

**I B.Tech I Semester Supplementary Examinations, Feb/Mar 2014
MATHEMATICS-I**

(Common to Civil Engineering, Electrical & Electronics Engineering,
Mechanical Engineering, Electronics & Communication Engineering,
Computer Science & Engineering, Chemical Engineering, Electronics &
Instrumentation Engineering, Bio-Medical Engineering, Information
Technology, Electronics & Computer Engineering, Aeronautical
Engineering, Bio-Technology, Automobile Engineering, Mining and
Petroleum Technology)

Time: 3 hours

Max Marks: 75

**Answer any FIVE Questions
All Questions carry equal marks**

1. (a) Solve $(x^2 + y^2 - a^2)x dx + (x^2 - y^2 - b^2)y dy = 0$. [7+8]
(b) If air is maintained at $20^\circ C$ and the temperature of the body cools from $80^\circ C$ to $60^\circ C$ in 10 minutes, find the temperature of the body after 30 minutes.
2. (a) Solve $(D^2 + a^2)y = \sec ax$
(b) Solve $(D^2 + 4)y = e^x + \sin 2x$ [8+7]
3. (a) If $V = \log(x^2 + y^2) + x - 2y$ find $\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial^2 V}{\partial x^2}, \frac{\partial^2 V}{\partial y^2}$.
(b) If $U = xe^{xy}$ where $x^2 + y^2 + 2xy = 1$, find $\frac{\partial^2 U}{\partial x^2}$. [8+7]
4. (a) Trace the curve $r = 2 + 3 \sin \theta$.
(b) Trace the curve $y^2(2a - x) = x^3$. [8+7]
5. (a) Find the surface of the solid generated by revolution of the lemniscate $r^2 = a^2 \cos^2 \theta$ about the initial line.
(b) Show that the whole length of the curve $x^2(a^2 - x^2) = 8a^2y^2$ is $\pi a\sqrt{2}$. [8+7]
6. (a) Show that $\int_0^{4a} \int_{\frac{y^2}{4a}}^y \frac{x^2 - y^2}{x^2 + y^2} dx dy = 8a^2 \left(\frac{\pi}{2} - \frac{5}{3} \right)$.
(b) Evaluate $\iint_R y dx dy$ where R is the domain bounded by y-axis, the curve $y = x^2$ and the line $x + y = 2$ in the first quadrants. [8+7]
7. (a) If $V = e^{xyz}(i+j+k)$, find curl V.
(b) Find the constants a and b so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, -1, 2) [8+7]
8. (a) Show that the area of the ellipse $x^2/a^2 + y^2/b^2 = 1$ is πab
(b) If $f = (2x^2 - 3z)i - 2xyj - 4xzk$, evaluate
(i) $\int_v \nabla \cdot f dV$ and
(ii) $\int_v \nabla \times f dV$ where V is the closed region bounded by $x = 0, y = 0, z = 0, 2x + 2y + z = 4$. [8+7]

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1. (a) Solve $e^y \left(1 + \frac{dy}{dx}\right) = e^x$
 (b) Show that the family of curves $\frac{x^2}{a^2+\lambda} + \frac{y^2}{a^2-\lambda} = 1$, where ' λ ' is a parameter is self orthogonal. [8+7]
2. (a) Solve $(D^2 + 9)y = 2 \cos^2 x$. (b) Solve $\frac{d^2y}{dx^2} + 4y = 2e^x \sin^2 x$. [8+7]
3. (a) Calculate the approximate value of $\sqrt{10}$ to four decimal places using Taylor's theorem.
 (b) Find 3 positive numbers whose sum is 600 and whose product is maximum. [8+7]
4. (a) Trace the curve $y = x^2(x^2 - 4)$. (b) Trace the curve $r = \cos \theta$. [8+7]
5. (a) The figure bounded by a parabola and the tangents at the extremities of its latus rectum revolves about the axis of the parabola, Find the volume of the solid thus generated. [8+7]
 (b) The segment of the parabola $y^2 = 4ax$ which is cutoff by the latus rectum revolves about the directrix. Find the volume of rotation of the annular region.
6. (a) Evaluate $\int \int (x+y)^2 dx dy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
 (b) Transform the following to Cartesian form and hence evaluate $\int_0^\pi \int_0^a r^3 \sin \theta dr d\theta$. [8+7]
7. (a) Prove that $\nabla r = \vec{r}/r$
 (b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, -1, 2). [8+7]
8. (a) Evaluate $\iint_S (yzi + zxj + xyk) \cdot dS$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant.
 (b) Evaluate $\oint_C (x^2 - 2xy)dx + (x^2y + 3)dy$ around the boundary of the region defined by $y^2 = 8x$ and $x = 2$. [8+7]

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1. (a) Solve $y(\sin x - y) dx = \cos x dy$
(b) If the temperature of air is maintained at $20^{\circ}C$ and the temperature of the body cools from $100^{\circ}C$ to $80^{\circ}C$ in 10 minutes, find the temperature of the body after 20 minutes. [8+7]
2. (a) Solve $(D^2 - 4D + 13)y = e^{2x}$
(b) Solve $(D^2 - 3D + 2)y = \cos hx$ [8+7]
3. (a) If $r + s + t = x$, $s + t = xy$, $t = xyz$, find $\frac{\partial(r,s,t)}{\partial(x,y,z)}$.
(b) Find the extreme points of $f(x, y) = xy + \frac{8}{x} + \frac{8}{y}$. [8+7]
4. (a) Trace the curve $y = 5 \cosh\left(\frac{x}{5}\right)$.
(b) Trace the curve $y^2 = (4 - x)(3 - x^2)$.. [8+7]
5. (a) Find the length of the arc of the curve $y = \log(\sec x)$ from $x = 0$ to $\frac{\pi}{3}$.
(b) Find the perimeter of the loop of the curve $3ay^2 = x(x-a)^2$. [8+7]
6. (a) Evaluate $\int \int r dr d\theta$ over the region bounded by the cardioid $r = a(1 + \cos\theta)$ and outside the circle $r = a$.
(b) Change the order of Integration & evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy dx$ [8+7]
7. (a) Prove that $(\mathbf{F} \times \nabla) \times \mathbf{r} = -2\mathbf{F}$
(b) Determine the constant a so that the vector $\mathbf{V} = (x+3y)\mathbf{i} + (y-z)\mathbf{j} + (x+az)\mathbf{k}$ is solenoidal. [8+7]
8. Apply Stokes theorem, to evaluate $\oint_C y dx + z dy + x dz$ where C is the curve of intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and $x + z = a$. [15]

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1. (a) Solve $(x+1) \frac{dy}{dx} - y = e^{3x}(x+1)^2$
 (b) Find the orthogonal trajectory of the family of curves $x^{2/3} + y^{2/3} = a^{2/3}$, where 'a' is a parameter [8+7]
2. (a) Solve $(D^3 - 6D^2 + 11D - 6)y = e^{-2x} + e^{-3x}$
 (b) Solve $\frac{d^2y}{dx^2} - 8 \frac{dy}{dx} + 15y = 0$ [8+7]
3. (a) If $a = \frac{yz}{x}$, $b = \frac{xz}{y}$, $c = \frac{xy}{z}$, find $\frac{\partial(x,y,z)}{\partial(a,b,c)}$.
 (b) Find the minimum value of $x^2 + y^2 + z^2$, give that $xyz = a^3$ [8+7]
4. (a) Trace the curve $r = \cos 4\theta$.
 (b) Trace the curve $y^2(1-x) = x^2(1+x)$. [8+7]
5. Prove that the volume of the solid generated by the revolution about the x -axis of the loop of the curve $x = t^2$, $y = t - \frac{1}{3}t^3$ is $\frac{3\pi}{4}$. [8+7]
6. (a) By changing the order of integration evaluate $\int_0^1 \int_0^{\sqrt{2-x^2}} \frac{x}{x^2+y^2} dy dx$.
 (b) Evaluate $\int_0^a \int_{a-x}^{\sqrt{a^2-x^2}} y \, dx \, dy$ by using change of order of integration . [8+7]
7. (a) If $V = e^{xyz}(i+j+k)$, find curl V.
 (b) Find the constants a and b so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1,-1,2) [8+7]
8. (a) Use divergence theorem to evaluate $\iint_S (x^3i + y^3j + z^3k) \cdot N ds$, and S is the surface of the sphere $x^2 + y^2 + z^2 = r^2$.
 (b) Using Green's theorem, Find the area bounded by the hypocycloid $x^{2/3} + y^{2/3} = a^{2/3}$, $a > 0$. Given that the parametric equations are $x = a \cos^3\theta$, $y = a \sin^3\theta$. [8+7]
