

Code No: R21016

R10
SET - 1
II B. Tech I Semester Supplementary Examinations, Oct/Nov- 2017
MATHEMATICS - III

(Com. to CE, CHEM, BT, PE)

Time: 3 hours

Max. Marks: 75

Answer any **FIVE** Questions
 All Questions carry **Equal** Marks

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1. a) Prove that  $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$ . (8M)

b) If  $m_1, m_2$  are roots  $J_n(x) = 0$ , then prove that  $\int_0^1 x J_n(m_1 x) J_n(m_2 x) dx = 0$ . (7M)

2. a) If  $u - v = (x - y)(x^2 + 4xy + y^2)$  and  $f(z) = u + iv$  is an analytic function of  $z = x + iy$  Find  $f(z)$  in terms of  $z$ . (8M)

b) If  $f(z) = w = u + iv$ , then prove that  $\frac{dw}{dz} = (\cos \theta - i \sin \theta) \frac{\partial w}{\partial r}$ . (7M)

3. a) If  $\tan \frac{x}{2} = \tanh \frac{y}{2}$  prove that i)  $\tan x = \sinh y$ , ii)  $\cos x \cosh y = 1$  (8M)

b) Prove that  $\log \left( \frac{a+ib}{a-ib} \right) = 2i \tan^{-1} \frac{b}{a}$ . Evaluate  $\cos \left[ i \log \left( \frac{a+ib}{a-ib} \right) \right]$  (7M)

4. a) Evaluate  $\int_c [(x - 2y)dx + (y^2 - x^2)dy]$  where  $C$  is the boundary of the (8M)

circle  $x^2 + y^2 = 4$  in the first octant.

b) Evaluate  $\int_c \left[ \frac{e^z}{z^3} + \frac{z^4}{(z+i)^2} \right] dz$  where  $c: |z| = 2$  using Cauchy's integral formula. (7M)

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5. a) Find the poles and residues of  $\frac{1}{z^2 - 1}$ . (8M)
- b) Expand the Laurent series of  $\frac{z^2 - 1}{(z + 2)(z + 3)}$  for  $|z| > 3$ . (7M)
6. a) Evaluate  $\int_C \frac{dz}{\sinh z}$ , where C is the circle  $|z| = 4$ , using residue theorem. (8M)
- b) Use the method of contour integration to evaluate  $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)^3} dx$  (7M)
7. a) State and prove Liouville's theorem. (8M)
- b) Determine the poles of the function  $f(z) = \frac{z^2}{(z - 1)^2(z + 1)}$  and the residue at each pole. Hence evaluate  $\oint_C f(z) dz$ , where C is the circle  $|z| = 2.5$ . (7M)
8. a) Find the image of the lines  $x = -3$  in the z-plane under the transformation  $w = z^2$ . (8M)
- b) Determine the bilinear transformation that maps the points  $1 - 2i$ ,  $2 + i$ ,  $2 + 3i$ , respectively into  $2 + 2i$ ,  $1 + 3i$ ,  $4$ . (7M)