Roll No. Total No. of Pages : 02

Total No. of Questions : 09

B.Tech.(ME) (2011 Onwards) (Sem.–5)

MATHEMATICS – III

Subject Code : BTAM-500

Paper ID : [A2127]

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.

SECTION-A

I. **Write briefly :**

a) Find the Fourier series of $f(x) = x$, $0 < x < 2\pi$.

b) Expand $f(x) = \sin x$ $(0 < x < \pi)$ in cosine series.

c) Find the Inverse Laplace transform of $\frac{s+2}{s^2-4s+13}$.

d) Find Inverse Laplace transform of $\frac{1}{s^2(s+1)}$.

e) Prove that $P_n'(1) = \frac{n(n+1)}{2}$ where $P_n(x)$ is the Legendre polynomial of order n.

f) Evaluate the integral $\int x^3 J_0(x)\,dx$ in terms of the Bessel's functions.

g) Form a partial differential equation from the relation $z = yf(x) + xg(y)$.

h) Solve $z(xp - yq) = y^2 - x^2$.

i) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although the Cauchy-Riemann equations are satisfied at the origin.

j) State Cauchy Integral Theorem.

SECTION-B

II. Obtain Fourier series of the function $f(x) = x^2, -\pi \le x \le \pi$ and hence show that

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\dots\dots\dots=\frac{\pi^2}{6}$$

III. State and prove convolution theorem for Laplace transform.

IV. Find the series solution of $xy'' + (1-x)y' + 3y = 0$, by Frobenius Method.

V. Using method of separation of variables, solve $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$, subject to the condition $u(x,0) = 6e^{-3x}$

VI. Evaluate $\int\limits_{|z|=3} \frac{e^{2z}}{(z+1)^4}\,dz$.

SECTION C

VII. a) Using Fourier series, solve the wave equation $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}, 0<x<L, t>0$, where a is a constant related to tension in the vibrating string of length L having fixed ends. The boundary conditions and initial conditions are :

$u(0, t) = u(L, t) = 0, t \ge 0,$

$u(x, 0) = f(x), 0 \le x \le L,$

$u_t(x, 0) = 0, 0 \le x \le L.$

b) Using Laplace transform, solve $y'' + 2y' + 5y = e^{-t}\sin t, y(0) = 1. y'(0) = 1.$

VIII. a) Show that $J_{-\frac{5}{2}}(x)=\sqrt{\frac{2}{\pi x}}\left\{\frac{1}{x^2}(3-x^2)\cos x+\frac{3}{x}\sin x\right\}$.

b) Show that the solution of the wave equation $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$, can be expressed in the form $y(x, t) = \phi(x + ct)+ \Psi(x - ct)$. If $u(x, 0) = f(x)$ and $\frac{\partial y}{\partial t}(x, 0) = 0$, show that $y(x, t) = 1/2[f(x + ct) + f(x - ct)]$

IX a) Find all the Taylor's and Laurent series of $1/[(z+1)^2(z+3)]$ about $z_0 = -1$.

b) Using Cauchy Residue theorem, evaluate $\int\limits_c \frac{dz}{(z^2+4)^2}$, where C is the curve $|z-i| = 2$.