# I B.Tech II Semester Supplementary Examinations, August 2014 MATHEMATICAL METHODS 

( Common to Mechanical Engineering, Electronics \& Communication Engineering, Chemical Engineering, Bio-Medical Engineering, Information
Technology, Electronics \& Computer Engineering, Mining and Petroliem Technology)
Time: 3 hours
Max Marks: 75
Answer any FIVE Questions
All Questions carry equal marks

1. (a) Find rank of $A=\left[\begin{array}{cccc}2 & 1 & 3 & 1 \\ 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6\end{array}\right]$ by reducing it to Normal Form
(b) Solve by Gauss seidal method, $x+4 y+15 z=24, x+12 y+z=26,10 x+y-2 z=10$
2. Find Eigen vectors of $A=\left[\begin{array}{lll}2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2\end{array}\right]$
3. Reduce the quadratic form $7 x^{2}+6 y^{2}+5 z^{2}-4 x y-4 y z$ to canonical from by diagonlzation. Also find the nature, index and signature and the linear transformation.
4. (a) Solve the equation $x^{3}+2 x^{2}+0.4=0$ using Newton's -Raphson's Method upto three decimal places.
(b) Show that the iteration scheme $\phi(x)=\frac{-1}{x^{2}-3}$ converges and hence find a real root of $f(x)=x^{3}-3 x+1=0$ near $x=3$.
5. (a) Use gauss forward interpolation formula to estimate $f(32)$, given $f(25)=$ $0.2707, f(30)=0.3027, f(35)=0.3386, f(40)=0.3794$.
(b) Find the interpolating polynomial $\mathrm{f}(\mathrm{x})$ from the table given below.

| $x$ | 0 | 1 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{f}(\mathrm{x})$ | 4 | 3 | 24 | 39 |

(a) The velocity v of a particle moving in a straight line covers a distance at time t . They are related as shown in the following Table. Find $\mathrm{v}(\mathrm{x})$ at $\mathrm{x}=10$ and $\mathrm{x}=15$.

| x | 0 | 10 | 20 | 30 | 40 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| v | 45 | 60 | 65 | 54 | 42 |

(b) Find the area bounded by the cure $y=x^{3}-x+1, x-$ axis between $x=0$ and $\mathrm{x}=1.2$ by using
(i) Trapezoidal Rule (ii) Simpson' $1 / 3$ rule.
7. (a) Solve $y^{1}=-x y^{2}, y(o)=2$ by modified Euler's method and hence find $y(o .1)$, $y(0.2)$
(b) Solve $\frac{d y}{d x}=\frac{y^{2}-x^{2}}{y^{2}+x^{2}}, y(0)=1$ by fourth order R-K method and hence find $\mathrm{y}(\mathrm{o} .2)$, $\mathrm{y}(\mathrm{o} .4)$
$[8+7]$
8. (a) Fit a least square parabola $y=a+b x+c x^{2}$ to the data $(-1,2),(0,1),(1,4)$
(b) By the method of least squares fit a straight line to the following data

| x | 5 | 10 | 15 | 15 | 20 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 15 | 19 | 23 | 26 | 30 |

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1. (a) Using Echelon form, find rank of $A=\left[\begin{array}{cccc}1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & 8\end{array}\right]$
(b) Solve system of equations $x+y+z=3,2 x+3 y+2 z=7,4 x+2 y+3 z=9$, using Gauss elimination method.
2. Verify Cayley - Hamilton theorem and find $A=\left[\begin{array}{ccc}1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1\end{array}\right]$
3. (a) Define quadratic form, rank and signature. Write the symmetric matrix corresponding to the quadratic form $x_{1} x_{3}+x_{2} x_{3}+x_{1} x_{3}+x_{1} x_{4}+x_{2} x_{4}+x_{3} x_{4}$.
(b) Discuss the nature of the quadratic form $x^{2}-y^{2}+4 z^{2}+4 x y+6 x z+2 y z[7+8]$
4. (a) Using Newton-Raphson's Method, find a positive root of $\operatorname{Cosx}-\mathrm{xe}^{x}=0$
(b) Find a real root of $f(x)=x+\tan x-1=0$ in the interval $(0,0.5)$ by using bisection method.
5. The following table gives the population of a town during the last six censuses. Estimate, using Newton's interpolation formula, the increase in the population during the period 1986, to 1988.

| year | 1911 | 1921 | 1931 | 1941 | 1951 | 1961 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Population (in thousands) | 12 | 15 | 20 | 27 | 39 | 52 |

(a) Compute $f^{\prime}(1)$ using the given data:

| X | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{f}(\mathrm{x})$ | 27 | 106.75 | 324 | 783.75 | 1621 |

(b) Using Simpson's $3 / 8^{\text {th }}$ rule evaluate $\int_{0}^{6} \frac{d x}{1+x^{2}}$ by dividing the range into 6 equal parts
7. (a) Solve $y^{1}=3 x^{2}+1$ by Euler's method and find $y$ at $x=2$ by taking $h=0.5$
(b) Solve by fourth order R-K method $y^{1}=x-y, y(1)=0.4$ and hence find $y(1.2)$
8. (a) Fit a second degree polynomial to the following data by the method of least squares

| x | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 1 | 1.8 | 1.3 | 2.5 | 6.3 |

(b) Fit a straight line of the form $y=a+b x$ to the following data

| x | 0 | 5 | 10 | 15 | 20 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 12 | 15 | 17 | 22 | 24 | 30 |

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1. (a) Find rank of matrix using Echelon form $A=\left[\begin{array}{cccc}1 & 2 & -4 & 5 \\ 2 & -1 & 3 & 6 \\ 8 & 1 & 9 & 7\end{array}\right]$
(b) Solve the equations using Gauss Jordan method $\mathrm{x}+5 \mathrm{y}+\mathrm{z}=9, \quad 2 \mathrm{x}+\mathrm{y}+3 \mathrm{z}=12, \quad 3 \mathrm{x}+\mathrm{y}+4 \mathrm{z}=16$
2. Verify Cayley - Hamilton theorem and find $A$ if $A=\left[\begin{array}{lll}2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2\end{array}\right]$
3. (a) Define quadratic form, rank and signature. Write the symmetric matrix corresponding to the quadratic form $x_{1} x_{3}+x_{2} x_{3}+x_{1} x_{3}+x_{1} x_{4}+x_{2} x_{4}+x_{3} x_{4}$.
(b) Discuss the nature of the quadratic form $x^{2}-y^{2}+4 z^{2}+4 x y+6 x z+2 y z[7+8]$
4. (a) Solve the equation $x^{3}+2 x^{2}+0.4=0$ using Newton's -Raphson's Method upto three decimal places.
(b) Find a real rootof $x=e^{-x}$, using Bisection method up to four iterations.
5. (a) Find the value of y from the following data at $\mathrm{x}=0.47$

| $X: X$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $Y:$ | 1 | 2 | 4 | 7 | 11 | 16 |

(b) Use Lagrange's interpolation formula, find $\mathrm{f}(4)$ from the following data.

| X | 1 | 2 | 5 | 6 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Y}=\mathrm{f}(\mathrm{x})$ | 2 | 8 | 17 | 20 | 35 |

6. (a) For the function $y=f(x)$ given by the following Table, find $y^{\prime}$ at $\mathrm{x}=0.04$ using the Bessel's formula.

| x | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 0.1023 | 0.1047 | 0.1071 | 0.1096 | 0.1122 | 0.1148 |

(b) Evaluate $\int_{0}^{4} e^{1 / x} d x$ by using the Simpson's $3 / 8^{\text {th }}$ rule, by dividing the interval into 3 equal parts.

$$
[8+7]
$$

7. (a)Solve $y^{1}=y+e^{x}, y(o)=0$ by modified Euler's method and find $y(? ?), y(? ?)$
(b) Solve $\mathrm{y}^{1}=-\mathrm{xy}^{2}, \mathrm{y}(\mathrm{o})=2, \mathrm{~h}=0.2$ by R-K method and hence find $\mathrm{y}(\mathrm{o} .2), \mathrm{y}(\mathrm{o} .4)$
8. (a) Find the best fit of the type $y=a e^{b x}$ to the data by the method of least squares

| x | 1 | 5 | 7 | 9 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 10 | 15 | 12 | 15 | 21 |

(b) Obtain the relation of the form $\mathrm{y}=\mathrm{ab}^{x}$ to the following data by the method of least squares

| x | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 8.3 | 15.4 | 33.1 | 65.2 | 127.4 |

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1. (a) Find rank of a Matrix using Echelon form where $A=\left[\begin{array}{cccc}1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 5 & 3 & 14 & 4\end{array}\right]$
(b) Show that equations $\mathrm{x}+\mathrm{y}+\mathrm{z}=6, \mathrm{x}+2 \mathrm{y}+3 \mathrm{z}=14 \mathrm{x}+4 \mathrm{y}+7 \mathrm{z}=30$ are consistent and solve them
2. Verify Cayley - Hamilton theorem and find $A^{4}$ if $A=\left[\begin{array}{ccc}1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1\end{array}\right]$
3. Find the transformation which will transform $4 x^{2}+3 y^{2}+z^{2}-8 x y-6 y z+4 z x$ into a sum of square and find the reduced from
4. (a) Using Newton-Raphson's method find the square root of a number and hence find the square root of 24 .
(b) Find a real root of the equation $\mathrm{x}=\mathrm{e}^{-x}$, using Bisection method $[8+7]$
5. (a) Find the value of y from the following data at $\mathrm{x}=0.47$

| $\mathrm{X}:$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Y}:$ |  | 1 | 2 | 4 | 7 | 11 |

(b) Use Lagrange's interpolation formula, find $\mathrm{f}(4)$ from the following data.

| $x$ | 1 | 2 | 5 | 6 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $x-f(x)$ | 2 | 8 | 17 | 20 | 35 |

6. (a) From the following data find $\mathrm{f}{ }^{\prime}(0)$

| x | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{f}(\mathrm{x})$ | 43 | 40 | 38 | 42 | 45 | 50 |

(b) By considering 4 strips, find the value of $\int_{3}^{7} x^{2} \log x d x$
7. (a) Solve $\mathrm{y}^{1}=\mathrm{xy}^{1 / 3}, \mathrm{y}(1)=1$ by Taylor series method and find $\mathrm{y}(1.1), \mathrm{y}(1.2)$
(b) Find an approximate value of $y$ for $x=0.1,0.2$ if $y^{1}=x+y$ and $y(1)=1$ by Picard's method and compare the solution with exact solution.
8. (a) Fit a second degree polynomial to the following data by the method of least squares

| x | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 1 | 1.8 | 1.3 | 2.5 | 6.3 |

(b) Fit a straight line of the form $\mathrm{y}=\mathrm{a}+\mathrm{bx}$ to the following data

| x | 0 | 5 | 10 | 15 | 20 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 12 | 15 | 17 | 22 | 24 | 30 |



