Roll No. Total No. of Pages : 02

Total No. of Questions : 09

B.Tech. (2011 Onwards) (Sem.-1)

ENGINEERING MATHEMATICS - I

Subject Code : BTAM-101

Paper ID : [A1101]

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C. have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.
5. Symbols used have their usual meanings. Statistical tables if demanded, may be provided.

SECTION-A

1. Solve the following :

a) Find asymptotes, parallel to axes, of the curve : $xy + 2y - 3x + 1 = 0$

b) Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be a given ellipse. Write its area in the form of a double integral.

c) Find the value of $\frac{\partial(r,\theta)}{\partial(x,y)}$, where $x = r\cos\theta$ & $y = r\sin\theta$.

d) If an error of 1% is made in measuring the side of a square, what is the percentage error in its area?

e) Is the function $f(x, y) = \frac{x^{1/4}+y^{1/4}}{x^{1/5}+y^{1/5}}$ homogeneous? If yes, what is its degree?

f) What is the value of $\int_0^3\int_0^1 (x^2+3y^2)\,dy\,dx$?

g) Give geometrical interpretation of $\int_0^1\int_{x^2}^{x} dy\,dx$

h) Show that for the vector field $\vec{F} = x^2 i + y^2 j + z^2 k, \vec{\nabla}\cdot(\vec{\nabla}\times\vec{F}) = 0$.

i) For $\phi(x,y,z) = x\cos(x+y+z)$, compute $\vec{\nabla}\times(\vec{\nabla}\phi)$.

j) Evaluate $\int_C xy\,ds$ over the curve C given by $x = 4\cos t, y = 4\sin t, z = -3; 0 \le t \le \pi/2$.

SECTION-B

2. Trace the following curves by giving their salient feature :

   a) $x^2y^2 = a^2(y^2 - x^2)$.

   b) $r = a(\sec\theta + \cos\theta)$, $a > 0$.

3. a) Find the length of the arc of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, $0 \le \theta \le \pi$.

   b) Use definite integral to find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

4. a) If $u = \log\left(\sqrt{x^2 + y^2 + z^2}\right)$, show that $(x^2 + y^2 + z^2)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 1$.

   b) State Euler's theorem for homogeneous functions and verify it for $u = \sin^{-1}(x/y) + \tan^{-1}(y/x)$.

5. a) Find the maximum and minimum distances from the origin to the curve $5x^2 + 6xy + 5y^2 - 8 = 0$.

   b) If $u = \frac{x}{y - z}, v = \frac{y}{z - x}, w = \frac{z}{x - y}$, show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 0$.

SECTION-C

6. a) Evaluate the following integral by changing the order of integration :

$$\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)\,dxdy.$$

   b) Evaluate the triple integral $\int_1^3 \int_{1/x}^1 \int_0^{\sqrt{xy}} xyz\,dzdydx$.

7. a) Find a unit vector normal to the surface $x^3 + y^2 + 3xyz = 3$ at the point $(1, 2, -1)$

   b) Show that the vector field $\vec{F}(x, y, z) = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is irrotational.

8. a) Calculate the work done by $\vec{F}(x, y, z) = i - yj + xyzk$ in moving the particle from $(0,0,0)$ to $(1,-1,1)$ along the curve $x = t$, $y = -t^2$, $z = t$ for $0 \le t \le 1$.

   b) Compute $\int_S \vec{F} \cdot \hat{N} ds$, where $\vec{F} = x\hat{i} + (z^2 - zx)\hat{j} - xy\hat{k}$ and S is the triangular surface with vertices $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 2)$.

9. What do you mean by conservative vector field? Find the scalar potential of the conservative vector field $\vec{F}(x, y) = (2x\cos 2y)i - (2x^2 \sin 2y + 4y^2)j$.