

Code No: R161203

R16

SET - 1

I B. Tech II Semester Regular Examinations, April/May – 2017

MATHEMATICS-III

(Com. to CE, EEE, ME, ECE, CSE, CHEM, EIE, IT, ECC, AE, AME, MM, PE, PCE, MET, AGE)

Time: 3 hours

Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)

2. Answering the question in **Part-A** is Compulsory

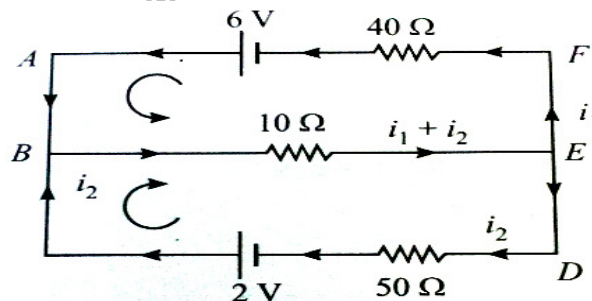
3. Answer any **FOUR** Questions from **Part-B**

PART -A

1. a) Find the rank of a matrix $A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 2 & 1 & -1 & 0 \\ 3 & 2 & 1 & 7 \end{bmatrix}$ (2M)
- b) Prove that if λ is an eigen value of a matrix A then λ^{-1} is an eigen value of the matrix A^{-1} if it exists. (2M)
- c) Evaluate $\int_0^1 \int_0^1 \int_{\sqrt{x^2+y^2}}^y xyz \, dz dy dx$. (2M)
- d) Find the value of $\Gamma\left(\frac{5}{2}\right)$. (2M)
- e) Find the angle between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2, -1, 2). (2M)
- f) If $\vec{F} = (5xy - 6x^2)\vec{i} + (2y - 4x)\vec{j}$ then evaluate $\int \vec{F} \cdot d\vec{R}$ along the curve $y = x^3$ from the point (1, 1) to (2, 8). (2M)
- g) Write the quadratic form corresponding to the symmetric matrix $\begin{bmatrix} 1 & 0 & 4 \\ 0 & -2 & -1 \\ 4 & -1 & 3 \end{bmatrix}$. (2M)

PART -B

2. a) Solve the system of equations $20x + y - 2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25$ by Gauss Jacobi method. (7M)
- b) Find the currents in the following circuit (7M)



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3. a) Verify Cayley-Hamilton theorem and find the inverse of the matrix (7M)
- $$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$
- b) Reduce the quadratic form $2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$ to canonical form by orthogonal transformation and hence find rank, index, signature and nature of the quadratic form. (7M)
4. a) Trace the curve $r^2 = a^2 \cos 2\theta$. (7M)
- b) Evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} x y^2 dy dx$ by changing the order of integration. (7M)
5. a) Express $\int_0^1 x^m (1-x^n)^p dx$ in terms of Γ functions and hence evaluate $\int_0^1 x^5 (1-x^3)^{10} dx$. (6M)
- b) Evaluate $\int_0^{\frac{\pi}{2}} \sin^5 \theta \cos^7 \theta d\theta$ by using β, Γ functions. (4M)
- c) Express $\int_0^4 \sqrt{x}(4-x)^{3/2} dx$ in terms of β function. (4M)
6. a) Show that the vector field $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ is conservative and find the scalar potential function corresponding to it. (7M)
- b) Show that $\nabla \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot (\nabla \times \vec{F}) - \vec{F} \cdot (\nabla \times \vec{G})$ (7M)
7. State Stoke's theorem and verify the theorem for $\vec{F} = (x+y)\vec{i} + (y+z)\vec{j} - x\vec{k}$ and S is the surface of the plane $2x + y + z = 2$, which is in the first octant. (14M)

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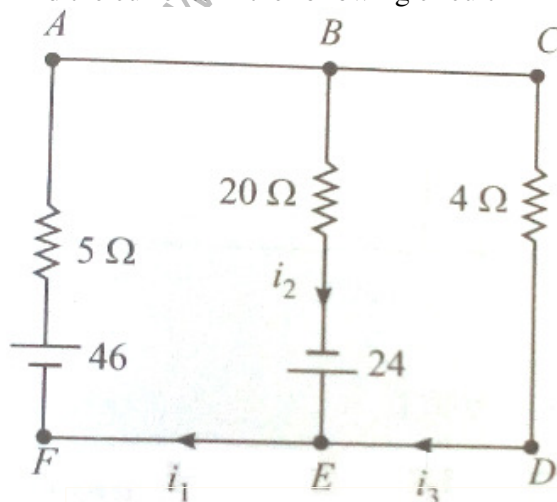
3. Answer any **FOUR** Questions from **Part-B**

PART -A

1. a) Determine the rank of a matrix $A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$. (2M)
- b) Use Cayley-Hamilton theorem to find A^8 if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. (2M)
- c) Evaluate $\int_0^1 \int_0^1 \int_0^y xyz \, dx dy dz$. (2M)
- d) Find the value of $\Gamma\left(-\frac{5}{2}\right)$. (2M)
- e) Find unit normal vector to the surface $x^2y + 2xz^2 = 8$ at the point $(1, 0, 2)$. (2M)
- f) If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz\vec{k}$ then evaluate $\int \vec{F} \cdot d\vec{R}$ from $(0, 0, 0)$ to $(1, 1, 1)$ along the path $x = t, y = t^2, z = t^3$. (2M)
- g) Write the quadratic form corresponding to the symmetric matrix $\begin{bmatrix} 0 & 5/2 & 3 \\ 5/2 & 7 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ (2M)

PART -B

2. a) Show that the system of equations is consistent $2x - y - z = 2, x + 2y + z = 2, 4x - 7y - 5z = 2$ and solve. (7M)
- b) Find the currents in the following circuit (7M)



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SET - 2

3. a) Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_1x_3$ to canonical form and hence state nature, rank, index and signature of the quadratic form. (7M)
- b) Determine the natural frequencies and normal modes of a vibrating system for which mass $m = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and stiffness $k = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$. (7M)
4. a) Trace the curve $y^2(2a - x) = x^3$. (7M)
- b) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing in to polar coordinates and hence deduce $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$. (7M)
5. a) Show that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (6M)
- b) Evaluate $\int_0^\pi \sin^4 \theta \cos^2 \theta d\theta$ by using β, Γ functions. (4M)
- c) Express $\int_0^1 \frac{1}{(1-x^3)^{1/3}} dx$ in terms of β function. (4M)
6. a) Show that the vector field $\vec{F} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$ is conservative and find the scalar potential function. (7M)
- b) Show that $\nabla(\nabla \cdot \vec{F}) = \nabla \times (\nabla \times \vec{F}) + \nabla^2 \vec{F}$. (7M)
7. State Greens theorem in plane and verify the theorem for $\oint_C [(y - \sin x)dx + \cos x dy]$, where C is the plane triangle formed by the lines $y = 0, x = \frac{\pi}{2}, y = \frac{2}{\pi}x$. (14M)

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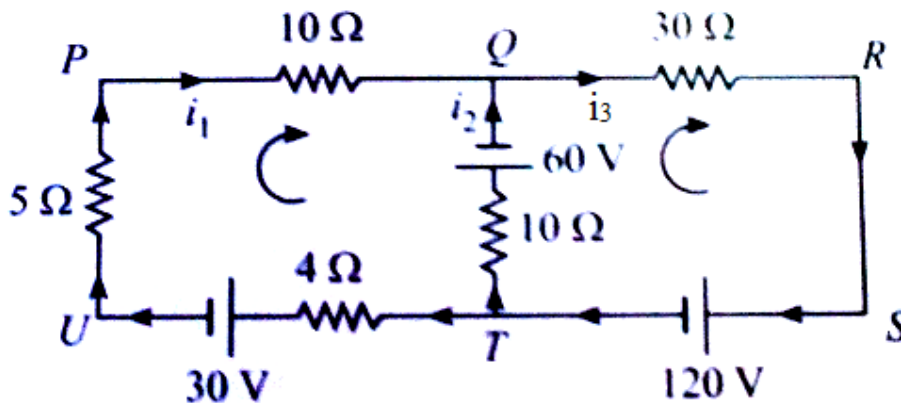
3. Answer any **FOUR** Questions from **Part-B**

PART -A

1. a) Determine the rank of a matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$. (2M)
- b) Use Cayley-Hamilton theorem and find A^{-1} if $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. (2M)
- c) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{a \sin \theta} \int_0^{\frac{(a^2-r^2)}{a}} r \, dz \, dr \, d\theta$. (2M)
- d) Show that $\beta\left(\frac{1}{2}, \frac{1}{2}\right) = \pi$. (2M)
- e) Find directional derivative of $\phi = xy^2 + yz^2$ at the point (2,-1,1) in the direction of the vector $\bar{i} + 2\bar{j} + 2\bar{k}$. (2M)
- f) If $\bar{F} = (x^2 - y)\bar{i} + (2xz - y)\bar{j} + z^2\bar{k}$ then evaluate $\int_C \bar{F} \cdot d\bar{R}$ where C is the straight line joining the points (0, 0, 0) to (1, 2, 4). (2M)
- g) Write the quadratic form corresponding to the symmetric matrix $\begin{bmatrix} 3 & 5 & 0 \\ 5 & 5 & 4 \\ 0 & 4 & 7 \end{bmatrix}$. (2M)

PART -B

2. a) Solve the system of equations $10x + y + z = 12$, $2x + 10y + z = 13$, $2x + 2y + 10z = 14$ by Gauss Seidel method. (7M)
- b) Find the currents in the following circuit (7M)



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SET - 3

3. a) Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ to canonical form by orthogonal transformation and hence find the rank, index signature and nature of the quadratic form. (7M)
- b) Find the natural frequencies and normal modes of a vibrating system (7M)
 $mx'' + kx = 0$ for mass $m = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and stiffness $k = \begin{bmatrix} 9 & -3 \\ -3 & 3 \end{bmatrix}$.
4. a) Trace the curve $a^2y^2 = x^2(a^2 - x^2)$. (7M)
- b) Evaluate $\int_0^1 \int_{\sqrt{y}}^{2-y} xy \, dx \, dy$ by changing the order of integration. (7M)
5. a) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. (6M)
- b) Evaluate $\int_0^{\frac{\pi}{2}} \sin^5 \theta \, d\theta$ by using β, Γ functions. (4M)
- c) Express $\int_0^1 \frac{x \, dx}{\sqrt{1+x^4}}$ in terms of β function. (4M)
6. a) Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ and $ax^2y + bz^3 = 4$ cut orthogonally at (1,-1,2). (7M)
- b) Show that $\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla \cdot \vec{F}) - \nabla^2 \vec{F}$. (7M)
7. State Gauss divergence theorem in plane and verify the theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + zy\vec{k}$ over the cube $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. (14M)

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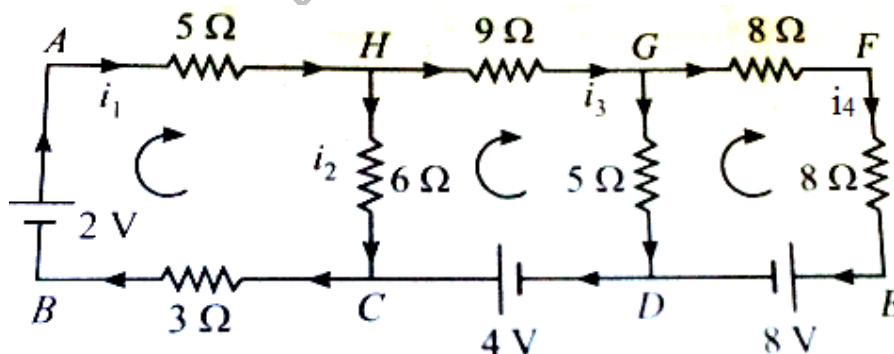
3. Answer any **FOUR** Questions from **Part-B**

PART -A

1. a) Find inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ by elementary operations. (2M)
 - b) Prove that if λ is an eigen value of a matrix A then $\frac{|A|}{\lambda}$ is an eigen value of $\text{adj}A$. (2M)
 - c) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz \, dx \, dy$. (2M)
 - d) Determine the value of $\beta(2, 3)$. (2M)
 - e) Show that $\nabla f g = f \nabla g + g \nabla f$. (2M)
 - f) If $\vec{F} = x^2 y^2 \vec{i} + y \vec{j}$ then evaluate $\int_C \vec{F} \cdot d\vec{R}$ where C is the curve $y^2 = 4x$ in the XY plane from (0, 0) to (4, 4). (2M)
 - g) Write the quadratic form corresponding to the symmetric matrix (2M)
- $$\begin{bmatrix} 2 & -3 & 5 \\ -3 & 2 & -2 \\ 5 & -2 & 2 \end{bmatrix}$$

PART -B

2. a) Solve the system of equations (7M)
- $$x + 10y + z = 6, 10x + y + z = 6, x + y + 10z = 6$$
- by Gauss Seidel method.
- b) Find the currents in the following circuit (7M)



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SET - 4

3. a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence find A^4 . (7M)
- b) Reduce the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3$ to canonical form and hence state nature, rank, index and signature of the quadratic form. (7M)
4. a) Trace the curve $r = a \sin 3\theta$. (7M)
- b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} y \sqrt{x^2+y^2} dy dx$ by transforming to polar coordinates. (7M)
5. a) Establish a relation between β and Γ functions. (6M)
- b) Evaluate $\int_0^{\frac{\pi}{2}} \cos^7 \theta d\theta$ by using β, Γ functions. (4M)
- c) Express $\int_0^1 \frac{x dx}{\sqrt{1-x^5}}$ in terms of β function. (4M)
6. a) Find the angle between the surfaces $ax^2 + y^2 + z^2 - xy = 1$ and conservative $bx^2y + y^2z + z = 1$ at $(1, 1, 0)$. (7M)
- b) Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$ is both solenoidal and irrotational. (7M)
7. a) State Greens theorem in plane and apply the theorem to evaluate $\oint_C x^2y dx + y^3dy$, where C is the closed path formed by $y = x$, $y = x^3$ from $(0, 0)$ to $(1, 1)$. (7M)
- b) Evaluate $\int_S \vec{F} \cdot \vec{ds}$ using Gauss divergence theorem, where $\vec{F} = 2xy\vec{i} + yz^2\vec{j} + z\vec{k}$ and S is the surface of the region bounded by $x = 0$, $y = 0$, $z = 0$, $x + 2z = 6$. (7M)