Subject Code: R13202/R13 **Set No - 1**

I B. Tech II Semester Supplementary Examinations April/May - 2017

MATHEMATICS-III

(Common to All Branches)

Time: 3 hours **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**

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PART A

1. a) Reduce the matrix $\begin{pmatrix} 5 & 3 & 4 \\ 2 & 2 & 1 \\ 1 & -1 & 2 \end{pmatrix}$ into Echelon form and hence find its rank.

b) If λ is an eigen value of A, then prove that the eigen value of $B = a_0A^2 + a_1A + a_2I$ is $a_0\lambda^2 + a_1\lambda + a_2$.

c) Evaluate $\iiint_V (xy + yz + zx)dV$ where V is the region of space bounded by $x = 0, x = 1, y = 0, y = 2, z = 0, z = 3$.

d) Evaluate $\int_0^{\frac{\pi}{2}} \sin^{\frac{7}{2}}\theta \cos^{\frac{3}{2}}\theta d\theta$.

e) If $\overline{F} = xy^2\overline{i} + 2x^2yz\overline{j} - 3yz^2\overline{k}$ find $div\overline{F}$ at $(1,-1,1)$.

f) Find work done by a force $\overline{F} = (x^2 - y^2 + x)\overline{i} - (2xy + y)\overline{j}$ which moves a particle in xy – plane from $(0,0)$ to $(1,1)$ along the parabola $y^2 = x$.

(4M+3M+4M+3M+4M+4M)

PART B

2. a) Find the rank of the matrix by reducing it to normal form $\begin{bmatrix} 1 & 2 & 2 & 4 \\ 2 & 3 & 4 & 6 \\ 3 & 5 & 6 & 10 \\ -1 & 1 & -2 & -2 \end{bmatrix}$.

b) Using Gauss Seidel method to solve 27x + 6y -z = 85, 6x + 15y +2 z = 72, x + y +54z = 110.

(8M+8M)

3. a) Find the eigenvalues and the corresponding eigen vectors of $\begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$.

b) Reduce the quadratic form $x^2 + 4y^2 + z^2 + 4xy + 6yz + 2zx$ to canonical form. Also find signature and rank of the quadratic form.

(8M+8M)

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4. a) Find the length of the curve $3x^2 = y^3$ between y=0 and y=1.

   b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2 - x^2 - y^2}\,dydx$. (8M+8M)

5. a) Evaluate $\int_0^{\frac{\pi}{2}} \sin^5\theta \cos^{7/2}\theta d\theta$ using Beta and Gamma functions.

   b) Show that $B(m, ½) = 2^{2m-1} B(m,m)$. (8M+8M)

6. a) Find the angle of intersection of the spheres $x^2 + y^2 + z^2 = 39$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z + 52 = 0$ at the point (4, -3, 2).

   b) Prove that curl $(\bar{a} \times \bar{b}) = \bar{a} div \bar{b} - \bar{b} div \bar{a} + (\bar{b}.\nabla)\bar{a} - (\bar{a}.\nabla)\bar{b}$. (8M+8M)

7. a) Evaluate $\int_C \bar{F}.dr$ where $\bar{F} = 3xy\bar{i} - y^2\bar{j}$ and C is the curve $y = 2x^2$ in xy-plane from (0, 0) to (1, 2).

   b) Evaluate $\iint_S \bar{F}.\bar{n}ds$ where $\bar{F} = 12x^2 y\bar{i} - 3yz\bar{j} + 2z\bar{k}$ and S is the portion of the plane $x + y + z = 1$ included in the first octant. (8M+8M)