Code No: R161203 **R16** SET - 1

I B. Tech II Semester Regular Examinations, April/May – 2017
MATHEMATICS-III
(Com. to CE, EEE, ME, ECE, CSE, CHEM, EIE, IT, ECC, AE, AME, MM, PE, PCE, MET, AGE)
Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is Compulsory
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Find the rank of a matrix $A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 2 & 1 & -1 & 0 \\ 3 & 2 & 1 & 7 \end{bmatrix}$ (2M)

b) Prove that if λ is an eigen value of a matrix A then λ^{-1} is an eigen value of the matrix A^{-1} if it exists. (2M)

c) Evaluate $\int_0^1 \int_0^1 \int_{\sqrt{x^2+y^2}}^{y} xyz\, dzdydx$. (2M)

d) Find the value of $\Gamma\left(\frac{5}{2}\right)$. (2M)

e) Find the angle between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2, -1, 2). (2M)

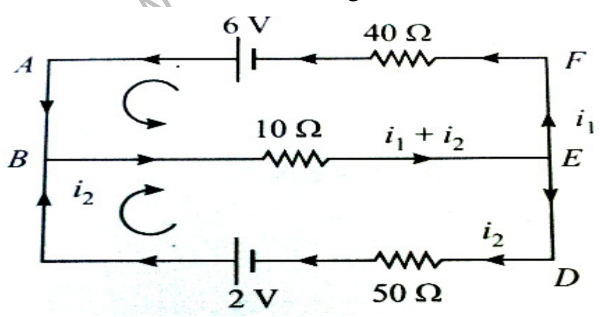
f) If $\bar{F} = (5xy - 6x^2)\bar{\imath} + (2y - 4x)\bar{\jmath}$ then evaluate $\int \bar{F}.\overline{dR}$ along the curve $y = x^3$ from the point (1, 1) to (2, 8). (2M)

g) Write the quadratic form corresponding to the symmetric matrix $\begin{bmatrix} 1 & 0 & 4 \\ 0 & -2 & -1 \\ 4 & -1 & 3 \end{bmatrix}$. (2M)

PART –B

2. a) Solve the system of equations $20x + y - 2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25$ by Gauss Jacobi method. (7M)

b) Find the currents in the following circuit (7M)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Code No: R161203 **R16** SET - 1

3. a) Verify Cayley-Hamilton theorem and find the inverse of the matrix (7M)
$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$
   b) Reduce the quadratic form $2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$ to canonical form by orthogonal transformation and hence find rank, index, signature and nature of the quadratic form. (7M)

4. a) Trace the curve $r^2 = a^2 cos2\theta$. (7M)
   b) Evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} x\, y^2 dydx$ by changing the order of integration. (7M)

5. a) Express $\int_0^1 x^m (1-x^n)^p\, dx$ in terms of Γ functions and hence evaluate $\int_0^1 x^5 (1-x^3)^{10}\, dx$. 6M)
   b) Evaluate $\int_0^{\frac{\pi}{2}} sin^5\theta\, cos^7\theta\ \ d\theta$ by using β, Γ functions. (4M)
   c) Express $\int_0^4 \sqrt{x}(4-x)^{3/2}\, dx$ in terms of β function. (4M)

6. a) Show that the vector field $\bar{F} = (x^2 - yz)\bar{\imath} + (y^2 - zx)\bar{\jmath} + (z^2 - xy)\,\bar{k}$ is conservative and find the scalar potential function corresponding to it. (7M)
   b) Show that $\nabla.(\bar{F} \times \bar{G}) = \bar{G}.(\nabla \times \bar{F}) - \bar{F}.(\nabla \times \bar{G})$ (7M)

7. State Stoke's theorem and verify the theorem for $\bar{F} = (x+y)\bar{\imath} + (y+z)\bar{\jmath} - x\bar{k}$ and S is the surface of the plane $2x + y + z = 2$, which is in the first octant. (14M)

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PART –A

1. a) Determine the rank of a matrix $A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$. (2M)

b) Use Cayley-Hamilton theorem to find A^8 if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. (2M)

c) Evaluate $\int_0^1 \int_0^1 \int_0^y xyz\, dxdydz$. (2M)

d) Find the value of $\Gamma\left(-\frac{5}{2}\right)$. (2M)

e) Find unit normal vector to the surface $x^2y + 2xz^2 = 8$ at the point (1, 0, 2). (2M)

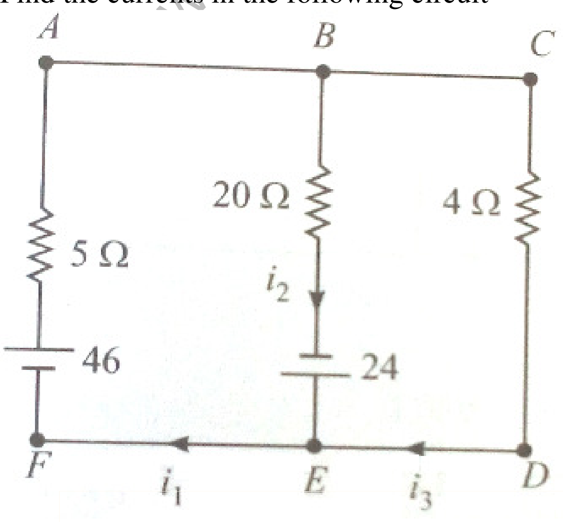
f) If $\bar{F} = (3x^2 + 6y)\bar{\imath} - 14yz\,\bar{\jmath} + 20xz\bar{k}$ then evaluate $\int \bar{F}.\overline{dR}$ from (0, 0, 0) to (1, 1, 1) along the path $x = t, y = t^2, z = t^3$. (2M)

g) Write the quadratic form corresponding to the symmetric matrix
$\begin{bmatrix} 0 & 5/2 & 3 \\ 5/2 & 7 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ (2M)

PART –B

2. a) Show that the system of equations is consistent (7M)
$2x - y - z = 2, x + 2y + z = 2, 4x - 7y - 5z = 2$ and solve.

b) Find the currents in the following circuit (7M)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Code No: R161203 **R16** SET - 2

3. a) Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_1x_3$ to canonical form and hence state nature, rank, index and signature of the quadratic form. (7M)

b) Determine the natural frequencies and normal modes of a vibrating system for which mass $m = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and stiffness $k = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$. (7M)

4. a) Trace the curve $y^2(2a - x) = x^3$. (7M)

b) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dxdy$ by changing in to polar coordinates and hence deduce $\int_0^\infty e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}$. (7M)

5. a) Show that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. 6M)

b) Evaluate $\int_0^{\frac{\pi}{2}} sin^4\theta\, cos^2\theta\ \ d\theta$ by using β, Γ functions. (4M)

c) Express $\int_0^1 \frac{1}{(1-x^3)^{1/3}} dx$ in terms of β function. (4M)

6. a) Show that the vector field $\bar{F} = (x^2 + xy^2)\bar{\imath} + (y^2 + x^2y)\bar{\jmath}$ is conservative and find the scalar potential function. (7M)

b) Show that $\nabla(\nabla.\bar{F}) = \nabla \times (\nabla \times \bar{F}) + \nabla^2\bar{F}$. (7M)

7. State Greens theorem in plane and verify the theorem for $\oint_C [(y - sinx)dx + cosx\, dy]$, where C is the plane triangle formed by the lines $y = 0, x = \frac{\pi}{2}$, $y = \frac{2}{\pi}x$. (14M)

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3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Determine the rank of a matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$. (2M)

b) Use Cayley-Hamilton theorem and find A^{-1} if $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. (2M)

c) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{a\sin\theta} \int_0^{\frac{(a^2-r^2)}{a}} r\, dzdr\, d\,\theta$. (2M)

d) Show that $\beta\left(\frac{1}{2},\frac{1}{2}\right) = \pi$. (2M)

e) Find directional derivative of $\phi = xy^2 + yz^2$ at the point (2,-1,1) in the direction of the vector $\bar{\imath} + 2\bar{\jmath} + 2\bar{k}$. (2M)

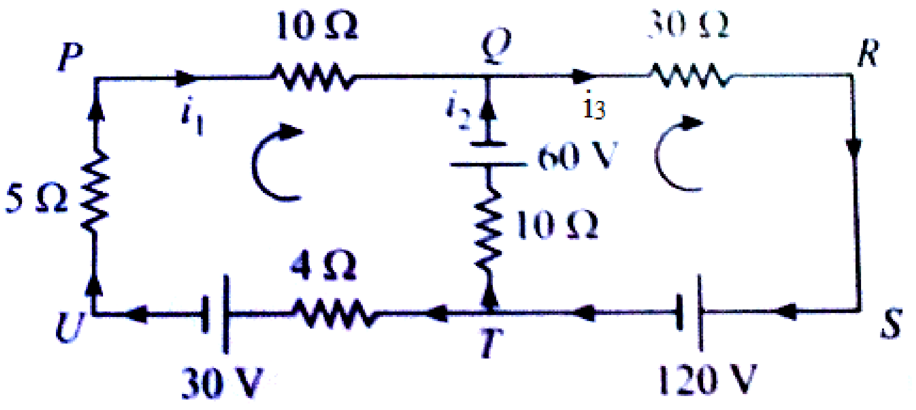
f) If $\bar{F} = (x^2 - y)\bar{\imath} + (2xz - y)\bar{\jmath} + z^2\bar{k}$ then evaluate $\int \bar{F}.\overline{dR}$ where C is the straight line joining the points (0, 0, 0) to (1, 2, 4). (2M)

g) Write the quadratic form corresponding to the symmetric matrix $\begin{bmatrix} 3 & 5 & 0 \\ 5 & 5 & 4 \\ 0 & 4 & 7 \end{bmatrix}$. (2M)

PART –B

2. a) Solve the system of equations $10x + y + z = 12,\ 2x + 10y + z = 13, 2x + 2y + 10z = 14$ by Gauss Seidel method. (7M)

b) Find the currents in the following circuit (7M)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Code No: R161203 **R16** SET - 3

3. a) Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ to canonical form by orthogonal transformation and hence find the rank, index signature and nature of the quadratic form. (7M)

   b) Find the natural frequencies and normal modes of a vibrating system $mx'' + kx = 0$ for mass $m = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and stiffness $k = \begin{bmatrix} 9 & -3 \\ -3 & 3 \end{bmatrix}$. (7M)

4. a) Trace the curve $a^2y^2 = x^2(a^2 - x^2)$. (7M)

   b) Evaluate $\int_0^1 \int_{\sqrt{y}}^{2-y} xy\, dxdy$ by changing the order of integration. (7M)

5. a) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. 6M)

   b) Evaluate $\int_0^{\frac{\pi}{2}} sin^5\theta \; d\theta$ by using β, Γ functions. (4M)

   c) Express $\int_0^1 \frac{x\, dx}{\sqrt{1+x^4}}$ in terms of β function. (4M)

6. a) Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ and $ax^2y + bz^3 = 4$ cut orthogonally at (1,-1,2). (7M)

   b) Show that $\nabla \times (\nabla \times \bar{F}) = \nabla(\nabla . \bar{F}) - \nabla^2 \bar{F}$. (7M)

7. State Gauss divergence theorem in plane and verify the theorem for $\bar{F} = 4xz\bar{\imath} - y^2\bar{\jmath} + zy\bar{k}$ over the cube $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. (14M)

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PART –A

1. a) Find inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ by elementary operations. (2M)

b) Prove that if λ is an eigen value of a matrix A then $\frac{|A|}{\lambda}$ is an eigen value of adjA. (2M)

c) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dzdxdy$. (2M)

d) Determine the value of $\beta(2,3\,)$. (2M)

e) Show that $\nabla fg = f\,\nabla g + g\,\nabla f$. (2M)

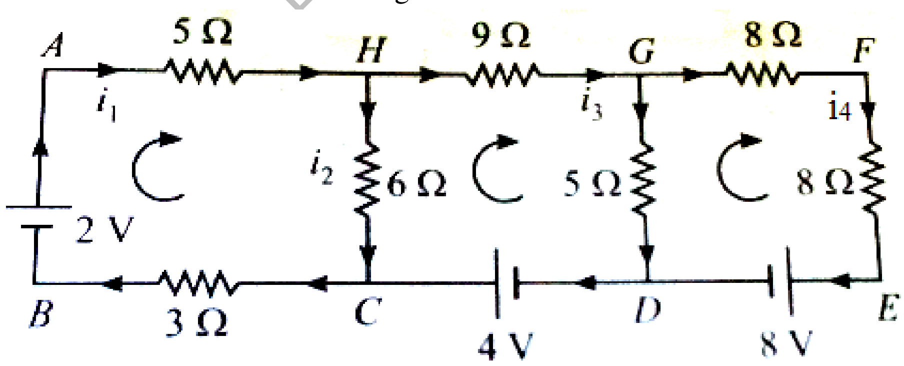
f) If $\bar{F} = x^2y^2\bar{\imath} + y\,\bar{\jmath}$ then evaluate $\int_C \bar{F}.\overline{dR}$ where C is the curve $y^2 = 4x$ in the XY plane from (0, 0) to (4, 4). (2M)

g) Write the quadratic form corresponding to the symmetric matrix (2M)
$\begin{bmatrix} 2 & -3 & 5 \\ -3 & 2 & -2 \\ 5 & -2 & 2 \end{bmatrix}$

PART –B

2. a) Solve the system of equations (7M)
$x + 10y + z = 6,\ \ 10x + y + z = 6, x + y + 10z = 6$ by Gauss Seidel method.

b) Find the currents in the following circuit (7M)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Code No: R161203 **R16** SET - 4

3\. a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence find A^4. (7M)

b) Reduce the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3$ to canonical form and hence state nature, rank, index and signature of the quadratic form. (7M)

4\. a) Trace the curve $r = a\, sin3\theta$. (7M)

b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2+y^2}\; dydx$ by transforming to polar coordinates. (7M)

5\. a) Establish a relation between β and Γ functions. 6M)

b) Evaluate $\int_0^{\frac{\pi}{2}} cos^7\theta\; d\theta$ by using β, Γ functions. (4M)

c) Express $\int_0^1 \frac{x\,dx}{\sqrt{1-x^5}}$ in terms of β function. (4M)

6\. a) Find the angle between the surfaces $ax^2 + y^2 + z^2 - xy = 1$ and conservative $bx^2y + y^2z + z = 1$ at (1, 1, 0). (7M)

b) Show that $\bar{F} = (y^2 - z^2 + 3yz - 2x)\,\bar{\imath} + (3xz + 2xy)\bar{\jmath} + (3xy - 2xz + 2z)\bar{k}$ is both solenoidal and irrotational. (7M)

7\. a) State Greens theorem in plane and apply the theorem to evaluate $\oint_C x^2y\,dx + y^3dy$, where C is the closed path formed by $y = x,\; y = x^3$ from (0, 0) to (1, 1). (7M)

b) Evaluate $\int_S \bar{F}.\overline{ds}$ using Gauss divergence theorem, where $\bar{F} = 2xy\,\bar{\imath} + yz^2\,\bar{\jmath} + z\,\bar{k}$ and S is the surface of the region bounded by $x = 0,\; y = 0,\; z = 0,\; x + 2z = 6$. (7M)