PART – B (5×16=80 Marks)

11. a) i) Using Laplace transform, an infinitely long string one end at $x = 0$ is initially at rest on the x – axis. The end $x = 0$ undergoes a periodic transverse displacement described by $A_0 \sin \omega t$, $t > 0$. Find displacement of any point on the string at any time t. (10)

ii) Find the inverse Laplace transform of $\frac{e^{-1/s}}{\sqrt{s}}$. (6)

(OR)

b) i) Prove that $L[\delta(t-a)f(t)] = e^{-as}f(a)$. (6)

ii) State and prove convolution theorem and hence find $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$. (10)

12. a) Solve the following potential problem in the semi-infinite strip described by

$u_{xx} + u_{yy} = 0$, $0 < x < \infty$, $0 < y < a$

$u(x, a) = 0$ $0 < x < \infty$

$u(x, y) = 0$ $0 < x < \infty$, $0 < y < a$ and $\frac{\partial u}{\partial x} \to 0$ as $x \to \infty$. (16)

(OR)

b) i) Find the Fourier transform of the normal distribution $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$. (8)

ii) Find the Fourier transform of $\frac{\partial^n u}{\partial x^n}$ of the funciton u(x, t), assuming u and first (n – 1) derivatives with respect to x vanish as $x \to \pm\infty$. (8)

13. a) i) Using Ritz method, find an approximate solution of the problem of the minimum of the functional $v[y(x)] = \int_0^1 \left(y'^2 + y^2 + 2xy\right)dx$; $y(0) = 0$, $y(2) = 0$ and compare it with the exact solution. (10)

ii) Find the extremals of the functional $v[y(x)] = \int_{x_1}^{x_2} \left(y'^2 + y^2 + 2ye^x\right)dx$. (6)

(OR)

b) i) A curve C joining the points $(x_1, y_1)$ and $(x_2, y_2)$ is revolved about the x-axis. Find the shape of the curve so that the surface thus generated is a minimum. (6)

ii) Find the extremals of the functional

$$I(y, z) = \int_0^1 \left(y'^2 + z'^2 + 2yz\right)dx;\ y(0) = 0,\ y(\pi/2) = -1,\ z(0) = 0,\ z(\pi/2) = 1.$$ (10)