**Subject Code: R13202/R13** **Set No - 1**

**I B. Tech II Semester Regular Examinations August - 2014**

# MATHEMATICS-III

**(Common to All Branches)**

**Time: 3 hours** **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**

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## PART-A

1.(i) Write down the properties of orthogonal matrix.

(ii) Write the nature of $2y_1^2 + 4y_2^2 + 5y_3^2$.

(iii) If A and B are non-singular matrices of same order, show that AB and BA have same eigen values.

(iv) Find the area of loop of the curve $r^2 = a^2 \cos 2\theta$

(v) Find the moment of inertia of a circle A of radius R relative to the centre O.

(vi) Evaluate $\int_0^\infty \frac{x^6(1-x^{10})dx}{(1+x)^{24}}$

(vii) If $\boldsymbol{F}$ is a conservative vector field show that curl $\boldsymbol{F}= 0$.

(viii) Write down the physical interpretation of Green's theroem.

[3+3+3+3+3+2+3+2]

## PART – B

2.(a) Reduce the matrix $\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & 5 \\ 1 & 3 & 2 & 0 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ to normal form and find its rank.

(b) Solve, by Gauss-Seidal method, the equations

$$9x - 2y + z - t = 50$$
$$x - 7y + 3z + t = 20$$
$$-2x + 2y + 7z + 2t = 22$$
$$x + y - 2z + 6t = 18 .$$

[8+8]

3. Diagonalise the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and hence find $A^4$.

[16]

4.(a) Find the volume of solid generated by the revolution of the cardioid $r = a(1+\cos\theta)$ about $\theta = 0$.

(b) Evaluate $\iint_R \left(\sqrt{xy} - y^2\right) dxdy$ where R is triangle with vertices at (0,0), (10,1), (1,1).

[8+8]

5.(a) Show that $\int_0^1 x^3 \left[\log\left(\frac{1}{x}\right)\right]^4 dx = \frac{3}{128}$ .

(b) Prove that $\int_0^4 \sqrt{x}(4-x)^{\frac{3}{2}}\, dx = 64\beta(\frac{3}{2}, \frac{5}{2})$.

[8+8]

6.(a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, -1, 2)

(b) Prove that $\nabla\left[\nabla.\frac{\vec{r}}{r}\right] = \frac{-2}{r^3}\vec{r}$

[8+8]

7.(a) Use Stokes theorem to evaluate the integral $\int_C \boldsymbol{A.dr}$ where $\boldsymbol{A} = 2\mathrm{y}^2 i + 3\mathrm{x}^2 j - (2x + z)k$, and C is the boundary of the triangle whose vertices are (0, 0, 0), (2, 0, 0), (2, 2, 0)

(b) Find the workdone in moving a particle in the force field $\boldsymbol{F} = 3\mathrm{x}^2 i + j + zk$ along the straight line from (0, 0, 0) to (2, 1, 3)

[8+8]

**Subject Code: R13202/R13** **Set No - 2**

**I B. Tech II Semester Regular Examinations August - 2014**

# MATHEMATICS-III

**(Common to All Branches)**

**Time: 3 hours** **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**

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## PART-A

1.(i) Express $\begin{bmatrix} 3 & 7 \\ 4 & 5 \end{bmatrix}$ as sum of a symmetric and skew-symmetric matrices.

(ii) When does a non homogeneous system consistent?

(iii) Define the latent root and latent vector.

(iv) Find the volume of a sphere of radius 'a'.

(v) Find the moment of inertia of a hallow sphere about a diameter. Its external and internal radii being 5 meters and 4 meters.

(vi) Evaluate $\int_0^\infty \sqrt{xe^{-x^3}}\, dx$

(vii) If $\boldsymbol{A}$ is a vector function, find Div (Curl $\boldsymbol{A}$ )

(viii) Write down the physical interpretation of Stoke's theroem.

[3+2+3+3+3+3+3+2]

## PART - B

2.(a) Reduce the matrix $\begin{bmatrix} 3 & 1 & 4 & 6 \\ 2 & 1 & 2 & 4 \\ 4 & 2 & 5 & 8 \\ 1 & 1 & 2 & 2 \end{bmatrix}$ to Echelon form and find its rank.

(b) Solve, by LU Decomposition method, the equations

$x + 2y + 3z = 10$
$3x + y + 2z = 13$
$2x + 3y + z = 13.$

[8+8]

3. Verify Cayley-Hamiltion theorem for $A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$ and hence find $A^{-1}$.

[16]

4.(a) Find the length of the loop of the curve $3ay^2 = x(x-a)^2$

(b) Find the volume of the solid generated by the revolution of the cardioid $r = a(1 + cos\theta)$ about the initial line $\theta = 0$.

[8+8]

5.(a) Show that $\int_0^1 [xlog(x)]^3\, dx = \frac{-3}{128}$ .

(b) Evaluate $4\int_0^\infty \frac{x^2 dx}{1+x^4}$ using $\beta - \Gamma$ functions. [8+8]

6. (a) Find the work done in moving a particle in the force field $\boldsymbol{F} = 2x^2 i + (2yz - x)j + yk$ along the space curve $x = 3t^2, y = t, z = 3t^2 - t$ from *t=0 to t=1.*

(b) Prove that $curl\ (a \times b) = a\ div\ b - b\ div\ a + (\vec{b}\ .\nabla)a - (a\ .\nabla)b$

7.(a) Verify the divergence theorem for $\boldsymbol{F} = 4\text{xy}i - \text{y}^2 j + xzk$, over the cube bounded by *x = 0, x = 1, y = 0, y = 1, z=0 and z = 1.*

(b) Evaluate $\iint_S \boldsymbol{A.n}\ ds$ where $\boldsymbol{A}$=*yzi + zxj + xyk* and S is the part of the sphere $x^2 + y^2 + z^2 = 9$ which lies in the first octant.

[8+8]

**Subject Code: R13202/R13**

**Set No - 3**

**I B. Tech II Semester Regular Examinations August - 2014**

# MATHEMATICS-III

**(Common to All Branches)**

**Time: 3 hours** **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**
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## PART-A

1.(i) Define rank of a matrix.

(ii) Write the nature of $-3y_1^2 - 2y_2^2 - y_3^2$

(iii) Find the matrix of the quadratic form $q = x^2 - 6xy + 3y^2$.

(iv) Find the length of the arc $ay^2 = x^3$ from the vertex to the ordinate *x=5a.*

(v) Find the moment of inertia of a circle A of radius R relative to the centre O.

(vi) Define $\beta$ and $\Gamma$ functions and write the relation between them.

(vii) Show that $V= 3y^4z^2i + 4x^3z^2j + 6x^2y^3k$ is solenoidal.

(viii) Write down the physical interpretation of Gauss's divergence theorem.

[3+3+3+3+3+2+3+2]

## PART – B

2.(a) Find the inverse of a matrix $\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$, using elementary operations.

(b) If consistent, solve the system of equations

$x + y + z + t = 4$
$x - z + 2t = 2$
$y + z - 3t = -1$
$x + 2y - z + t = 3.$

[8+8]

3.(a) Find the latent values and latent roots of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$.

(b) Verify Cayley-Hamilton theorem and hence find $A^{-1}$ if $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$.

[8+8]

4.(a) Find the perimeter of the cardioids $r = a(1 - \cos\theta)$.

(b) Find the moment of inertia of the area bounded by the curve $r^2 = a^2 \cos 2\theta$ about its axis.

[8+8]

5.(a) Evaluate $\int_0^\infty 3^{-4x^2}\, dx$ .

(b) Evaluate $\int_0^a x^4\sqrt{a^2 - x^2}\, dx.$ [8+8]

6. (a) Find the directional derivative of $\emptyset(x, y, z) = xy^2 + yz^2$ at the point (2, -1, 1) in the direction of $i + 2j + 2k$

   (b) Prove that $Div(A \times B) = B.curl\ A - A.curl\ B$

[8+8]

7.(a) Evaluate using the divergence theorem $\iint_S (\boldsymbol{F}.\boldsymbol{n})\boldsymbol{ds}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = b^2$ in the first octant and $\boldsymbol{F} = \mathrm{y}i + \mathrm{z}j + xk$

   (b) If $\boldsymbol{A} = (3\mathrm{xy} - 2\mathrm{y}^2)i + (x - y)j$, evaluate $\int_C \boldsymbol{A}.\boldsymbol{dr}$ along the curve C in xy –plane given by $y = x^3$ from the point (0, 0) to (2, 8)

[8+8]

**Set No - 4**

**Subject Code: R13202/R13**

# I B. Tech II Semester Regular Examinations August - 2014

# MATHEMATICS-III

**(Common to All Branches)**

**Time: 3 hours** **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**

*****

## PART-A

1.(i) Show that $\begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix}$ is idempotent.

(ii) When does the non homogeneous system consistent?

(iii) Define positive definite, negative definite and indefinite.

(iv) Find the volume of a sphere of radius 'a'.

(v) Find the surface area of the solid generated by the revolution about the x-axis of the area bounded by the curves $y = f(x)$, the x-axis the ordinates $x = a$, $x = b$.

(vi) Define Gamma function and Beta function and write the relation between them.

(vii) Find the normal to the surface $x^2 + y^2 + 2z^2 = 26$ at the point (2, 2, 3)

(viii) Write the statement of Green's theroem.

[3+3+3+3+3+2+3+2]

## PART - B

2.(a) If $A = \begin{bmatrix} 1 & -1 & -1 & 2 \\ 4 & 2 & 2 & -1 \\ 2 & 2 & 0 & -2 \end{bmatrix}$, find two non-singular matrices P and Q such that PAQ is in the normal form.

(b) Test for consistency and solve

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5.$$

[8+8]

3. Reduce the quadratic form $q = x_1^2 + 2x_2^2 + 3x_3^2 + 4x_1x_2 - 2x_2x_3 + 6x_3x_1$ into a canonical form by diagonalising the matrix of the quadratic form.

[16]

4.(a) Trace the curve $y = \frac{x^2+2x}{x+1}$.

(b) Find the volume of the solid generated by the revolution of the curve $xy^2 = 4(2 - x)$ about *y-axis.*

[8+8]

5.(a) Evaluate $\int_0^2 x^7(16 - x^4)^{10}\, dx$ .

(b) Evaluate $4\int_0^\infty \frac{x^2 dx}{1+x^4}$ using $\beta - \Gamma$ functions.

[8+8]

6. (a) Show that the vector $[(x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k]$ is irrotational and find the scalar potential.

   (b) Find the acute angle between the surface $xy^2z = 2$ and $x^2 + y^2 + z^2 = 6$ at the point (2, 1, 1). [8+8]

7.(a) Verify the divergence theorem for $\boldsymbol{F} = 4\text{xy}i - \text{y}^2 j + xzk$, over the cube bounded by *x = 0, x = 1, y = 0, y = 1, z=0 and z = 1.*

   (b) Evaluate $\iint_S (\boldsymbol{curl\ A}).\boldsymbol{n}\ ds$ where $\boldsymbol{A}$=*yi +(x - 2 z)j - xyk* and S is the surface of the sphere $x^2 + y^2 + z^2 = 4$ above the *xy-plane.*

[8+8]