

I B.Tech II Semester Supplementary Examinations, August 2014
MATHEMATICS- II

(Common to Civil Engineering, Electrical & Electronics Engineering,
 Mechanical Engineering, Electronics & Communication Engineering,
 Computer Science & Engineering, Chemical Engineering, Electronics &
 Instrumentation Engineering, Bio-Medical Engineering, Information
 Technology, Electronics & Computer Engineering, Aeronautical
 Engineering, Bio-Technology, Automobile Engineering, Mining and
 Petroleum Technology)

Time: 3 hours

Max Marks: 75

Answer any FIVE Questions
 All Questions carry equal marks

1. (a) Find the Laplace transform of $\cos^3 3t$
 (b) Find the Laplace transform of $f(t) = |t - 1| + |t + 1|, t \geq 0$ [7+8]
2. (a) Find $L^{-1}[s^2/(s^2 + 4)(s^2 + 25)]$.
 (b) Find $L^{-1}\{\tan^{-1}(s + 1)\}$. [7+8]
3. (a) Find cosine series and sine series for the function $f(x) = x^2$ in $(0, \pi)$ find sum of the series $1/1^2 - 1/2^2 + 1/3^2 - \dots$.
 (b) Find cosine series and sine series for the function $f(x) = \pi - x$ in $(0, \pi)$ [8+7]
4. Find the finite Fourier sine and cosine transform of $f(x) = \frac{\pi}{3} - x + \frac{x^2}{2\pi}$ where $0 < x < \pi$ [15]
5. (a) Solve $x^2 p + y^2 q = xz$
 (b) Solve $(3y + 2z)p + (4z - 3x)q = -(2x + 4y)$ [8+7]
6. A homogenous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is $u(x, 0) = \begin{cases} x; & 0 \leq x \leq 50 \\ 100 - x; & 50 \leq x \leq 100 \end{cases}$.
 Find the temperature $u(x, t)$ at any time. [15]
7. (a) Find the inverse Z-transform of $\frac{z^2}{(z - \frac{1}{2})(z - \frac{1}{4})}$
 (b) Evaluate $Z^{-1}\left(\frac{z}{z^2 + 11z + 24}\right)$. [8+7]
8. (a) Evaluate $\int_0^1 x^3 \sqrt{1-x} dx$, using Beta and Gamma functions.
 (b) Prove that $\int_0^\infty x^{2n-1} e^{-ax^2} dx = \frac{\Gamma(n)}{2a^n}, a > 0, n > 0$. [8+7]

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- (a) Find the Laplace transform of
 (i) $\cos (at+b)$
 (ii) $\sin(at-b)$
 (b) Find the Laplace transform of
 (i) $e^{4t} \cos 4t$
 (ii) $t^4 e^{5t}$ [7+8]
- (a) Find $L^{-1} \left\{ \frac{(s^2-1)}{(s^2+1)^2} \right\}$.
 (b) Using Laplace transforms, solve $(D^2 + 1)x = t \cos 2t$, given that
 $x = \frac{dx}{dt} = 0$ at $t = 0$. [7+8]
- Find the half range Fourier cosine series of $f(x) = \sin(\pi x/L)$ in the range $0 < x < L$ [15]
- (a) Find the fourier transform of $f(x)$ defined by $f(x) = x, |x| = a = 0, |x| > a$
 (b) Find the fourier transform of $f(x)$ defined by $f(x) = \cos x, |x| = a, f(x) = 0,$
 $|x| = a > 0$ [8+7]
- (a) Solve $p^2 + q^2 = x^2 + y^2$
 (b) Solve $yz p - xz q = xy$ [8+7]
- A bar 100 cm long with insulated sides has its ends kept at 0°C and 100°C respectively until steady state conditions prevail. Then the temperature 0°C is raised to 30°C and that of 100°C is lowered to 60°C . Find the subsequent temperature distribution. [15]
- (a) Find the inverse Z-transform of $\frac{z^2}{(z-\frac{1}{2})(z-\frac{1}{4})}$
 (b) Evaluate $Z^{-1} \left(\frac{z}{z^2+11z+24} \right)$. [8+7]
- (a) Prove that $\Gamma(m)\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$
 (b) Prove that $\int_0^\infty x^n e^{-a^2 x^2} dx = \frac{1}{2a^{n+1}}\Gamma\left(\frac{n+1}{2}\right), n > -1$ [8+7]

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1. (a) Find the Laplace transform of
 - (i) $\cosh(at+b)$
 - (ii) $\sinh(at-b)$
 (b) Find the Laplace transform of
 - (i) $e^{4t} \cos 4t$
 - (ii) $t^4 e^{5t}$[7+8]
2. (a) Find $L^{-1} \left[\log \left(\frac{s^2+1}{s^2+9} \right) \right]$.
 (b) Find inverse Laplace transform of $\frac{1}{s(s^2-a^2)}$ [7+8]
3. (a) Express $f(x)=x$ as a half range Fourier cosine series in $(-\pi, \pi)$
 (b) Express $f(x)=x^2$ as a Fourier series in $(-\pi, \pi)$ [8+7]
4. Find the fourier cosine and sine transform of $f(x)$ defined by $f(x) = e^{-ax}/x$ and hence evaluate that $\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x} \sin sx \, dx$ [15]
5. (a) Form the partial differential equation by eliminating arbitrary function from $f(x^2+y^2, x^2-z^2)=0$
 (b) Solve $2p + 3q = 1$ [8+7]
6. The ends A and B of a rod 20cm long have the temperature at 30°C and 80°C until steady states prevail. The temperatures of the ends are changed at 40°C and 60°C respectively. Find the temperature distribution in the rod at time t . [15]
7. (a) Find the inverse Z-transform of $\frac{z^2}{(z-\frac{1}{2})(z-\frac{1}{4})}$
 (b) Evaluate $Z^{-1} \left(\frac{z}{z^2+11z+24} \right)$. [8+7]
8. (a) Show that $\Gamma(n) = \frac{1}{n} \int_0^\infty e^{-x} x^{n-1} dx, n > 0$
 (b) Evaluate $\int_0^1 x^7 (1-x)^5 dx$ using Beta and Gamma functions. [8+7]

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1. (a) Find $L(t(3 \sin 2t - 2 \cos 2t))$
 (b) Find $L(t^2 e^{-2t} \cos t)$ [7+8]
2. (a) State and prove Convolution theorem for inverse Laplace transform
 (b) Show that $f * g = g * f$ where $f(t) = t^3$, $g(t) = 2t$ [7+8]
3. The intensity of an alternating current after passing through a rectifier is given by $f(x) = i_0 \sin x$ for $0 = x = \pi$, $f(x) = 0$ for $\pi = x = 2\pi$ where $i_0 = \text{maximum current}$ and the period is 2π . Express $f(x)$ as a Fourier series [15]
4. Find the Fourier cosine and sine transform of $f(x)$ defined by $f(x) = e^{-ax}/x$ and hence evaluate that $\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} \sin sx \, dx$ [15]
5. (a) Solve $p^2 + q^2 = x^2 + y^2$
 (b) Solve $yz \, p - xz \, q = xy$ [8+7]
6. A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is vibrating by giving to each of its points a velocity $\lambda x(l-x)$, find the displacement of the string at any distance 'x' from one end at any time 't'. [15]
7. (a) If $Z(u_n) = \bar{u}(z)$, then prove that $Z(n u_n) = -z \frac{d}{dz} [\bar{u}(z)]$
 (b) Evaluate $Z^{-1} \left(\frac{z^2}{z^2 - 5z + 6} \right)$, using convolution theorem. [8+7]
8. (a) Given $\int_0^\infty \frac{x^{n-1}}{1+x} dx = \frac{\pi}{\sin n\pi}$, show that $\Gamma(n) \Gamma(1-n) = \frac{\pi}{\sin n\pi}$ and deduce $\Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{3}{4}\right)$
 (b) Prove that $\int_0^{\pi/2} [\sqrt{\tan \theta} + \sqrt{\sec \theta}] d\theta = \frac{1}{2} \Gamma\left(\frac{1}{4}\right) \left\{ \Gamma\left(\frac{3}{4}\right) + \frac{\sqrt{\pi}}{\Gamma\left(\frac{3}{4}\right)} \right\}$ [8+7]
