

GUJARAT TECHNOLOGICAL UNIVERSITY
BE-I - SEMESTER- 1st - EXAMINATION – SUMMER 2018

Subject Code: 110008

Date: 21-05-2018

Subject Name: MATHS-I

Time: 02:30 pm to 05:30 pm

Total Marks: 70

Instructions:

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) (1) Find the value of k so that the function given below is continuous at $x = 0$ 03

$$f(x) = \begin{cases} \frac{\sin 3x}{2x} & x \neq 0 \\ k & x = 0 \end{cases}$$

(2) State Sandwich theorem on limit of sequences and using it find 04

$$\lim_{x \rightarrow 0} g(x), \text{ if } 3 - x^3 \leq g(x) \leq 3 \sec x, \forall x \in R$$

(b) If $u = f(r)$, where $r^2 = x^2 + y^2$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$ 07

Q.2 (a) (1) Evaluate $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$ 03

(2) Find the area of the region between the x-axis and the graph of 04

$$f(x) = x^3 - x^2 - 2x, \quad -1 \leq x \leq 2$$

(b) State (1) Rolle's Theorem 07

(2) The Mean Value Theorem

Find the value of c using Mean Value Theorem, for the function

$$f(x) = 1 - x^2, \text{ in } 0 \leq x \leq 2$$

Q.3 (a) (1) Find the gradient of $f(x, y, z) = 2z^3 - 3(x^2 + y^2)$ at the point (1,1,1). 03

(2) Change the order of integration in the integral 04

$$\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx \quad \text{and evaluate it.}$$

(b) Trace the curve $r = a(1 + \cos \theta)$; $a > 0$ 07

Q.4 (a) (1) Find the curl of $\vec{F} = (x^2 - z)\hat{i} + xe^z\hat{j} + xy\hat{k}$ 03

(2) If $u = (x^2 + y^2 + z^2)^m$ then find $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$. 04

(b) Find the local extreme values of the function 07

$$f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$$

Show that the motion is irrotational.

(2) Expand $\sin\left(\frac{\pi}{4} + x\right)$ in powers of x . Hence find the value of $\sin 46^\circ$. 04

(b) Find the point on the plane $x + 2y + 3z = 13$ closest to the point $(1,1,1)$. 07

Q.6 (a) State modified Euler's Theorem. Show that, if $u = \tan^{-1}\left(\frac{x^2 + y^2}{x - y}\right)$ 07

$$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = \frac{1}{4} \sin 4u - \frac{1}{2} \sin 2u$$

(b) State Green's Theorem. Using Green's theorem, evaluate 07

$$\oint_C e^{-x} (\sin y \, dx + \cos y \, dy) \quad \text{where } C \text{ is the rectangle with vertices } (0,0), (\pi,0), (\pi, \pi/2), (0, \pi/2).$$

Q.7 (a) Write the statement of Cauchy's integral test. Test the convergence of the series 07

$$\sum_{n=2}^{\infty} \frac{1}{n (\log n)^a}, \quad \text{for } 0 \leq a \leq 1.$$

(b) Find the volume of the solid of revolution of the area bounded by the curve $y = xe^x$ and the straight lines $x = 1, y = 0$ 07

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