Code No: R10102

R10

SET - 1

I B. Tech I Semester Supplementary Examinations, May - 2017 **MATHEMATICS-I**

(Com. to All Branches)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions All Questions carry **Equal** Marks

1. a) Solve
$$\frac{dy}{dx} = \frac{1}{(1+y^2)} \left(e^{\tan^{-1}x} - y \right)$$
. (7M)

(8M)b) A body is heated to 110°C and placed in air at 10°C. After 1 hour its temperature is 60° C. How much additional time is required for it to cool to 30° C?

2. a) Solve
$$(D^2 + 2)y = x^2e^{3x} + e^x \cos 2x$$
, where $D = \frac{d}{dx}$. (7M)

b) Solve
$$(D^2 + 3D + 2)y = e^{-x} + x^2 + \cos x$$
, where $D = \frac{d}{dx}$. (8M)

3. a) If
$$u = x + y + z$$
, $uv = y + z$, $uvw = z$, find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. (7M)

b) Examine the function $f(x, y) = \sin x + \sin y + \sin(x + y)$ for extreme. (8M)

4. a) Trace the curve
$$x^3 + y^3 = 3axy$$
. (7M)

b) Trace the curve
$$r^2 = a^2 \cos 2\theta$$
. (8M)

(7M)Find the length of the arc of parabola $y^2 = 4ax$ cut-off by latus rectum.

b) Find the surface area of the solid generated by the revolution of the asteroid (8M) $x = a \cos^3 t$, $y = a \sin^3 t$ about the y - axis.

6. a) Evaluate
$$\int_{0}^{\infty} \int_{0}^{\infty} e^{-(x^2+y^2)} dx dy$$
 by changing to polar coordinates. (5M)
b) Evaluate $\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x+y+z) dx dy dz$. (5M)

b) Evaluate
$$\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x+y+z) dx dy dz$$
. (5M)

c) Change the order of integration and hence evaluate
$$I = \int_{0}^{a} \int_{x/a}^{\sqrt{x/a}} (x^2 + y^2) dx dy$$
. (5M)

7. a) Evaluate divergence of $(2x^2z i - xy^2z j + 3yz^2 k)$ at the point (1, 1, 1). (5M)

b) Show that
$$\nabla^2 r^n = n(n+1)r^{n-2}$$
 where $r^2 = x^2 + y^2 + z^2$. (5M)

c) Evaluate Curl of
$$\overline{V} = yz i + 3zx j + z k$$
 at the point $(2, 3, 4)$. (5M)

8. Verify Stoke's theorem for a vector field defined $\overline{F} = -y^3 i + x^3 j$, in the region (15M) $x^2 + y^2 \le 1$, z = 0.