

Subject Code: R13202/R13

Set No - 1

I B. Tech II Semester Supplementary Examinations April/May - 2017

MATHEMATICS-III

(Common to All Branches)

Time: 3 hours

Max. Marks: 70

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**

PART A

1. a) Reduce the matrix $\begin{pmatrix} 5 & 3 & 4 \\ 2 & 2 & 1 \\ 1 & -1 & 2 \end{pmatrix}$ into Echelon form and hence find its rank.
 - b) If λ is an eigen value of A , then prove that the eigen value of $B = a_0 A^2 + a_1 A + a_2 I$ is $a_0 \lambda^2 + a_1 \lambda + a_2$.
 - c) Evaluate $\iiint_V (xy + yz + zx) dV$ where V is the region of space bounded by $x = 0, x = 1, y = 0, y = 2, z = 0, z = 3$.
 - d) Evaluate $\int_0^{\frac{\pi}{2}} \sin^{\frac{7}{2}} \theta \cos^{\frac{3}{2}} \theta d\theta$.
 - e) If $\vec{F} = xy^2 \vec{i} + 2x^2 yz \vec{j} - 3yz^2 \vec{k}$ find $\text{div} \vec{F}$ at $(1, -1, 1)$.
 - f) Find work done by a force $\vec{F} = (x^2 - y^2 + x) \vec{i} - (2xy + y) \vec{j}$ which moves a particle in xy -plane from $(0, 0)$ to $(1, 1)$ along the parabola $y^2 = x$.
- (4M+3M+4M+3M+4M+4M)

PART B

2. a) Find the rank of the matrix by reducing it to normal form $\begin{bmatrix} 1 & 2 & 2 & 4 \\ 2 & 3 & 4 & 6 \\ 3 & 5 & 6 & 10 \\ -1 & 1 & -2 & -2 \end{bmatrix}$.
 - b) Using Gauss Seidel method to solve $27x + 6y - z = 85, 6x + 15y + 2z = 72, x + y + 54z = 110$.
- (8M+8M)
3. a) Find the eigenvalues and the corresponding eigen vectors of $\begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$.
 - b) Reduce the quadratic form $x^2 + 4y^2 + z^2 + 4xy + 6yz + 2zx$ to canonical form. Also find signature and rank of the quadratic form.
- (8M+8M)

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4. a) Find the length of the curve $3x^2 = y^3$ between $y=0$ and $y=1$.

b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2} dy dx$. (8M+8M)

5. a) Evaluate $\int_0^{\frac{\pi}{2}} \sin^5 \theta \cos^{\frac{7}{2}} \theta d\theta$ using Beta and Gamma functions.

b) Show that $B(m, \frac{1}{2}) = 2^{2m-1} B(m, m)$. (8M+8M)

6. a) Find the angle of intersection of the spheres $x^2 + y^2 + z^2 = 39$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z + 52 = 0$ at the point $(4, -3, 2)$.

b) Prove that $\text{curl}(\vec{a} \times \vec{b}) = \vec{a} \text{div} \vec{b} - \vec{b} \text{div} \vec{a} + (\vec{b} \cdot \nabla) \vec{a} - (\vec{a} \cdot \nabla) \vec{b}$. (8M+8M)

7. a) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = 3xy\vec{i} - y^2\vec{j}$ and C is the curve $y = 2x^2$ in xy-plane from $(0, 0)$ to $(1, 2)$.

b) Evaluate $\iint_S \vec{F} \cdot \vec{n} ds$ where $\vec{F} = 12x^2 y\vec{i} - 3yz\vec{j} + 2z\vec{k}$ and S is the portion of the plane $x + y + z = 1$ included in the first octant. (8M+8M)