Seat No.: ________ Enrolment No.________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER– 1st / 2nd • EXAMINATION – SUMMER 2018

Subject Code:110014 **Date: 21-05-2018**

Subject Name: Calculus

Time: 02:30 pm to 05:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	**(a) (1)**	Evaluate $\lim_{x\to 1}\left(\frac{1}{\log x}-\frac{x}{x-1}\right)$	**03**
	(2)	Show that the sequence $\left\{\frac{n}{n^2+1}\right\}$ is monotonic decreasing and bounded. Is it convergent?	**04**
	(b)	Find expansion of $\tan\left(x+\frac{\pi}{4}\right)$ is ascending powers of x upto terms in x^4 and find approximately the value of $\tan(43^\circ)$	**07**
Q.2	**(a)(1)**	Find expansion of $\log(1+x)$.	**03**
	(2)	Test the convergence of the series $1+\frac{2^2}{2!}+\frac{3^2}{3!}+\frac{4^2}{4!}+\dots$	**04**
	(b)	Determine absolute or conditional convergence of the series. $\sum_{n=1}^{\infty}(-1)^n\frac{n^2}{n^3+1}$	**07**
Q.3	**(a) (1)**	Evaluate $\int_1^{\infty}\frac{1}{x^2}dx$	**03**
	(2)	Find the linearization of $f(x,y,z)=xy+yz+xz$ at the point (1,0,0)	**04**
	(b)	Trace the curve $r=a(1-\cos\theta),\ a>0$	**07**
Q.4	**(a)(1)**	Show that $f(x,y)=x^2+2y$ is continuous at (1,2).	**03**
	(2)	If $u=\tan^{-1}\left(\frac{x}{y}\right)$ *where* $x^2+y^2=a^2$ find $\frac{du}{dx}$.	**04**
	(b)	State Euler's theorem. If $u=tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$,prove that $x^2\frac{\partial^2u}{\partial x^2}+2xy\frac{\partial^2u}{\partial x\partial y}+y^2\frac{\partial^2u}{\partial z^2}=-2sin^3ucosu$	**07**
Q.5	**(a) (1)**	If $u=x^2y^3,\ x=\log t,\ y=e^t$ find $\frac{du}{dt}$	**03**
	(2)	Find the equation of the tangent plane and normal line to the surface $x^2+y^2+z^2=3$ at the point (1,1,1).	**04**
	(b)	Change the order of integration and Evaluate for $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}}xy\ dydx$	**07**

Q.6 **(a)** **(1)** Evaluate $\int_{-1}^{1}\int_{0}^{2}\int_{0}^{1}\left(xz - y^{3}\right)dzdydx$ **03**

(2) State fundamental theorem of calculus. Use first fundamental theorem of calculus to find area under the curve f(x) given as an integrand. **04**

$\int_{1}^{2} \log x \, dx$

(b) Find the extreme values of the function $x^{3} + 3xy^{2} - 3x^{2} - 3y^{2} + 7$ **07**

Q.7 **(a)** **(1)** Evaluate $\int_{1}^{2}\int_{0}^{1}(1 + 3xy)\, dxdy$ **03**

(2) If $x = r\cos\theta,\ y = r\sin\theta$ find $\frac{\partial(x, y)}{\partial(r,\theta)}$ and $\frac{\partial(r,\theta)}{\partial(x, y)}$ **04**

(b) Find the volume generated by revolving the area bounded by $2y = x^{2}, x = 4, y = 0$ about x-axis. **07**

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