

**GUJARAT TECHNOLOGICAL UNIVERSITY**

**BE - SEMESTER-VIII (NEW) - EXAMINATION – SUMMER 2018**

**Subject Code: 2181911**

**Date: 30/04/2018**

**Subject Name: Finite Elements Method(Department Elective II)**

**Time: 10:30 AM to 01:00 PM**

**Total Marks: 70**

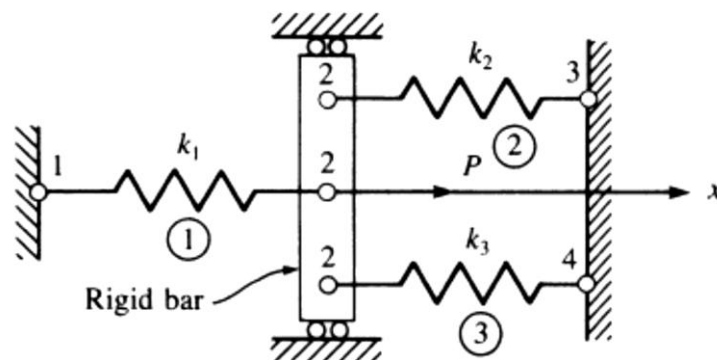
**Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1**
- (a) Can the FEM handle a wide range of problems, i.e., solve general PDEs? Enlist two advantages of FEM. **03**
  - (b) List four applications of FEM and computer programs used for the FEM. **04**
  - (c) List and briefly describe the process of the Finite Element Method. **07**
- Q.2**
- (a) What are the characteristics of shape function? Why polynomials are generally used as shape function? **03**
  - (b) Draw three 2D and 3D types of finite element. **04**
  - (c) Derive the Stiffness Matrix for a Spring Element. **07**

**OR**

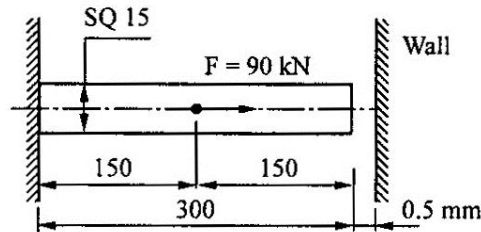
- (c) (a) Formulate the global stiffness matrix and equations for solution of the unknown global displacement and forces. The spring constants for the elements are  $k_1$ ,  $k_2$ , and  $k_3$ ;  $P$  is an applied force at node 2. **07**
- (b) Using the direct stiffness method, formulate the same global stiffness matrix and equation as in part (a).



**Figure 1**

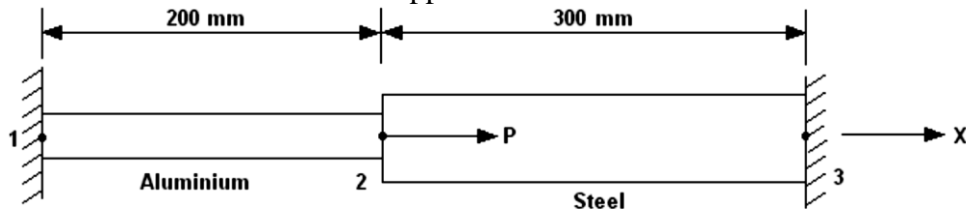
- Q.3**
- (a) Distinguish between essential boundary conditions and natural boundary conditions. Give their examples. **03**
  - (b) Discuss the penalty approach for FEM. **04**
  - (c) A tapered bar 1200 mm long, having cross-sectional area 450 mm<sup>2</sup> at one end and 150 mm<sup>2</sup> at other end is fixed at the larger end. It is subjected to an axial load of 35 kN. Calculate the stress on a model bar having three finite elements 400 mm long. Assume modulus of elasticity,  $E = 2 \times 10^5$  N/mm<sup>2</sup> circular cross section at both end. **07**

- Q.3** (a) For the loading system as shown in Figure 2, determine the element stiffness matrix and global stiffness matrix. Assume modulus of elasticity as  $80 \times 10^3 \text{ N/mm}^2$  **03**



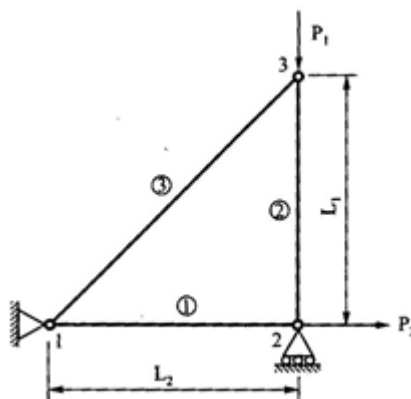
**Figure 2**

- (b) For above Q. 3 (a) determine the displacements, stresses and support reaction using penalty approach. **04**
- (c) Axial load  $P = 300 \text{ kN}$  is applied at  $20^\circ \text{C}$  to the rod as shown in Figure 3. The temperature is then raised to  $60^\circ \text{C}$ . The coefficient of thermal expansion for Aluminium is  $23 \times 10^{-6}$  per  $^\circ \text{C}$  and Steel is  $11.7 \times 10^{-6}$  per  $^\circ \text{C}$ .  $A_{\text{Al}} = 900 \text{ mm}^2$ ,  $A_{\text{Steel}} = 1200 \text{ mm}^2$ ,  $E_{\text{Al}} = 70 \times 10^9 \text{ N/m}^2$ ,  $E_{\text{Steel}} = 200 \times 10^9 \text{ N/m}^2$ . Using FEM, Determine the nodal displacement and element stresses and the reaction forces at the supports. **07**



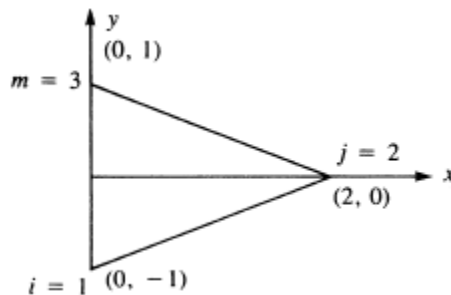
**Figure 3**

- Q.4** (a) Write the shape function and stiffness matrix for one-dimensional finite element formulation of the fluid-flow problem. **03**
- (b) Derive the element stiffness matrix of truss element and write the stress calculation formula for truss. **04**
- (c) A three bar truss is shown in Figure 4. The modulus of elasticity of the material is  $300 \times 10^3 \text{ N/mm}^2$ . The area of the bar used for the truss is  $60 \text{ mm}^2$  for all the elements. The length  $L_1 = 750 \text{ mm}$  and  $L_2 = 100 \text{ mm}$ . The load  $P = 20 \text{ kN}$  and  $P_2 = 25 \text{ kN}$ . Determine the element stiffness matrix for each element and the global stiffness matrix. **07**



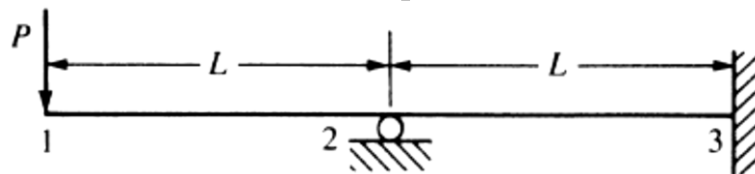
**Figure 4**

- Q.4** (a) List out the application of axisymmetric elements. **03**  
 (b) Discuss the terms “plain stress” and “plain strain” problems. **04**  
 (c) Evaluate the stiffness matrix for the element shown in Figure 5. The coordinates are shown in units of inches. Assume plane stress conditions. Let  $E = 30 \times 10^6$  psi,  $\nu = 0.25$ , and thickness  $t = 1$  in. Assume the element nodal displacements have been determined to be  $u_1 = 0$ ,  $v_1 = 0.0025$  in.,  $u_2 = 0.0012$  in.,  $v_2 = 0$ ,  $u_3 = 0$ , and  $v_3 = 0.0025$  in. Determine the element stresses. **07**



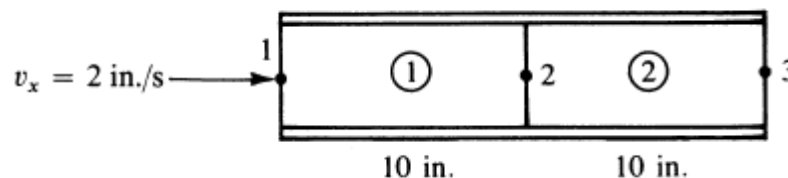
**Figure 5** Plane stress element for stiffness matrix evaluation

- Q.5** (a) Write the point force, body force and surface traction force using natural coordinate system. **03**  
 (b) Write the four shape function equations for a beam element. **04**  
 (c) Using the direct stiffness method, solve the problem of the propped cantilever beam subjected to end load  $P$  in Figure 6. The beam is assumed to have constant  $EI$  and length  $2L$ . It is supported by a roller at mid length and is built in at the right end. Propped cantilever beam shown in below Figure 6 **07**



**Figure 6**  
OR

- Q.5** (a) Write the consistent and lumped mass matrices for 1D element. **03**  
 (b) List out applications of the axisymmetric elements. **04**  
 (c) For the smooth pipe shown discretized in Figure 7 with uniform cross section of  $1 \text{ in}^2$ , determine the flow velocities at the center and right end, knowing the velocity at the left end is  $v_x = 2 \text{ in./s}$ . **07**



**Figure 7** Discretized pipe for fluid-flow problem

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