Roll No. Total No. of Pages : 03

Total No. of Questions : 09

B.Tech. (2011 Onwards) (Sem.–2)

ENGINEERING MATHEMATICS – II

Subject Code : BTAM-102

Paper ID : [A1111]

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C. have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

1. **Write briefly :**

a) Find the rank of the matrix $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & -3 & 0 \\ 3 & -3 & 1 \end{pmatrix}$

b) If λ be an Eigen value of a non-singular matrix A then show that $\frac{|A|}{\lambda}$ is an Eigen value of Adj (A)

c) Use Demoivre's theorem to prove that $\cos^6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$.

d) Find the modulus of the complex number $(1 - i)^{1+i}$.

e) Test the convergence/divergence of the series $\sum_{n=1}^{\infty} \frac{5^n}{4^n + 3}$.

f) Let $\sum_{n=1}^{\infty} a_n$ is convergent series of non-negative numbers, $a_n \neq 1$, $a_n > 0$ for all n. What can be said about the convergence of the series $\sum_{n=1}^{\infty} \frac{a_n}{1-a_n}$

g) Find the Wronskian of the functions 1, $\sin x$, $\cos x$.

h) Find the general solution of the equation $4x^2y'' + y = 0$.

i) Solve the differential equation $ye^{xy}\,dx + (xe^{xy} + 2y)dy = 0$.

j) For what values of x do the series $\frac{1}{1-x} + \frac{1}{2(1-x)^2} + \frac{1}{3(1-x)^3} + \ldots\ldots\infty$, converges?

SECTION-B

2. a) Find the complete solution of the differential $y'' + 3y' + 2y = x\,e^x \sin x$. 4

b) Use method of variation of parameters to find the general solution of the differential equation

$$y'' - y = \frac{2}{1+e^x}.$$ 4

3. a) Find the complete solution of the differential equation

$$(1 + x)^2 y'' + (1 + x)y' + y = 4 \cos(\log(1 + x))$$

by using operator method. 3

b) Find the solution of the equation $y + px = x^4 p^2$, where $p = \frac{dy}{dx}$ 5

4. a) Solve the differential equation $\frac{dy}{dx} = \frac{y}{x+\sqrt{xy}}$ 4

b) Solve the following simultaneous differential equation

$$\frac{dx}{dt} + 2y + \sin t = 0,\ \frac{dy}{dt} - 2x - \cos t = 0 \text{ given that } x(0) = 0,\ y(0) = 1$$ 4

5. a) An $e.m.f$ E sin pt is applied at $t = 0$ to a circuit containing a capacitance C and inductance L. The current i satisfies the equation $L\frac{di}{dt} + \frac{1}{C}\int i\,dt = E \sin pt$ 5

If $p^2 = 1/LC$ and initially the current i and the charge q are zero, then show that the current i any time t in the circuit is given by $(Et / 2L) \sin pt$

b) Solve the equation $\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x$ 3

SECTION-C

6. a) Find the Eigen values and the corresponding Eigen vectors of the matrix $A = \begin{pmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$. 4

b) Use the rank method to find the values of λ for which the system of equations 4

$3x - y + 4z = 3;\ x + 2y - 3z = -2;\ 6x + 5y + \lambda z = -3;$ has

i) Unique solution

ii) Infinitely many solutions. Determine the solution in each case.

7. a) For what values of α and β the series 3

$1 + \frac{1+\alpha}{1+\beta} + \frac{(1+\alpha)(1+2\alpha)}{(1+\beta)(1+2\beta)} + \frac{(1+\alpha)(1+2\alpha)(1+3\alpha)}{(1+\beta)(1+2\beta)(1+3\beta)} + ...\infty$ converges/diverges.

b) Test the convergence/divergence of the following series : 3,2

i) $\left[\frac{2^2}{1^2} - \frac{2}{1}\right]^{-1} + \left[\frac{3^3}{2^3} - \frac{3}{2}\right]^{-2} + \left[\frac{4^4}{3^4} - \frac{4}{3}\right]^{-3} + ...\infty$

ii) $\sum_{n=1}^{\infty} \sin\frac{1}{n}$

8. a) Find all the roots of the equation $z^7 + z^4 + z^3 + 1 = 0$. 4

b) If $\tan(x + iy) = e^{i\theta}$, then find the value y in terms of θ. 4

9. a) Use C + i S method to find the sum of the series 4

$$\sin^2\alpha - \frac{1}{2}\sin^2\alpha\sin 2\alpha + \frac{1}{3}\sin^3\alpha\sin 3\alpha + ...\infty.$$

b) Separate real and imaginary parts of log $\sin(x + iy)$. 4