Code No: R161202 **R16** SET - 1

**I B. Tech II Semester Regular Examinations, April/May - 2017**
**MATHEMATICS-II (MM)**
(Com. to CE, EEE, ME, CHEM, AE, BIO, AME, MM, PE, PCE, MET)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answer **ALL** the question in **Part-A**
3. Answer any **FOUR** Questions from **Part-B**

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## PART –A

1. a) Explain the Bisection method. (2M)
   b) Prove that $\Delta = E - 1$. (2M)
   c) Write Newton's forward interpolation formula. (2M)
   d) Write Trapezoidal rule and Simpson's 3/8th rule. (2M)
   e) Write the Fourier series for $f(x)$ in the interval $(0, 2\pi)$. (2M)
   f) Write One dimensional wave equation with boundary and initial conditions. (2M)
   g) If $F(s)$ is the complex Fourier transform of $f(x)$ , then prove that (2M)
   $F\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$.

## PART –B

2. a) Using bisection method, obtain an approximate root of the equation $x^3 - x - 1 = 0$. (7M)
   b) Develop an Iterative formula to find the square root of a positive number $N$ using Newton-Raphson method. (7M)

3. a) Evaluate $\Delta^2 \left(\tan^{-1} x\right)$. (6M)
   b) Using Newton's forward formula, find the value of $f(1.6)$, if (8M)

| $x$ | 1 | 1.4 | 1.8 | 2.2 |
|---|---|---|---|---|
| $f(x)$ | 3.49 | 4.82 | 5.96 | 6.5 |
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Code No: R161202 **R16** SET - 1

4\. a) Compute the value of $\int_{0.2}^{1.4}\left(\sin x-\log x+e^{x}\right)dx$ using Simpson's $\frac{3}{8}$ th rule. (7M)

b) Using the fourth order Runge – Kutta formula, find $y(0.2)$ *and* $y(0.4)$ given that $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2},\ y(0)=1$. (7M)

5\. a) Find a Fourier series to represent $f(x)=x-x^2$ in $-\pi\le x\le\pi$ . Hence show that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots\cdots=\frac{\pi^2}{12}$. (7M)

b) Obtain the half range sine series for $f(x)=e^{x}$ *in* $0<x<1$. (7M)

6\. a) Solve by the method of separation of variables $4u_x+u_y=3u$ *and* $u(0,y)=e^{-5y}$. (7M)

b) A tightly stretched string with fixed end points $x=0$ and $x=L$ is initially in a position given by $y=y_0\sin^3\left(\frac{\pi x}{L}\right)$ if it is released from rest from this position, find the displacement $y(x,t)$. (7M)

7\. a) Express the function $f(x)=\begin{cases}1 & for\ |x|\le 1\\ 0 & for\ |x|\ge 1\end{cases}$ as a Fourier integral. Hence evaluate $\int_0^\infty\frac{\sin\lambda\cos\lambda x}{\lambda}d\lambda$. (7M)

b) Find the Fourier transform of $f(x)=\begin{cases}1-x^2 & for\ |x|\le 1\\ 0 & for\ |x|>1\end{cases}$ hence evaluate $\int_0^\infty\left(\frac{x\cos x-\sin x}{x^3}\right)\cos\frac{x}{2}\,dx$. (7M)

Code No: R161202 **R16** SET - 2

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**PART –A**

1. a) Explain the Method of false position. (2M)
   b) Prove that $\nabla = 1 - E^{-1}$. (2M)
   c) Write Newton's backward interpolation formula. (2M)
   d) Write Simpson's 1/3rd and 3/8th rule. (2M)
   e) Write the Fourier series for $f(x)$ in the interval $(0, 2L)$. (2M)
   f) Write the suitable solution of one dimensional wave equation. (2M)
   g) If $F(s)$ is the complex Fourier transform of $f(x)$, then prove that $F\{f(x-a)\} = e^{ias} F(s)$. (2M)

**PART -B**

2. a) Using bisection method, compute the real root of the equation $x^3 - 4x + 1 = 0$. (7M)
   b) Develop an Iterative formula to find the cube root of a positive number $N$ using Newton-Raphson method. (7M)

3. a) Evaluate $\Delta\left(e^x \log 2x\right)$. (6M)
   b) Using Newton's forward formula compute $f(142)$ from the following table: (8M)

| $x$ | 140 | 150 | 160 | 170 | 180 |
|---|---|---|---|---|---|
| $f(x)$ | 3.685 | 4.854 | 6.302 | 8.076 | 10.225 |

4. a) Evaluate, $\int_0^2 e^{-x^2} dx$ by using Trapezoidal rule and Simpson's $\frac{1}{3}$ rd rule taking $h = 0.25$. (7M)
   b) Find the value of $y$ at $x = 0.1$ by Picard's method, given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$. (7M)
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Code No: R161202 **R16** SET - 2

5. a) Given that $f(x) = \begin{cases} -\pi\ , & -\pi < x < 0 \\ x\ , & 0 < x < \pi \end{cases}$ . Find the Fourier series for $f(x)$ . (7M)

Also deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots\cdots = \frac{\pi^2}{8}$.

b) Express $f(x) = x$ as a half-range cosine series in $0 < x < 2$. (7M)

6. a) Solve by the method of separation of variables (7M)
$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given $u(x,0) = 6e^{-3x}$.

b) A string of length $L$ is initially at rest in equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = b\sin^3\left(\frac{\pi x}{L}\right)$. Find displacement $y(x,t)$. (7M)

7. a) Express $f(x) = \begin{cases} 1 & for\ 0 \le x \le \pi \\ 0 & for\ \ x > \pi \end{cases}$ as a Fourier sine integral and hence evaluate (7M)
$\int_0^\infty \frac{1-\cos(\pi\lambda)}{\lambda} \sin(x\lambda)\, d\lambda$.

b) Find the Fourier sine and cosine transform of (7M)
$f(x)=e^{-ax}, a > 0, x > 0$.

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**PART –A**

1. a) Explain the Newton-Raphson method. (2M)
b) Prove that $\delta = E^{1/2} - E^{-1/2}$. (2M)
c) Write Lagrange's interpolation formula for unequal intervals. (2M)
d) Explain Taylor's series method for solving IVP $\frac{dy}{dx} = f(x, y) \;\; with \;\; y(x_0) = y_0$. (2M)
e) Write the Fourier series for $f(x)$ in the interval $(-\pi, \pi)$. (2M)
f) Write the suitable solution of one dimensional heat equation. (2M)
g) If $F(s)$ is the complex Fourier transform of $f(x)$, then prove that (2M)
$F\{ f(x)\cos ax \} = \frac{1}{2}\left[ F(s+a) + F(s-a) \right]$.

**PART –B**

2. a) Using Regula-Falsi method, compute the real root of the equation $x^3 - 4x - 9 = 0$. (7M)
b) Develop an Iterative formula to find $\frac{1}{N}$. Using Newton-Raphson method. (7M)

3. a) Evaluate $\Delta \left( \frac{x^2}{\cos 2x} \right)$. (6M)
b) Compute $f(27)$ Using Lagrange's formula from the following table: (8M)

| $x$ | 14 | 17 | 31 | 35 |
|---|---|---|---|---|
| $f(x)$ | 68.7 | 64.0 | 44.0 | 39.1 |
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Code No: R161202 **R16** SET - 3

4. a) Evaluate $\int_0^{0.6} e^{-x^2}dx$ by using Simpson's $\frac{1}{3}$ rd rule taking seven ordinates. (7M)

b) Given that $\frac{dy}{dx} = 2+\sqrt{xy}$ , $y(1)=1$ . (7M)

Find $y(2)$ in steps of **0.2** using the Euler's method.

5. a) Find the Fourier series for the function $f(x) = \begin{cases} x & , \ 0 \le x \le \pi \\ 2\pi - x , & \pi \le x \le 2\pi \end{cases}$. (7M)

Also deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+ \cdots\cdots = \frac{\pi^2}{8}$.

b) Obtain the Fourier expansion of $f(x) = x\sin x$ as a cosine series in $(0,\pi)$. (7M)

6. Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ in a rectangle in the $xy$-plane, $0 \le x \le a$ *and* $0 \le y \le b$ satisfying the following boundary condition $u(0,y)=0$, $u(a,y)=0$, $u(x,b)=0$ *and* $u(x,0)=f(x)$. (14M)

7. a) Find the Fourier sine transform of the function (7M)

$$f(x) = \begin{cases} x & , 0 < x < 1 \\ 2-x & , 1 < x < 2 \\ 0 & , \ x > 2 \end{cases}.$$

b) Find the Fourier cosine integral and Fourier sine integral of (7M)
$f(x) = e^{-kx}, k > 0$.

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## PART –A

1. a) Explain Iteration method. (2M)
   b) Prove that $\mu = \frac{1}{2}\left(E^{1/2} + E^{-1/2}\right)$. (2M)
   c) Prove that $\Delta^3 y_2 = \nabla^3 y_5$. (2M)
   d) Explain Runge-Kutta method of fourth order for solving IVP $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$. (2M)
   e) Write the Fourier series for $f(x)$ in the interval $(-L, L)$. (2M)
   f) Write the various possible solutions of two-dimensional Laplace equation. (2M)
   g) If $F(s)$ and $G(s)$ are the complex Fourier transform of $f(x)$ and $g(x)$, then prove that $F\{\, a\, f(x) + b\, g(x)\,\} = a\, F(s) + b\, G(s)$. (2M)



## PART –B

2. a) Find a positive real root of the equation $x^4 - x - 10 = 0$ using Newton-Raphson's method. (7M)
   b) Explain the bisection method. (7M)

3. a) Evaluate $\Delta^2(\cos 2x)$. (6M)
   b) Using Newton's backward formula compute $f(84)$ from the following table: (8M)

| $x$ | 40 | 50 | 60 | 70 | 80 | 90 |
|---|---|---|---|---|---|---|
| $f(x)$ | 184 | 204 | 226 | 250 | 276 | 304 |
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Code No: R161202 **R16** SET - 4

4. a) Evaluate $\int_0^1 e^{-x^2} dx$ by using Trapezoidal rule with $n = 10$. (7M)

   b) Obtain Picard's second approximate solution of the initial value problem (7M)
   $\frac{dy}{dx} = \frac{x^2}{y^2+1}$, $y(0) = 0$.

5. a) Obtain the Fourier series $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval 0 < x < 2π. Deduce that (8M)
   $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\cdots = \frac{\pi^2}{6}$.

   b) Express $f(x) = x$ as a half-range cosine series in $0 < x < 2$. (6M)

6. a) Solve by the method of separation of variables (7M)
   $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ *and* $u(0, y) = 8e^{-3y}$.

   b) Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in the $xy$-plane, $0 \le x \le a$ *and* $0 \le y \le b$ satisfying the following boundary condition $u(x,0) = 0$, $u(x,b) = 0$, $u(0, y) = 0$ *and* $u(a, y) = f(y)$. (7M)

7. a) Find the Fourier cosine integral and Fourier sine integral of (7M)
   $f(x) = e^{-ax} - e^{-bx}$, $a > 0, b > 0$.

   b) Find the Fourier transform of $e^{-a^2x^2}$, $a > 0$. Hence deduce that $e^{-\frac{x^2}{2}}$ is self reciprocal in respect of Fourier transform. (7M)