

Subject Code: 2110014
Date: 21-05-2018
Subject Name: Calculus
Time: 02:30 pm to 05:30 pm
Total Marks: 70
Instructions:

1. Question No. 1 is compulsory. Attempt any four out of remaining Six questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 Objective Question (MCQ)
Mark
(a)
07

1. The sequence $\left\{\frac{\cos 2n}{n}\right\}$ converges to
 (a) 1 (b) 0 (c) 2 (d) -1
2. Sum of the series $\sum_{n=0}^{\infty} \frac{4}{2^n}$ is
 (a) 4 (b) 2 (c) 8 (d) 16
3. The coefficient of x^4 in the expansion of $\cos x$ is
 (a) $\frac{1}{4!}$ (b) $-\frac{1}{4!}$ (c) $\frac{1}{4}$ (d) $-\frac{1}{4}$
4. $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x =$ _____
 (a) 0 (b) 1 (c) e (d) ∞
5. The curve $x^3 + y^3 = 3xy$ is symmetric about
 (a) x-axis (b) y-axis (c) line $y = x$ (d) origin
6. Asymptote parallel to x-axis of the curve $3x^3 + xy^2 + xy = 0$ is
 (a) $y=3$ (b) $y=1$ (c) $y=0$ (d) not possible
7. The curve $r^2 = a^2 \cos 2\theta$ is not symmetric about
 (a) initial line (b) line $\theta = \frac{\pi}{4}$ (c) line $\theta = \frac{\pi}{2}$ (d) pole

(b)
07

1. If $u = y \tan^{-1}(x/y) + x \cot^{-1}(y/x)$, then $xu_x + yu_y =$ _____
 (a) 0 (b) u (c) 2u (d) 3u
2. For an implicit function $f(x, y) = c$, the value of $\frac{dy}{dx}$ is
 (a) $\frac{f_x}{f_y}$ (b) $\frac{f_y}{f_x}$ (c) $-\frac{f_x}{f_y}$ (d) $-\frac{f_y}{f_x}$
3. $\lim_{(x,y) \rightarrow (0,1)} \frac{y^2 \tan^{-1} x}{x} =$ _____
 (a) 0 (b) 1 (c) -1 (d) ∞
4. $\int_0^1 \int_0^y e^{x/y} dx dy =$ _____
 (a) $\frac{e-1}{2}$ (b) $e-1$ (c) e (d) $e+1$
5. If $u = x - y$ and $v = x + y$, then the value of $J = \frac{\partial(x,y)}{\partial(u,v)}$ is _____
 (a) 1 (b) -1 (c) $1/2$ (d) $1/4$

The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent when

(a) $p = 1$

(b) $p < 1$

(c) $p > 1$

(d) $p = 0$

7. The series $\sum_{n=1}^{\infty} \frac{n-1}{n+1}$ is _____

(a) divergent

(b) convergent and sum 1

(c) convergent and sum 2

(d) none of these

Q.2 (a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n+1}{n^3-3n+2}$ **03**

(b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{4^n+1}{5^n}$ **04**

(c) Evaluate (i) $\lim_{x \rightarrow 0} \frac{2x-x\cos x-\sin x}{2x^3}$; (ii) $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$. **07**

Q.3 (a) If $u = \log(x^2 + y^2)$, verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. **03**

(b) If $u = r^m$, prove that $u_{xx} + u_{yy} + u_{zz} = m(m+1)r^{m-2}$, where $r^2 = x^2 + y^2 + z^2$. **04**

(c) Find the maxima and minima of the function **07**
 $f(x, y) = x^2y - xy^2 + 4xy - 4x^2 - 4y^2$.

Q.4 (a) If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, show that $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$. **03**

(b) Evaluate $\iint (6x^2 + 2y) dx dy$ over the region R bounded between $y = x^2$ and $y = 4$. **04**

(c) Change the order of integration and evaluate $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy dx$. **07**

Q.5 (a) If $z = xy^2 + x^2y$, $x = at^2$, $y = 2at$, find $\frac{dz}{dt}$. **03**

(b) If $u = e^{x^2+y^2-xy}$, then prove that **04**

(i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u \ln u$;

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2u \ln u (2 \ln u + 1)$.

(c) (i) Check the absolute and conditional convergence of the series **07**

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n}}.$$

(ii) Find the radius and interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(-5)^n x^n}{n!}.$$

Q.6 (a) Evaluate $\iint x dA$, over the region R bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. **03**

(b) Expand $\cos\left(\frac{\pi}{4} + x\right)$ in powers of x by Taylor series. Hence find the value of $\cos 46^\circ$. **04**

(c) Evaluate $\int_0^4 \int_{y/2}^{(y/2)+1} \frac{2x-y}{2} dx dy$ by applying the transformations **07**
 $u = \frac{2x-y}{2}, v = \frac{y}{2}$.

Q.7 (a) Evaluate the improper integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$. **03**

(b) Find the volume of the solid generated by revolving the region **04**
bounded by $y = \sqrt{x}$ and the lines $y = 2, x = 0$ about the line $y = 2$.

(c) Trace the curve $y^2(a-x) = x^3, a > 0$. **07**