max Marks: 80 Time: 3 hours Answer any FIVE Questions All Questions carry equal marks **** 1. (a) Evaluate $\int_{0}^{2\pi} \frac{\sin^2 \theta \, d\theta}{a^+ b \cos \theta}$ using residue theorem. (b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ using residue theorem. [8+8](a) Find the poles and residue at each pole of the function $\csc^2 z$ 2. (b) Evaluate $\int_C \frac{ze^{iz} dz}{(z^2+9)^2}$ where C is the circle |z| = 4 by residue theorem. |8+8|(a) If f(z) is an analytic function, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$. 3. (b) If tan log (x+iy) = a + i b where $a^2 + b^2 \neq 1$ prove that tan log (x²+ y²) = $\frac{2a}{1-a^2+b^2}$ [8+8]4. (a) Show that $\int_{0}^{1} x^{m} (\log x)^{n} dx = \frac{(-1)^{n} n!}{(m+1)^{n+1}}$ where n is a positive interger and m>-1 (b) Show that $\beta(\mathbf{m},\mathbf{n}) \Rightarrow \int_{0}^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}} dy$ (c) Show that $\int_{-\infty}^{\infty} x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$ [6+5+5](a) Prove that $1 + \frac{1}{2} P_1(\cos\theta) + \frac{1}{3} P_2(\cos\theta) + \dots = \log\left(\frac{1 + \sin\frac{\theta}{2}}{\sin\frac{\theta}{2}}\right)$ 5.(b) Prove that $\frac{1}{2}xJ_n = (n+1)J_{n+1} - (n+3)J_{n+3} + (n+5)J_{n+5} - \frac{x}{2}J_{n+6}[8+8]$ (a) State and prove Laurent's theorem. 6. (b) Obtain the Laurent series of the function $\frac{7z-2}{(z+1)(z)(z-2)}$ about z = -1. [8+8]7. (a) Evaluate $\int_C \frac{z^2 - z - 1}{z(z - i)^2} dz$ with C: $|z - \frac{1}{2}| = 1$, using Caucy's integral theorem. (b) Evaluate $\int_{1-i}^{2+4i} z^2 dz$ along i. the parabola x=t, y=t² where $1 \le t \le 2$ ii. the stright line joining (1, 1) and (2, 4)iii. the stright lines from (1, 1) to (2, 1) and then to (2, 4)What is the inference and justify. [8+8]

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Set No. 3

8. (a) In the transformation z=(i-w)/(i+w), show that the positive half of the w-plane given by $w\geq 0$ corresponds to the circle $|z|\leq 1$ in the z- plane.

(b) Show that the transformation $w=(z+a^2)/z$ transforms circles with origin at the centre in the z –plane in the co-axial concentric confocal ellipses in the w-plane.

[8+8]

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