Code No: RR220202 **RR** **Set No. 2**

II B.Tech II Semester Examinations,December 2010
MATHEMATICS - III
Common to AE, ICE, MMT, ETM, E.CONT.E, EIE, ECE, EEE

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Find the poles and residue at each pole of the function cosec^{2}z
   (b) Evaluate $\int_C \frac{ze^{iz}\ dz}{(z^2+9)^2}$ where C is the circle $|z| = 4$ by residue theorem. [8+8]

2. (a) If f(z) is an analytic function, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$.
   (b) If tan log (x+iy) = a + i b where a^2 + b^2 $\neq$1 prove that tan log (x^2+ y^2) = $\frac{2a}{1-a^2-b^2}$ [8+8]

3. (a) State and prove Laurent's theorem.
   (b) Obtain the Laurent series of the function $\frac{7z-2}{(z+1)(z)(z-2)}$ about z= -1. [8+8]

4. (a) Evaluate $\int_C \frac{z^2-z-1}{z(z-i)^2} dz$ with C: $|z - \frac{1}{2}| = 1$, using Caucy's integral theorem.
   (b) Evaluate $\int_{1+i}^{2+4i} z^2\, dz$ along
      i. the parabola x=t, y=t^2 where 1 ≤ t ≤ 2
      ii. the stright line joining (1, 1) and (2, 4)
      iii. the stright lines from(1, 1) to (2, 1) and then to (2, 4)

   What is the inference and justify. [8+8]

5. (a) Prove that $1+\frac{1}{2}\, P_1\,(\cos\theta)\ +\ \frac{1}{3}\, P_2\,(\cos\theta)\ + ... =\ \log\left(\frac{1+\sin\frac{\theta}{2}}{\sin\frac{\theta}{2}}\right)$
   (b) Prove that $\frac{1}{2}\, xJ_n\ =\ (n+1)\, J_{n+1}\ -(n+3)\, J_{n+3}\ +(n+5)\, J_{n+5}\ -\ \frac{x}{2}\, J_{n+6}$ [8+8]

6. (a) In the transformation z=(i-w)/(i+w) , show that the positive half of the w-plane given by w≥0 corresponds to the circle |z| ≤1 in the z- plane.
   (b) Show that the transformation w=(z+a^2)/z transforms circles with origin at the centre in the z –plane in the co-axial concentric confocal ellipses in the w-plane.

   [8+8]

7. (a) Show that $\int_0^1 x^m\,(\log x)^n\, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is a positive interger and m>-1
   (b) Show that β(m,n)= $\int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}}dy$

(c) Show that $\int\limits_{0}^{\infty} x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$ [6+5+5]

8. (a) Evaluate $\int\limits_{0}^{2\pi} \frac{\sin^2\theta \, d\theta}{a^+ b\cos\theta}$ using residue theorem.

(b) Evaluate $\int\limits_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)\,(x^2+4)}$ using residue theorem. [8+8]

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Code No: RR220202 **RR** **Set No. 4**

II B.Tech II Semester Examinations,December 2010
MATHEMATICS - III
Common to AE, ICE, MMT, ETM, E.CONT.E, EIE, ECE, EEE
Time: 3 hours **Max Marks: 80**
Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Prove that $1+\frac{1}{2}\,P_1\,(\cos\theta)\,+\,\frac{1}{3}\,P_2\,(\cos\theta)\,+\,....\,=\,\log\left(\frac{1+\sin\frac{\theta}{2}}{\sin\frac{\theta}{2}}\right)$
   (b) Prove that $\frac{1}{2}\,xJ_n\,=\,(n+1)\,J_{n+1}\,-(n+3)\,J_{n+3}\,+(n+5)\,J_{n+5}\,-\frac{x}{2}\,J_{n+6}$ [8+8]

2. (a) Evaluate $\int\limits_0^{2\pi}\frac{\sin^2\theta\,d\theta}{a^+b\cos\theta}$ using residue theorem.
   (b) Evaluate $\int\limits_{-\infty}^{\infty}\frac{x^2dx}{(x^2+1)\,(x^2+4)}$ using residue theorem. [8+8]

3. (a) Show that $\int\limits_0^1 x^m\,(\log x)^n\,dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is a positive interger and m>-1
   (b) Show that β(m,n)= $\int\limits_0^{\infty}\frac{y^{n-1}}{(1+y)^{m+n}}dy$
   (c) Show that $\int\limits_0^{\infty}x^4e^{-x^2}dx = \frac{3\sqrt{\pi}}{8}$ [6+5+5]

4. (a) State and prove Laurent's theorem.
   (b) Obtain the Laurent series of the function $\frac{7z-2}{(z+1)(z)(z-2)}$ about z= -1. [8+8]

5. (a) Find the poles and residue at each pole of the function cosec^{2}z
   (b) Evaluate $\int\limits_C\frac{ze^{iz}\ dz}{(z^2+9)^2}$ where C is the circle $|z|\,=\,4$ by residue theorem. [8+8]

6. (a) In the transformation z=(i-w)/(i+w) , show that the positive half of the w-plane given by w≥0 corresponds to the circle |z| ≤1 in the z- plane.
   (b) Show that the transformation w=(z+a ^{2})/z transforms circles with origin at the centre in the z –plane in the co-axial concentric confocal ellipses in the w-plane.

[8+8]

7. (a) Evaluate $\int\limits_C\frac{z^2-z-1}{z(z-i)^2}dz$ with C: $|z-\frac{1}{2}|=1$, using Caucy's integral theorem.
   (b) Evaluate $\int\limits_{1+i}^{2+4i}z^2\,dz$ along
      i. the parabola x=t, y=t^2 where $1 \le t \le 2$
      ii. the stright line joining (1, 1) and (2, 4)
      iii. the stright lines from(1, 1) to (2, 1) and then to (2, 4)

What is the inference and justify. [8+8]

8. (a) If f(z) is an analytic function, show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

   (b) If tan log (x+iy) = a + i b where $a^2 + b^2 \neq 1$ prove that
   tan log $(x^2 + y^2) = \frac{2a}{1-a^2-b^2}$ [8+8]

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Code No: RR220202 **RR** **Set No. 1**

II B.Tech II Semester Examinations,December 2010
MATHEMATICS - III
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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) If f(z) is an analytic function, show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$.

   (b) If tan log (x+iy) = a + i b where a^2 + b^2 $\neq$1 prove that tan log (x^2+ y^2) = $\frac{2a}{1-a^2-b^2}$ [8+8]

2. (a) Prove that $1+\frac{1}{2}\,P_1\,(\cos\theta)\,+\,\frac{1}{3}\,P_2\,(\cos\theta)\,+\,....\,=\,\log\left(\frac{1+\sin\frac{\theta}{2}}{\sin\frac{\theta}{2}}\right)$

   (b) Prove that $\frac{1}{2}\,xJ_n\;=\;(n+1)\,J_{n+1}\,-(n+3)\,J_{n+3}+(n+5)\,J_{n+5}\,-\,\frac{x}{2}\,J_{n+6}$ [8+8]

3. (a) Evaluate $\int\limits_C \frac{z^2-z-1}{z(z-i)^2}dz$ with C: $|z-\frac{1}{2}|=1$, using Caucy's integral theorem.

   (b) Evaluate $\int\limits_{1+i}^{2+4i} z^2\,dz$ along

      i. the parabola x=t, y=t^2 where 1 ≤ t ≤ 2

      ii. the stright line joining (1, 1) and (2, 4)

      iii. the stright lines from(1, 1) to (2, 1) and then to (2, 4)

   What is the inference and justify. [8+8]

4. (a) Find the poles and residue at each pole of the function cosec^{2}z

   (b) Evaluate $\int\limits_C \frac{ze^{iz}\;dz}{(z^2+9)^2}$ where C is the circle $|z|\,=\,4$ by residue theorem. [8+8]

5. (a) In the transformation z=(i-w)/(i+w) , show that the positive half of the w-plane given by w≥0 corresponds to the circle |z| ≤1 in the z- plane.

   (b) Show that the transformation w=(z+a^2)/z transforms circles with origin at the centre in the z –plane in the co-axial concentric confocal ellipses in the w-plane.

   [8+8]

6. (a) Evaluate$\int\limits_0^{2\pi}\frac{\sin^2\theta\,d\theta}{a^+b\cos\theta}$ using residue theorem.

   (b) Evaluate $\int\limits_{-\infty}^{\infty}\frac{x^2dx}{(x^2+1)\,(x^2+4)}$ using residue theorem. [8+8]

7. (a) Show that $\int\limits_0^1 x^m\,(\log x)^n\,dx=\frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is a positive interger and m>-1

   (b) Show that β(m,n)= $\int\limits_0^{\infty}\frac{y^{n-1}}{(1+y)^{m+n}}dy$

(c) Show that $\int_0^\infty x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$ [6+5+5]

8. (a) State and prove Laurent's theorem.
   (b) Obtain the Laurent series of the function $\frac{7z-2}{(z+1)(z)(z-2)}$ about z= -1. [8+8]

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Code No: RR220202 **RR** **Set No. 3**

II B.Tech II Semester Examinations,December 2010
MATHEMATICS - III
Common to AE, ICE, MMT, ETM, E.CONT.E, EIE, ECE, EEE
Time: 3 hours **Max Marks: 80**
Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Evaluate $\int_0^{2\pi} \frac{\sin^2\theta\, d\theta}{a^+ b\cos\theta}$ using residue theorem.

   (b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)\,(x^2+4)}$ using residue theorem. [8+8]

2. (a) Find the poles and residue at each pole of the function cosec^{2}z

   (b) Evaluate $\int_C \frac{ze^{iz}\ dz}{(z^2+9)^2}$ where C is the circle $|z| = 4$ by residue theorem. [8+8]

3. (a) If f(z) is an analytic function, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

   (b) If tan log (x+iy) = a + i b where a^2 + b^2 $\neq$1 prove that
   tan log (x^2+ y^2) = $\frac{2a}{1-a^2-b^2}$ [8+8]

4. (a) Show that $\int_0^1 x^m (\log x)^n\, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is a positive interger and m>-1

   (b) Show that β(m,n)= $\int_0^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}} dy$

   (c) Show that $\int_0^{\infty} x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$ [6+5+5]

5. (a) Prove that $1+\frac{1}{2}\,P_1\,(\cos\theta)\, +\, \frac{1}{3}\,P_2\,(\cos\theta)\, + ... =\, \log\left(\frac{1+\sin\frac{\theta}{2}}{\sin\frac{\theta}{2}}\right)$

   (b) Prove that $\frac{1}{2}\,xJ_n\, =\, (n+1)\,J_{n+1}\, -(n+3)\,J_{n+3}\, + (n+5)\,J_{n+5}\, - \frac{x}{2}\,J_{n+6}$ [8+8]

6. (a) State and prove Laurent's theorem.

   (b) Obtain the Laurent series of the function $\frac{7z-2}{(z+1)(z)(z-2)}$ about z= -1. [8+8]

7. (a) Evaluate $\int_C \frac{z^2-z-1}{z(z-i)^2} dz$ with C: $|z - \frac{1}{2}| = 1$, using Caucy's integral theorem.

   (b) Evaluate $\int_{1+i}^{2+4i} z^2\, dz$ along

      i. the parabola x=t, y=t^2 where $1 \leq t \leq 2$
      ii. the stright line joining (1, 1) and (2, 4)
      iii. the stright lines from(1, 1) to (2, 1) and then to (2, 4)

   What is the inference and justify. [8+8]

8. (a) In the transformation z=(i-w)/(i+w) , show that the positive half of the w-plane given by w$\geq$0 corresponds to the circle |z| $\leq$1 in the z- plane.

   (b) Show that the transformation w=(z+a^2)/z transforms circles with origin at the centre in the z –plane in the co-axial concentric confocal ellipses in the w-plane.

[8+8]

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