Seat No.: _____ Enrolment No.:_____

# GUJARAT TECHNOLOGICAL UNIVERSITY

**SPFU-SEMESTER-1st/ 2nd EXAMINATION-Summer 2018**

**Subject Code: MTH001** **Date: 17-05-2018**
**Subject Name: CALCULUS**
**Time: 02:30 PM to 05:30 PM** **Total Marks: 70**
**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

## Objective Section (MCQ)

**Q.1** The sum of the series $\sum_{n=1}^{\infty} \frac{1}{2^n}$ is **1**

(A)0 (B) 1 (C) -1 (D) $\frac{1}{2}$

**Q.2** $\sum_{n=0}^{\infty} \frac{cosn\pi}{2^n}$ *is ...* **1**

(A) Convergent (B) Divergent (C) Oscillatory (D) None of these

**Q.3** $\int_1^3 \int_0^4 e^{x+y}\, dx\, dy =$ **1**

(A) $(e^3 - e)\,(e^4 - 1)$ (B) $(e^3 - e^2)\,(e^4 - 1)$ (C) 1 (D) 0

**Q.4** $\frac{\partial}{\partial x}(x^2 + 3xy) =$ **1**

(A) 1 (B) $3x + 2y$ (C) $x + y$ (D) $2x + 3y$

**Q.5** $f(x,y) = \tan^{-1}\frac{x}{y} + \sin^{-1}\frac{y}{x}$ is homogeneous function with degree **1**

(A) -1 (B) 0 (C) 1 (D) None of these

**Q.6** If $u = f(x,y)$ is homogeneous function of degree $n$ then **1**

$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} =$

(A) $n(n-1)u$ (B) 0 (C) $n(n^2-1)u$ (D) $nu$

**Q.7** **1**

The series $\sum a_n$ where $a_n = \begin{cases} \frac{n}{2^n} & n \text{ is odd} \\ \frac{1}{2^n} & n \text{ is even} \end{cases}$ is…….

(A) Divergent (B) Convergent (C) Alternative (D) None of above

**Q.8** The homogeneous function f(x, y) of degree $n$ can written as **1**

(A) $x^n \varphi\left(\frac{y}{x}\right)$ (B) $y^n \varphi\left(\frac{y}{x}\right)$ (C) $x^n \varphi\left(\frac{x}{y}\right)$ (D) None of these

**Q.9** The series $\sum_{n=1}^{\infty} \frac{1}{n^{0.001}}$ is **1**

(A) Divergent (B) Convergent (C) Alternative (D) None of above

**Q.10** $\int_0^1 \int_0^x e^{\frac{y}{x}}\, dx\, dy =$ **1**

(A) $e^3 - 1$ (B) $\frac{e-1}{2}$ (C) 1 (D) $\frac{e-1}{3}$

**Q.11** If $\varphi(x,y,z) = 0$, then $\left(\frac{\partial z}{\partial y}\right)_x \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial y}{\partial x}\right)_z =$ **1**

(A) 0 (B) 1 (C) -1 (D) 2

**Q.12** If $\lim_{n\to\infty} u_n \neq 0$ for some series $\sum u_n$ then the series is **1**

(A)Convergent (B) Divergent (C) Oscillatory (D) None of these

**Q.13** $\int_1^e \int_1^e \int_1^e \frac{1}{xyz}\, dx\, dy\, dz =$ **1**

(A) 0 (B) 1 (C) -1 (D) None of these

**Q.14** If $f(x,y)$ is continuous then **1**

(A) $f_x = f_y$ (B) $f_x \neq f_y$ (C) $f_{xy} = f_{yx}$ (D) $f_{xy} \neq f_{yx}$

**Q.15** Area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is **1**

(A) $\pi a^2 b$ (B) $\pi ab^2$ (C) $\pi a^2 b^2$ (D) $\pi ab$

**Q.16** The series 1-1+1-1+1-1+1-1+…. **1**

(A) Convergent (B) Divergent (C) Oscillatory (D) None of these

**Q.17** If $n^{\frac{1}{n}} \to 1$ as $n \to \infty$ ,then the series $\sum_{n=1}^{\infty}\left(n^{\frac{1}{n}}-1\right)^n$ **1**

(A) Convergent (B) Divergent (C) Oscillatory (D) None of these

**Q.18** The series $\frac{1}{1\cdot 2\cdot 3}+\frac{3}{2\cdot 3\cdot 4}+\frac{5}{3\cdot 4\cdot 5}+\cdots\cdots$ is **1**

(A) Convergent (B) Divergent (C) Oscillatory (D) None of these

**Q.19** For $u=f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$, the value of $xu_x+yu_y+zu_z$ is **1**

(A)1 (B) 0 (C) $-u$ (D) $nu$

**Q.20** The sum of the series $\sum_{n=0}^{\infty}\frac{4^n+5^n}{6^n}$ is **1**

(A) 9 (B) $\frac{9}{6}$ (C) 1 (D) $\frac{9}{2}$

**Q.21** If implicit function $f(x,y)=c$ then $\frac{dy}{dx}=$ **1**

(A) $-\frac{f_x}{f_y}$ (B) $-\frac{f_y}{f_x}$ (C) $f_x f_y$ (D) None of these

**Q.22** If change the order of integration of $\int_0^1\int_0^x f(x,y)\,dy\,dx$ then **1**
$\int_0^1\int_0^x f(x,y)\,dydx=$

(A) $\int_0^x\int_0^1 f(x,y)\,dx\,dy$ (B) $\int_0^1\int_0^x f(x,y)\,dx\,dy$

(C) $\int_0^1\int_1^y f(x,y)\,dx\,dy$ (D) $\int_0^1\int_y^1 f(x,y)\,dx\,dy$

**Q.23** Equation of tangent plane of $z=x$ at (2,0,2) is **1**

(A) $z=x$ (B) $z+y+x=2$ (C) $z+x=0$ (D) $z=x^2$

**Q.24** The series $\frac{1}{1^p}-\frac{1}{2^p}+\frac{1}{3^p}-\frac{1}{4^p}+\cdots$ converges if **1**

(A) $p>0$ (B) $p<0$ (C) $p=-1$ (D) None of these

**Q.25** If $w=x^2+y^2$ , $x=r-s$ , $y=r+s$ then $\frac{\partial w}{\partial r}$ is **1**

(A) $2r$ (B) $2s$ (C) $4r$ (D) $2(r+s)$

**Q.26** The series $1+x+2x^2+3x^3+\cdots..$ is convergent if **2**

(A) $x=-3$ (B) $-1<x<1$ (C) $x=e$ (D) $x=\pi$

**Q.27** Express 5.232323… as a ratio of two integers is **2**

(A) $\frac{521}{99}$ (B) $\frac{525}{13}$ (C) $\frac{520}{99}$ (D) $\frac{518}{99}$

**Q.28** $\lim_{(x,y)\to(0,0)}\frac{x^2-xy}{x+y}=$ **2**

(A) 2 (B) 1 (C) 0 (D) $-1$

**Q.29** $\int_0^1\int_2^{4-2x} dx\,dy=$ **2**

(A) 4 (B) 1 (C) 0 (D) $-1$

**Q.30** $\int_1^e\int_1^e\int_1^e \ln x\,\ln y\,\ln z\;\;dx\,dy\,dz=$ **2**

(A) 0 (B) 1 (C) -1 (D) None of these

## Subjective Section

**Attempt any five:**

**Q.1** If $u = \text{cosec}^{-1}\left(\sqrt{\frac{x^{\frac{1}{2}}+y^{\frac{1}{2}}}{x^{\frac{1}{3}}+y^{\frac{1}{3}}}}\right)$ show that **7**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144}(13 + tan^2 u).$$

**Q.2** Find the extreme values of $x^3 + y^3 - 3axy,\ a > 0$. **7**

**Q.3** Test the convergence of $\sum_{n=1}^{\infty} \frac{1}{1^2+2^2+3^2+\cdots+n^2}$. **7**

**Q.4** For what values of $x$ does the series $\sum_{n=1}^{\infty} \frac{3^n \cdot x^n}{n!}$ converge? and find radius of convergence. **7**

**Q.5** Sketch the region of integration and evaluate the $\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y} dydx$ by reversing its order. **7**

**Q.6** Evaluate $\iint_R xy\, dA$, where R is the region bounded by the lines y=x, y=2x and x+y=2. **7**

**Q.7** Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dxdy$ by changing to polar coordinates. **7**