IV - UNIT

INTERNAL COMBUSTION

HEAT ENGINES

Any type of engine or machine which derives heat energy from the combustion of fuel or any other source and converts this energy into mechanical work is termed as a heat engine.

Heat engines may be classified into two main classes as follows:

1. External Combustion Engines.
2. Internal Combustion Engines.

1. External Combustion Engines (E.C. Engines)

In this case, combustion of fuel takes place outside the cylinder as in case of steam engines where the heat of combustion is employed to generate steam which is used to move a piston in a cylinder. Other examples of external combustion engines are hot air engines, steam turbine and closed cycle gas turbine. These engines are generally used for driving locomotives, ships, generation of electric power etc.

2. Internal Combustion Engines (I.C. Engines)

In this case, combustion of the fuel with oxygen of the air occurs within the cylinder of the engine. The internal combustion engines group includes engines employing mixtures of combustible gases and air, known as gas engines, those using lighter liquid fuel or spirit known as petrol engines and those using heavier liquid fuels, known as oil compression ignition or diesel engines.

CLASSIFICATION OF I.C. ENGINES

Internal combustion engines may be classified as given below:

1. According to cycle of operation:
   (i) Two stroke cycle engines
   (ii) Four stroke cycle engines.
2. According to cycle of combustion:
   (i) Otto cycle engine (combustion at constant volume)
   (ii) Diesel cycle engine (combustion at constant pressure)
   (iii) Dual-combustion or Semi-Diesel cycle engine (combustion partly at constant volume and partly at constant pressure).
3. According to arrangement of cylinder:
   (i) Horizontal engine
   (ii) Vertical engine
   (iii) V-type engine
   (iv) Radial engine etc.
4. According to their uses:
   (i) Stationary engine
   (ii) Portable engine
   (iii) Marine engine
   (iv) Automobile engine
   (v) Aero engine etc.
5. According to the fuel employed and the method of fuel supply to the engine cylinder:

(i) Oil engine
(ii) Petrol engine
(iii) Gas engine
(iv) Kerosene engine
(v) Carburettor, hot bulb, solid injection and air injection engine.

6. According to the speed of the engine:
   (i) Low speed engine
   (ii) Medium speed engine
   (iii) High speed engine
7. According to method of ignition
   (i) Spark ignition
   (ii) Compression ignition
8. According to method of cooling the cylinder
   (i) Air cooled engine
   (ii) Water cooled engine
9. According to method of Governing
   (i) Hit miss governed engine
   (ii) Quality governed engine
   (iii) Quantity governed engine
10. According to valve arrangement
   (i) Over head valve engine
   (ii) L-head type engine
   (iii) T-head type engine
   (iv) F-head type engine
11. According to number of cylinders
   (i) Single cylinder engine
   (ii) Multi cylinder engine

APPLICATION OF I.C. ENGINES

The I.C. engines are generally used for:

(i) Road vehicles (e.g., scooter, motorcycle, buses etc.)
(ii) Air craft
(iii) Locomotives
(iv) Construction in civil engineering equipment such as bull-dozer, scraper, power shovels etc.
(v) Pumping sets
(vi) Cinemas
(vii) Hospital
(viii) Several industrial applications.

Note: Prime movers in all construction equipment are invariable I.C. engines, unless of course, when drive is electric. Use of steam source for this equipment is almost absolute.

BASIC IDEA OF I.C. ENGINES

The basic idea of internal combustion engine is shown in Fig. 1 The cylinder which is closed at one end is filled with a mixture of fuel and air. As the crank shaft turns it pushes cylinder. The piston is forced up and compresses the mixture in the top of the cylinder. The mixture is set alight and, as it burns, it creates a gas pressure on the piston, forcing it down the cylinder. This motion is shown by arrow '1'. The piston pushes on the rod which pushes on the crank. The crank is given rotary (turning) motion as shown by the arrow '2'. The fly wheel fitted on the end of the crank shaft stores energy and keeps the crank turning steadily.

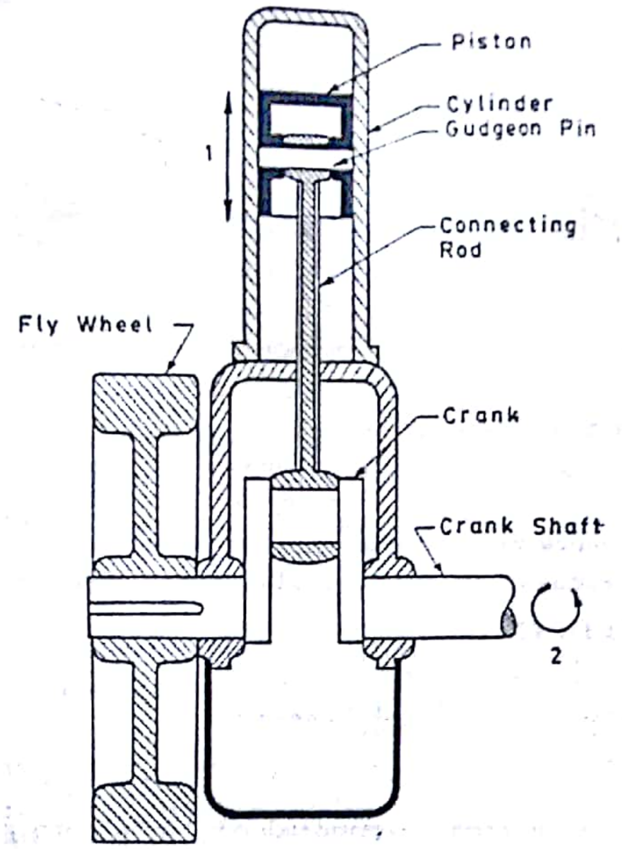

Fig 1: Basic idea of I.C.Engine

DIFFERENT PARTS OF IC ENGINES

Here follows the details of the various parts of an internal combustion engine.
A cross section of an air cooled I C engine with principal parts is shown in fig 1.

A. Parts common to both petrol and diesel engine:

1. Cylinder
2. Cylinder head
3. Piston
4. Piston rings
5. Gudgeon pin
6. Connecting rod
7. Crank shaft
8. Crank
9. Engine bearing
10. Crank case
11. Flywheel
12. Governor
13. Valves and valve operating machine

B.Parts for petrol engines only:

1. Spark plugs
2. Carburettor
3. Fuel pump

C. Parts fro diesel engine only:

1. Fuel pump
2. Injector

FLY WHEEL:

A fly wheel (steel or cast iron disc) secured on the crank shaft performs the following functions:

(a) Brings the mechanism out of dead centers.

(b) Stores energy required to rotate the shaft during preparatory strokes.

(c) Makes crank shaft rotation more uniform.

(d) Facilitates the starting of the engine and overcoming of short time over loads as, for example, when the machine is started from rest.

The weight of the flywheel depends upon the nature of variation of the pressure. The flywheel for a double steam engine is lighter than that of a single acting one. Similarly, the flywheel for a two stroke cycle is lighter than a flywheel used for a four stroke cycle engine. Lighter flywheels are used for multi cylinder engine.

GOVERNOR

A governor may be defined as a device for regulating automatically output of a machine by regulating the supply of working fluid. When the speed decrease due to increase in load the supply valve is opened by mechanism operated by the governor and the engine therefore speeds up again to its original speed. If the speed increase due to a decrease of load the governor mechanism close the supply valve sufficiently of engine speed due to change of load.

COMPARISON BETWEEN A FLYWHEEL AND GOVERNOR:

	Flywheel	Governor
1	It is provided on engine and fabrication machines viz., rolling mills, punching machine, shear machines, presses etc.	It is provided on prime movers such as engine and turbines.
2	Its function is to store the available mechanical energy when it is in excess of the load requirement and to part with the same when the available energy is less that required by the load.	Its functions is to regulate the supply of driving fluid producing energy, according to the load requirement so that at different loads almost a constant speed is maintained.
3	It working continuously from cycle to cycle	It works intermittently i.e., only when there is change in load.
4	In engine it takes care of fluctuations of speed during thermodynamic cycle.	It takes care of fluctuations of speed due to variation of load over long range of working engines and turbines.
5	In fabrication machines it is very economical to use it in that it reduces capital investment on prime movers and their running expenses.	But for governor, there would have been unnecessarily more consumption of driving fluid. Thus it economics its consumption

TERMS CONNECTINTED WITH IC ENGINE:

Bore: The inside diameter of the cylinder is called bores.

Stroke: as the piston reciprocating inside the engine cylinder it has got limited upper and lower positions beyond which it cannot move and reversal of motion takes place at these place at these limiting positions.

The linear distance along the cylinder axis between two limiting positions, is called stroke.

Top dead center (TDC): the top most of the position towards cover end side of the cylinder is called top dead center. In case of horizontal engines, this known as linear dead center.

Bottom dead center: the lowest position of the piston towards the crank end side of the cylinder is called bottom dead center. In case of horizontal engines it is called outer dead center.

Clearance Volume: The volume contained in the cylinder above the top of the piston, when the piston is at top dead center, is called the clearance volume.

Swept volume: The volume swept through by the piston in moving in moving between top dead center and bottom dead center, is called swept volume or piston displacement. Thus when piston is at bottom dead center, total volume= swept volume + clearance volume.

Compression ratio: It is ratio of total cylinder volume to clearance volume

$$r = \frac{V_s + V_c}{V_c}$$

Where Vs = Swept volume Vc = Clearance volume

The compression ratio varies from 5:1 to 11:1 (average value 7:1 to 9:1) in S.I engine and from 12:1 to 24:1 (average value 15:1 to 18:1) in C.I engines.

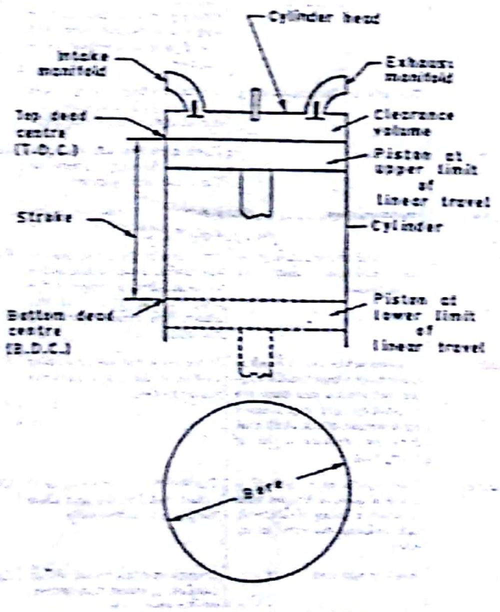

Fig 2: Terms relating I.C.Engines

Piston speed: The average speed of the piston is called piston speed.

Piston speed = 2 L N

Where L = Length of the stroke.

N = Speed of the engine in rpm.

FOUR STROKE CYCLE ENGINS

Here follows the description of the four stroke Otto and diesel cycle engines.

Otto engine: The Otto four stroke cycle refers to its use in petrol engines, gas engines, light oil engines and heavy oil engines in which the mixture of air and fuel are drawn in the engine cylinder. Since ignition in these engines is due to a spark, therefore they are also called spark ignition engines.

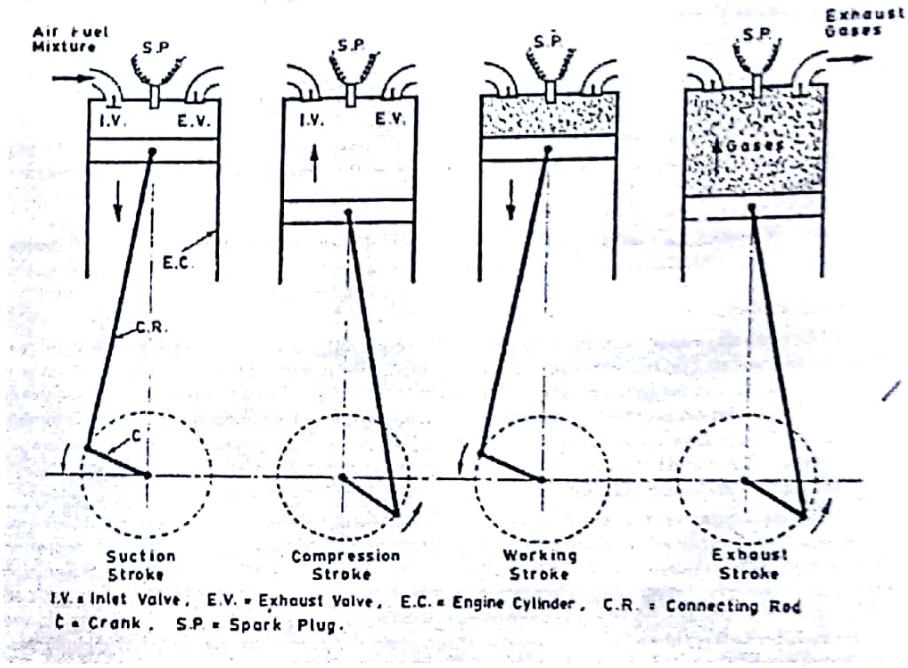

Fig 3: Four stroke Otto cycle engine

The various strokes of a four stroke (Otto) cycle engine are detailed below

1. **Suction Stroke:** During this stroke (also known as induction stroke) the piston moves from top dead center (T.D.C) to bottom dead center (B.D.C); the inlet valve opens and proportionate fuel air mixture is sucked in the engine cylinder. This operation is represented by the line 5-1. The exhaust valve remains closed through out the stroke.
2. **Compression Stroke:** In the stroke, the piston moves(1-2) towards (T.D.C) and compresses the enclosed fuel air mixture drawn in the engine cylinder during suction. The pressure of the mixture rises in the cylinder to a valve of about 8 bar. Just before the end of the stroke the operating plug initiates a spark which ignite the mixture and combustion takes place at constant volume (2-3). Both the inlet and exhaust valves remain closed during the stroke

3. **Expansion or Working Stroke:** when the mixture is ignite by the spark plug the hot gases are produced which drive or throw the piston from T.D.C to B.D.C and thus the work is obtained in this stroke. It is during this stroke when we get work from the engine; the other three strokes namely suction, compression and exhaust being idle. The flywheel mounted on the engine shaft stores energy during this stroke and supplies it during the idle strokes. The expansion of the gases is shown by 3-4. Both the valves remain closed during the start of this stroke but when the piston just reaches the B.D.C the exhaust valve opens.

4. **Exhaust Stroke:** This is the last stroke of the cycle. Here the gases from which the work has been collected become useless after the completion of the expansion stroke and are made to escape through exhaust valve to the atmosphere. This removal of gas is accomplished during this stroke. The piston moves from B.D.C. to T.D.C. and the exhaust gases are driven out of the engine cylinder; this is also called scavenging. This operation is represented by the line (1-5).

The actual indicator diagram off our stroke Otto cycle engine. It may be noted that line 5-1 is below the atmospheric pressure line.

This is due to the fact that owing to restricted area of the inlet passages the entering fuel air mixture cannot cope with the speed of the piston. The exhaust line 4-5 is slightly above the atmospheric pressure line. This is due to restricted exhaust passages which do not allow the exhaust gases to leave the engine-cylinder quickly.

The loop which has area 4-5-1 is called negative loop; it gives the pumping loss due to admission of fuel air mixture and removal of exhaust gases. The area 1234 is the total or gross negative work from the area 1234 i.e., gross work.

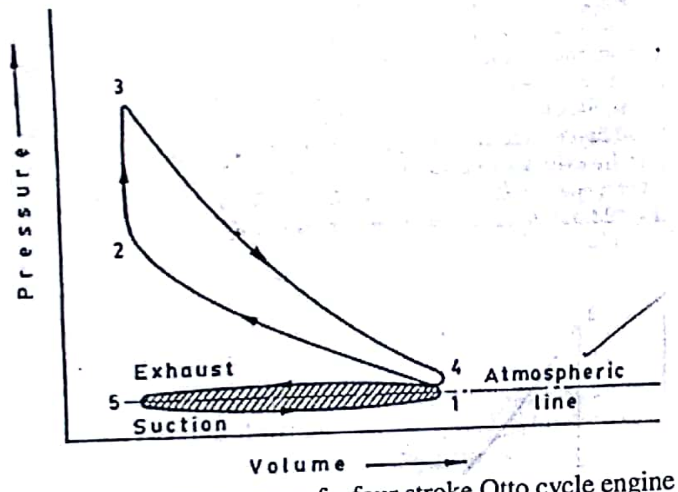

Fig 4: Actual p-V diagram of a four stroke Otto cycle engine

DIESEL ENGINES (four stroke cycle): As is the case of Otto four stroke; this cycle too is completed in four strokes as follows.

1. Suction Stroke: With the movement of the piston from T.D.C. to B.D.C. during this stroke, the inlet valve opens and the air at atmospheric pressure is drawn inside the engine cylinder; the exhaust valve however remains closed. This operation is represented by the line 5-1.

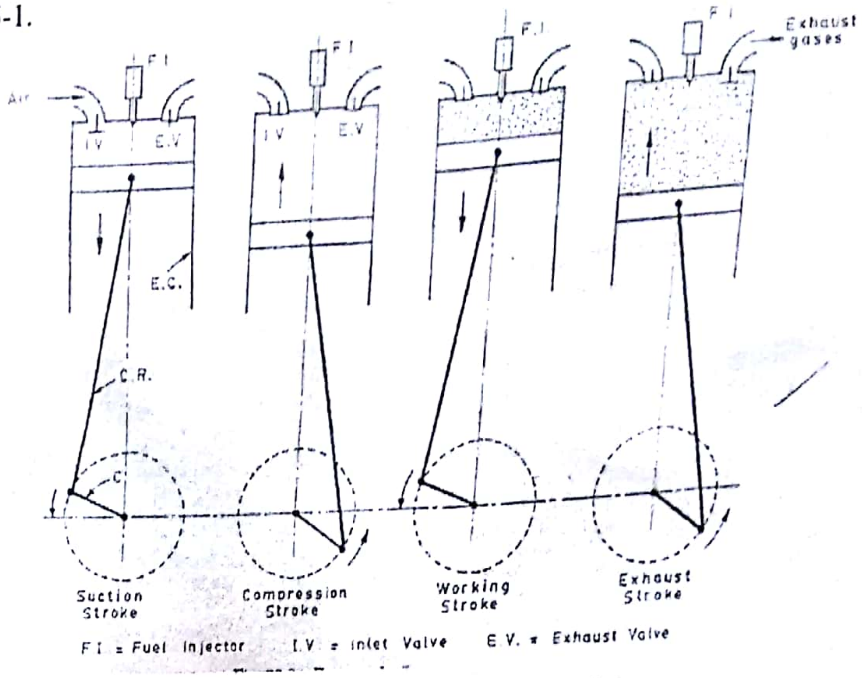

Fig 5: Four stroke Diesel cycle engine

2. Compression Stroke: The air drawn at atmospheric pressure during the suction stroke is compressed to high pressure and temperature (to the value of 35 bar and 600°C respectively) as the piston moves from B.D.C. to T.D.C. This operation is represented by 1-2. Both the inlet and exhaust valves do not open during any part of this stroke

3. Expansion or Working Stroke: As the piston starts moving from T.D.C. a matered quantity of fuel is injected into the hot compressed air in fine sprays by the fuel injector and it (fuel) starts burning at constant pressure shown by the line 2-3. At the point 3 fuel supply is cut off. The fuel is injected at the end of compression stroke but in actual practice the ignition of the fuel starts before the end of the compression stroke. The hot gases of the cylinder expand adiabatically to point 4, thus doing work on the piston. The expansion is shown by 3-4.

4. Exhaust Stroke: The piston moves from the B.D.C. to T.D.C. and the exhaust gases escape to the atmosphere through the exhaust valve. When the piston reaches the T.D.C. the exhaust valve closes and the cycle is completed. This stroke is represented by the line 1-5.

The actual indicator diagram for a four-stroke Diesel cycle engine. It may be noted that line 5-1 is below the atmospheric pressure line. This is due to the fact that owing to the restricted area of the inlet passages the entering air can't cope with the speed of the piston. The exhaust line 4-5 is slightly above the atmospheric line. This is because of the restricted exhaust passages which do not allow the exhaust gases to leave the engine cylinder quickly

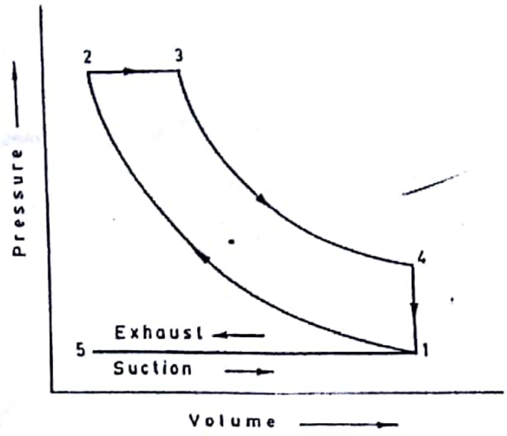

Fig 6: Theoretical p-V diagram of a four stroke Diesel cycel

.The loop of area 4-5-1 is called negative loop; it gives the pumping loss due to admission of air and removal of exhaust gases. The area 1234 is the total or gross work obtained from the piston and net work can be obtained by subtracting area 451 from area 1234.

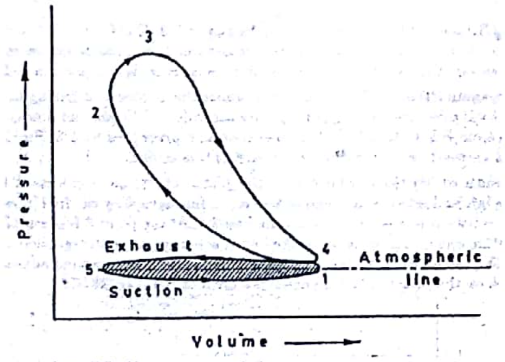

Fig 7: Actual p-V diagram of four stroke Diesel cycle

Valve Timing Diagrams (Otto and Diesel engines)

1. **Otto engine:** Theoretical valve timing diagram for four stroke “Otto cycle” engines which is self explanatory.

In actual practice, it is difficult to open and close the valve instantaneously; so as to get better performance of the engine the valve timings are modified. In Fig. 8 is shown an actual valve timing diagram. The inlet valve is opened 10° to 30^0 in advance of the T.D.C. position to enable the fresh charge to enter the cylinder and to help the burnt gases at the same time, to escape to the atmosphere. The suction of the mixture continues up to 30°-40^0 or even 60^0 after B.D.C. position. The inlet valve closes and the compression of the entrapped mixture starts

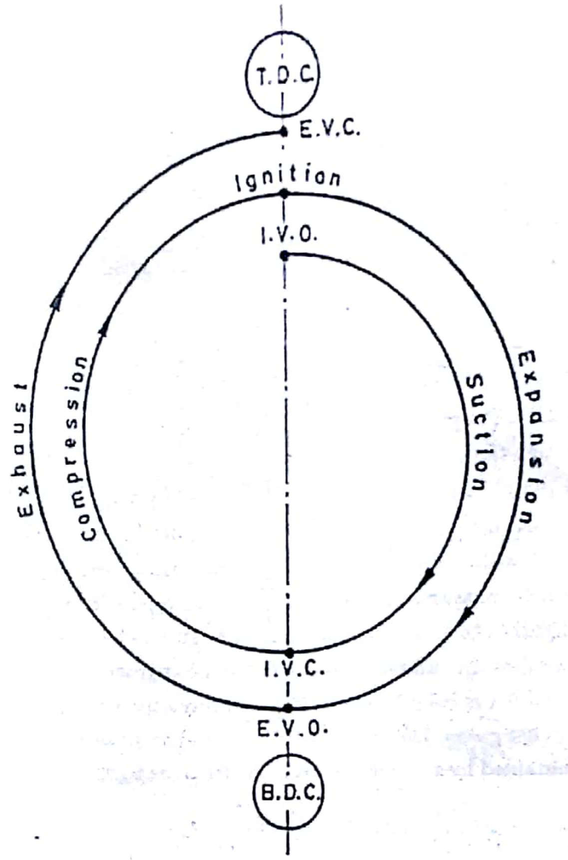

Fig 8: Theoretical valve timing diagram (four stroke Otto cycle engine)

The sparking plug produces a spark 30° to 40^0 before the T.D.C. position; thus fuel gets more time to burn. The pressure becomes maximum nearly 10^0 past the T.D.C. position. The exhaust valve opens 30^0 to 60^0 before the B.D.C. position and the gases are driven out of the cylinder by piston during its upward movement. The exhaust valve closes when piston is nearly 10^0 past T.D.C. position.

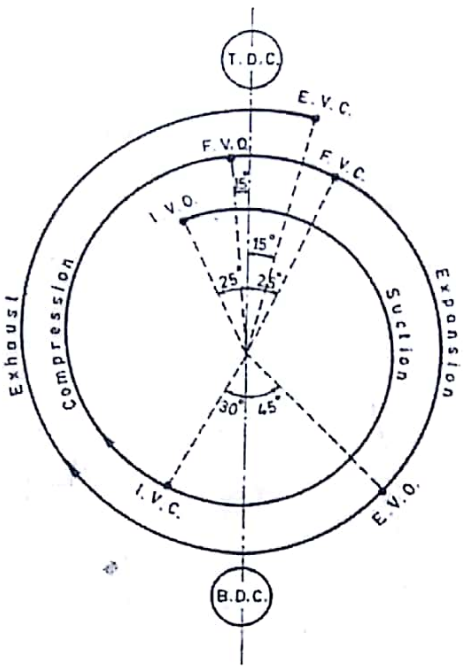

Fig 9: Actual valve timing diagram (Four stroke Diesel cycle engine)

2. **Diesel engines:** The valve timing diagram of a lour stroke "Diesel cycle" engine (theoretical valve timing diagram, is however the same as Fig. 8).

Inlet valve opens 10^0 to 25° in advance of T.D.C. position and doses 25^0 to 50° after the B.D.C. position. Exhaust valve opens 30° to 50° in advance of B.D.C. position and closes 10^0 to 15° after the T.D.C. position. The fuel injection takes place 5° to 10^0 before T.D.C. position and continues up to 15° to 25° near T.D.C. position

TWO STROKE CYCLE ENGINES

In 1878, Dugald-clerk, a British engineer introduced a cycle which could be completed in two strokes of piston rather than four strokes as is the case with the four stroke cycle engines. The engines using this cycle were called two stroke cycle engines. In this engine suction and exhaust strokes are eliminated. Here instead of valves, ports are used. The exhaust gases are driven out from engine cylinder by the fresh charge of fuel entering the cylinder nearly at the end of the working stroke.

Fig. 10 shows a two stroke petrol engine (used in scooters, motor cycles etc.). The cylinder L is connected to a closed crank chamber C.C. During the upward stroke of the piston M, the gases in L are compressed and at the same time fresh air and fuel (petrol) mixture enters the crank chamber through the valve V. When the piston moves downwards, V closes and the mixture in the crank chamber is compressed.

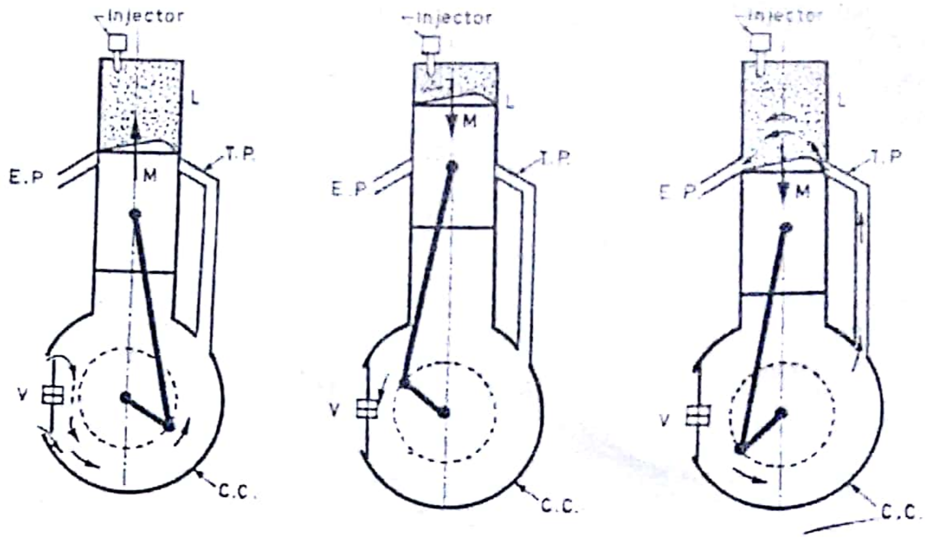

Fig 10: Two stroke cycle engine

Refer Fig. 10 (i), the piston is moving upwards and is compressing an explosive change which has previously been supplied to L. Ignition takes place at the end of the stroke. The piston then travels downwards due to expansion of the gases (Fig. 10 (i)) and near the end of this stroke the piston uncovers the exhaust port (E.P.) and the burnt exhaust gases escape through this port (Fig. 10 (iii)). The transfer port (T.P.) then is uncovered immediately, and the compressed charge from the crank chamber flows into the cylinder and is deflected upwards by the hump provided on the head of the piston. It may be noted that the incoming air petrol mixture helps the removal of gases from the engine cyclinder; if in case these exhaust gases do not leaves the cylinder, the fresh charge gets diluted and efficiency of the engine will decrease. The piston then again starts moving from B.D.C to T.D.C and the charge gets compressed when E.P (exhaust port) and T.P are covered by the piston; thus the cycle is repeated.

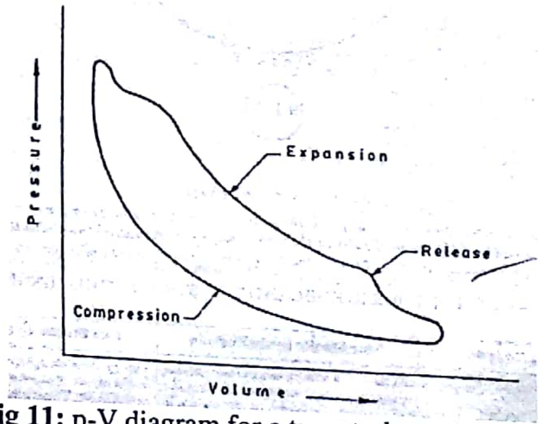

Fig 11: p-V diagram for a two stroke cycle engine

Show the P-V diagram for a two stroke cycle engine. It is only for the main cylinder or the top side of the piston.self explanatory port timing diagram for a two stroke cycle engine.

In a two stroke Diesel cycle engine all the operations are the same as in the spark ignition (Otto cycle) engine with the differences; firstly in this case, only air is admitted into instead of air fuel mixture and secondly fuel injector is fitted to supply the fuel instead of a sparking plug.

COMPARISON OF FOUR STROKE AND TWO STROKE CYCLE ENGINES

S.No	Aspects	Four Stroke Cycle Engines	Two Stroke Cycle Engines
1	Completion of cycle	The cycle is completed in four strokes of the piston or in two revolutions of the crankshaft. Thus one power stroke is obtained in every two revolutions of the crankshaft.	The cycle is completed in two strokes of the piston or in one revolutions of the crankshaft. Thus one power stroke is obtained in each revolutions of the crankshaft.
2	Flywheel required heavier or lighter	Because of the above turning movement is not so uniform and hence heavier flywheel is needed.	More uniform turning movement and hence lighter flywheel is needed.
3	Power produced for same size of engine	Again because of one power stroke for two revolutions, power produced for same size of engine is small or for the same power the engine is heavy and bulky.	Because of one power stroke for one revolution, power produced for same size of engine in more (theoretically twice, actual about 1.3 times) or for the same power the engine is light and compact.
4	Cooling and lubrication reuirements	Because of one power stroke in two revolution lesser cooling and lubrication requirements. Lesser rate of wear and tear.	Because of one power stroke in one revolution greater cooling and lubrication requirements. greater rate of wear and tear.
5	Valve and valve mechanism	The four stroke engine contains valve and valve mechanism.	Two stroke engine have no valves but only ports (some two stroke engines are fitted with conventional exhaust valves)
6	Initial cost	Because of the heavy weight and complication of valve mechanism, higher is the initial cost	Because of light weight and simplicity due to absence of valve mechanism, chapter in initial cost.

7	Volumetric efficiency	Volumetric efficiency more due to more time of induction.	Volumetric efficiency less due to lesser time for induction.
8	Thermal and part load efficiency	Thermal efficiency higher, part load efficiency better than two stroke cycle engine.	Thermal efficiency lower, part load efficiency lesser than four stroke cycle.
9	Applications	Used where efficiency is important; in cars, buses, trucks tractors, industrial engines, aeroplane, power generators etc.	In two stroke petrol engine some fuel is exhausted during scavenging. Used where(a) low coast (b) compactness and light weight important. Two stroke (air cool) petrol engine used in very small sizes only, lawn movers, scooters motor cycles (lubricating oil mixed with petrol). Two stroke diesel engines used in very large sizes more than 60 cm bore, for ship propulsion because of flow weight and compactness

COMPARISON OF SPARK IGNITION (S.I) AND COMBUSTIION IGNITION (C.I) ENGINES:

S.No	Aspects	S.I Engines	C.I Engines
1	Thermodynamic cycle	Otto cycle	Diesel cycle….for slow speed engines Dual cycle…… for high speed engines
2	Fuel used	Petrol	Diesel
3	Air fuel ratio	10:1 to 20:1	18:1 to 100:1
4	Compression	Up to 11: average valve 7 to 9; upper limit of compression ratio fixed by anti knock quality of fuel.	12 to 24; Average valve 15 to 18; Upper limit of compression ratio is limited by thermal and mechanical stresses
5	Combustion	Spark ignition	Compression ignition
6	Fuel supply	By carburetor…cheap method	By injection …explosive method.

7	Operating pressure 1. Compression pressure 2. Maximum pressure	7 bar to 15 bar 45 bar to 60 bar	30 bar to 50 bar 60 bar to 120 bar
8	Operating speed	High speed:2000 to 6000r.pm	Low speed:400 r.p.m Medium speed: 400 to 1200 r.p.m High speed : 1200 to 3500 r.p.m
9	Control power	Quantity governing …by throttle	Quality governing …by rack.
10	Calorific valve	44 MJ/Kg	42 MJ/Kg
11	Cost of running	High	Low
12	Maintenance cost	Minor maintenance required	Major overall required but less frequently
13	Supercharging	Limiting by detonation. Used only in aircraft engines.	Limited by blower power and mechanical and thermal stresses. Widely used.
14	Two stroke operation	Less suitable, fuel loss in scavenging. But small two stroke engines are used in mopeds, scooters and motor cycles due to their simplicity and low cost.	No fuel loss in scavenging. More suitable.
15	High power	No	Yes
16	Uses	Mopeds, scooters, motor-cycles, simple engine passenger cars, air crafts etc.	Buses, trucks locomotives, tractors, earth moving machinery and stationary generating plants.

COMPARISON BETWEEN A PETROL ENGINE AND A DIESEL ENGINE

S.No	Petrol engine	Diesel engine
1	Air petrol mixture is sucked in the engine cylinder during suction stroke.	Only air is sucked during suction stroke.
2	Spark plug is used	Employs an injector.
3	Power is produced by spark ignition.	Power is produced by compression ignition
4	Thermal efficiency up to 25%	Thermal efficiency up to 40%
5	Occupies less space.	Occupies more space.
6	~~More running cost~~	~~Less running cost~~

7	Light in weight	Heavy in weight
8	Fuel(petrol) costlier	Fuel (diesel) cheaper
9	Petrol being volatile is dangerous	Diesel is non dangerous as it is non-volatile
10	Pre-ignition possible	Pre-ignition not possible
11	Works on Otto cycle	Works on Diesel cycle
12	Less dependable	More dependable
13	Used in cars and motor cycles	Used in heavy duty vehicles like trucks, buses and heavy machinery

Notes-5

PERFORMANCE OF I.C. ENGINES

Engine performance is an indication of the degree of success with which it does its assigned job i.e., conversion of chemical energy contained in the fuel into the useful mechanical work.

In evaluation of engine performance certain basic parameters are chosen and the effect of various operating conditions, design concepts and modifications on these parameters are studied.

The basic performance parameters are numerated and discussed below:

1. Power and mechanical efficiency
2. Mean effective pressure and torque
3. Specific output
4. Volumetric efficiency
5. Fuel-air ratio
6. Specific fuel consumption
7. Thermal efficiency and heat balance
8. Exhaust smoke and other emissions
9. Specific weight.

1. Power and mechanical efficiency

(i) Indicated power. The total power developed by combustion of fuel in the combustion chamber is called indicated power.

$$I.P = \frac{n p_{mi} L A N k \times 10}{6} \text{ Kw} \qquad \text{--------(1)}$$

Where, n = Number of cylinders,

P_{mi} = Indicated mean effective pressure, bar,

L = Length of stroke, m,

A = Area of piston, m^2, and

k = ½ for 4-stroke engine

= 1 for 2-stroke engine.

(ii) Brake power (B.P.). The power developed by an engine at the output shaft is called the brake power.

$$B.P = \frac{2\pi NT \times 10}{6 \times 1000} \text{ Kw} \qquad \text{--------(2)}$$

Where, N = speed in rpm, and

T = torque in N-m.

The difference between I.P. and B.P. is called frictional power, F.P.

F.P. = I.P. - B.P. ---------(3)

The ratio of B.P. to I.P. is called mechanical efficiency

Mechanical efficiency, $\eta_{Mech} = \frac{B.P}{I.P}$ ---------(4)

2. Mean effective pressure and torque:

Mean effective pressure is defined as hypothetical pressure which is thought to be acting on the piston throughout the power stroke. If it is based on I.P. it is called indicated mean effective pressure ($I_{m.e.p.}$ or P_m) and if based on B.P. it is called brake mean effective pressure ($B_{m.e.p.}$ or P_{mb}) Similarly, frictional mean effective pressure ($F_{m.e.p.}$ or P_m) can be defined as :

$$F_{m.e.p.} = I_{m.e.p.} - B_{m.e.p} \quad \text{---------(5)}$$

The torque and mean effective pressure are related by the engine size. Since the power (P) of an engine is dependent on its size and speed, therefore it is not possible to compare engine on the basis of either power or torque. Mean effective pressure is the true indication of the relative performance of different engines.

3. Specific output

It is defined as the brake output per unit of piston displacement and is given by:

Specific output $\frac{B.P}{A \times L}$

= Constant x P_{mb} x rpm. ------------(6)

For the same piston displacement and brake mean effective pressure (P_{mb}) an engine running at higher speed will give more output.

4. Volumetric efficiency

It is defined as the ratio of actual volume (reduced to N. T.P.) of the charge drawn in during the suction stroke to the swept volume of the piston.

The average value of this efficiency is from 70 to 80 per cent but in case of supercharged engine it may be more than 100 per cent, if air at about atmospheric pressure is forced into the cylinder at a pressure greater than that of air surrounding the engine.

5. Fuel-air ratio

It is the ratio of the mass of fuel to the mass of air in the fuel-air mixture.

"Relative fuel air ratio" is defined as the ratio of the actual fuel air ratio to that of stoichiometric fuel-air ratio required to burn the fuel supplied.

6. Specific fuel consumption (s.f.c.)

It is the mass of fuel consumed per kW developed per hour, and is a criterion of economical power production.

$$s.f.c = \frac{m_f}{B.P} \text{ Kg/kWh}$$

7. Thermal efficiency and heat balance

Thermal efficiency: It is the ratio of indicated work done to energy supplied by the fuel.

If m_f = Mass of fuel used in kg/sec., and

C = Calorific value of fuel (lower),

Then indicated thermal efficiency (based on I.P.),

$$\eta_{th.}(I) = \frac{I.P}{m_f x C} \quad \text{-------------(7)}$$

and brake thermal efficiency (based on B.P.)

$$\eta_{th.}(B) = \frac{B.P}{m_f x C} \quad \text{-------------(8)}$$

Heat balance sheet

The performance of an engine is generally given by heat balance sheet.

To draw a heat balance sheet for I.C. engine, it is run at constant load. Indicator diagram is obtained with the help of an indicator. The quantity of fuel used in a given time and its calorific value, the amount, inlet and outlet temperatures of cooling water and the weight of exhaust gases are recorded. After calculating I.P. and B.P. the heat in different items is found as follows:

Heat supplied by fuel

For petrol and oil engines, heat supplied = m_f x C, where m_f, and C are mass used per minute (kg) and lower calorific value (kJ or kcal) of the fuel respectively.

For gas engines, heat supplied = V x C, where V and C is volume at N.T.P. (m^3/min.) and lower calorific value of gas respectively.

(i) Heat absorbed in I.P.

Heat equivalent of I.P. (per minute) = I.P. x 60 kJ -----------(9)

(ii) Heat taken away by cooling water

If, m_w = Mass of cooling water used per minute,

t_1 = Initial temperature of cooling water, and

t_2 = Final temperature of cooling water,

Then, heat taken away by water = $m_w \times C_w \times (t_2 - t_1)$ ----------(10)

where C_w = Specific heat of water.

(iii) Heat taken away by exhaust gases

If, m_e = Mass of exhaust gases (kg/min),

C_{pg} = Mean specific heat at constant pressure,

t_e = Temperature of exhaust gases, and

t_r = Room (or boiler house) temperature,

Then heat carried away by exhaust gases $= m_e \times c_{pg}(t_e - t_r)$ ----------(11)

Note: The mass of exhaust gases can be obtained by adding together mass of fuel supplied and mass of air supplied.

The heat balance sheet from the above data can be drawn as follows:

Item	kJ	Per cent
Heat supplied by fuel		
(i) Heat absorbed in I.P.	...	...
(ii) Heat taken away by cooling water	...	...
(iii) Heat carried away by exhaust gases	...	...
(iv) Heat unaccounted for (by difference)	...	...

8. Exhaust smoke and other emissions

Smoke is an indication of incomplete combustion. It limits the output of an engine if air pollution control is the consideration. Exhaust emissions have of late become a matter of grave concern and with the enforcement of legislation on air pollution in many countries, it has become necessary to view them as performance parameters.

9. Specific weight

It is defined as the weight of the engine in kg for each B.P. developed. It is an indication of the engine bulk.

Basic Measurements:

To evaluate the performance of an engine following basic measurements are usually under taken:

1. Speed
2. Fuel consumption
3. Air consumption
4. Smoke density
5. Exhaust gas analysis
6. Brake power
7. Indicated power and friction power
8. Heat going to cooling water
9. Heat going to exhaust.

1. Measurement of speed

The speed may be measured by:

(i) Revolution counters

(ii) Mechanical tachometer

(iii) Electrical tachometer.

2. Fuel measurement

The fuel consumed by an engine can be measured by the following methods:

(i) Fuel flow method

(ii) Gravemetric method

(iii) Continuous flow meters.

3. Measurement of air consumption

The air consumption can be measured by the following methods:

(i) Air box method

(ii) Viscous-flow air meter.

(i) Air box method

Fig. shows the arrangement of the system. It consists of air-tight chamber fitted with a sharp edged orifice of known co-efficient of discharge. The orifice is located away from the suction connection to the engine. Due to the suction of engine, there is a pressure depression in the air box or chamber which causes the flow through the orifice. For obtaining a steady flow, the volume of chamber should be sufficiently large compared with the swept volume of the cylinder, generally

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500 to 600 times the swept volume. It is assumed that the intermittent suction of the engine will not effect the air pressure in the air box as volume of the box is sufficiently large, and pressure in the box remains same.

A water manometer is used to measure the pressure difference causing the flow through the orifice. The depression across the orifice should not exceed 100 to 150 mm of water.

Let A = Area of orifice, m^2,

d = Diameter of orifice, cm,

h_w = Head of water in cm causing the flow,

C_d = Co-efficient of discharge for orifice,

ρ_a = Density of air in kg/m^3 under atmospheric conditions, and

P_w = Density of water in kg/m^3.

Head in meters of air (If) is given by:

$$H.\rho_a = \frac{h_w}{100}\rho_w$$

$$H = \frac{h_w}{100} x \frac{\rho_w}{\rho_a} = \frac{h_w}{100} x \frac{1000}{\rho_a} = \frac{10h_w}{\rho_a} \text{ m for air}$$

The velocity of air passing through the orifice is given by,

$$C_a = \sqrt{2gHST} \; m/s = \sqrt{2g\frac{10h_w}{\rho_a}} \; m/s$$

The volume of air passing through the orifice,

$$V_a = C_d \, xAxC_a = C_d A\sqrt{2g\frac{10h_w}{\rho_a}} = 14AC_d\sqrt{\frac{h_w}{\rho_a}} \; m/s$$

$$= 840 \, AC_d\sqrt{\frac{h_w}{\rho_a}} \; m^3/\min$$

Mass of air passing through the orifice is given by

$$m_a = V_a \, \rho_a = 14 \, x \frac{\pi d^2}{4x100^2} xC_d\sqrt{\frac{h_w}{\rho_a}} \, \rho_a \; m/s$$

$$= 0.0011 C_d x d^2 \sqrt{h_w \rho_a} \quad Kg/s$$

$$= 0.066 \; C_d \, x \, d^2 \sqrt{h_w \rho_a} \quad Kg/\min$$

(ii) Viscous-flow air meter

Alcock viscous-flow air meter is another design of air meter. It is not subjected to the error of the simple types of flow meters. With the air-box the flow is proportional to the square root of the pressure difference across the orifice. With the Alcock meter the air flows through a form of honeycomb so that flow is viscous. The resistance of the element is directly proportional to the air velocity and is measured by means of an inclined manometer. Felt pads are fitted in the manometer connections to damp out fluctuations. The meter is shown in Fig. 23.69.

The accuracy is improved by fitting a damping vessel between the meter and the engine to reduce the effect of pulsations.

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4. Measurement of exhaust smoke

The following smoke meters are used:

(i) Bosch smoke meter

(ii) Hatridge smoke meter

(iii)PHS smoke meter.

5. Measurement of exhaust emission

Substances which are emitted to the atmosphere from any opening down stream of the exhaust part of the engine are termed as exhaust emissions. Some of the more comm. only used instruments for measuring exhaust components are given below:

(i) Flame ionisation detector

(ii) Spectroscopic analysers

(iii) Gas chromatography.

6. Measurement of B.P.

The B.P. of an engine can be determined by a brake of some kind applied to the brake pulley of the engine. The arrangement for determination of B.P. of the engine is known as dynamometer.

The dynamometers are classified into following two classes:

(i) Absorption dynamometers

(ii) Transmission dynamometers.

(i) Absorption dynamometers. Absorption dynamometers are those that absorb the power to be measured by friction. The power absorbed in friction is finally dissipated in the form of heat energy.

Common forms of absorption dynamometers are:

- Prony brake
- Rope brake

- Hydraulic brake
- Fan brake
- Electrical brake dynamometers

- Eddy current dynamometer
- Swinging field d.c. dynamometer.

(ii) **Transmission dynamometers.** These are also called torquemeters. These are very accurate and are used where continuous transmission of load is necessary. There are used mainly in automatic units.

Here we shall discuss Rope brake dynamometer only:

Rope brake dynamometer

Refer Fig. 23.70. A rope is wound round the circumference of the brake wheel. To prevent the rope from slipping small wooden blocks (not shown in the Fig. 23.70) are laced to rope. To one end of the rope is attached a spring balance (S) and the other end carries the load (W). The speed of the engine is noted from the tachometer (revolution counter).

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If W = weight at the end of the rope, N,

S = spring balance reading, N,

N = engine speed, r.p.m.,

Db = diameter of the brake wheel, m,

d = diameter of the rope, m., and

(Db + d) = effective diameter of the brake wheel,

Then work/revolution = Torque x angle turned per revolution

$$= (W - S) \times \left(\frac{D_b + d}{2}\right) \times 2\pi = (W - S)(D_b + d) \times \pi$$

Work done /min $= (W - S)\pi(D_b + d) \times N$

$$= \frac{(W - S)\pi(D_b + d) \times N}{60}$$

$$B.P = \frac{(W - S)\pi(D_b + d) \times N}{60 \times 1000} \text{ Kw}$$ ---------()

$$= \frac{(W - S)\pi D_b N}{60 \times 1000}$$ ....If d is neglected

$$= \frac{T \times 2\pi N}{60 \times 1000} \text{Kw}$$ ---------()

Rope brake is cheap and easily constructed but not very accurate because of changes in friction co-efficient of the rope with temperature.

Measurement of Indicated power (I.P.)

The power developed in the engine cylinder or at the piston is necessarily greater than that at the crankshaft due to engine losses. Thus,

I.P. = B.P. + engine losses.

Indicated power is usually determined with the help of a p- V diagram taken with the help of an indicator. In case indicated power cannot be measured directly, it is made possible by measuring the brake power and also the engine losses. If the indicator diagram is available, the indicated power may be computed by measuring the area of diagram, either with a planimeter or by ordinate method, and dividing by the stroke measurement in order to obtain the mean effective pressure (m.e.p.).

$$p_{mi} = \frac{\textit{Net} \text{ area of diagram in mm}^2}{\textit{Length} \text{ of diagram in mm}} x \text{ Spring constnt}$$

Where p_{mi} is in bar.

(The spring constant is given in bar per mm of vertical movement of the indicator stylus.)

Engine indicators

The main types of engine indicators are:

1. Piston indicator
2. Balanced diaphragm type indicator
   (i) The Farnborough balanced engine indicator
   (ii) Dickinson-Newell indicator
   (iii)MIT balanced pressure indicator
   (iv)Capacitance-type balance pressure indicator.

3. Electrical indicators

In addition to this, optical indicators are also used.

Calculation of indicated power (I.P.):

If, P_{mi} = Indicated mean effective pressure, bar,

A = Area of piston, m^2,

L = Length of stroke, m,

N = Speed of the engine, r.p.m.,

k = ½ for 4-stroke engine,

= 1 for 2-stroke engine.

Then, Force on the piston $= P_{mi} \times A \times 10^5$ N

Work done per working stroke = Force x length of stroke

$= P_{mi} \times A \times 10^5 \times L$ N-m

Work done per second = Work done per stroke

x number of working stroke per second

$= P_{mi} \times A \times 10^5 \times N/60 \times k$ N-m/s or J/s

$$= \frac{p_{mi} \times LANk \times 10^5}{60 \times 1000} \text{ Kw}$$

Indicated power, I.P. $= = \frac{p_{mi} \times LANk \times 10}{6}$ Kw

If n is the number of cylinders, then

$$= \frac{n\, p_{mi} \times LANk \times 10}{6} \text{ Kw}$$

Morse test

This test is only applicable to multi-cylinder engines.

The engine is run at the required speed and the torque is measured. One cylinder is cut out, by shorting the plug if an S.I. engine is under test, or by disconnecting an injector if a C.I. engine is under test. The speed falls because of the loss of power with one cylinder cut out, but is restored by reducing the load. The torque is measured again when the speed has reached its original value. If the values of I.P. of the cylinders are denoted by I_1, I_2 I_3 and I_4 (considering a four-cylinder engine), and the power losses in each cylinder are denoted by L_1, L_2, L_3 and L_4, then the value of B.P, B at the test speed with all cylinders firing is given by

$$B = (I_1 - L_1) + (I_2 - L_2) + (I_3 - L_3) + (I_4 - L_4) \quad \text{----------}$$

If number 1 cylinder is cut out, then the contribution I1 is lost; and if the losses due to that cylinder remain the same as when it is firing, then the B.P., B_I now obtained at the same speed is

$$B_I = (0 - L_1) + (I_2 - L_2) + (I_3 - L_3) + (I_4 - L_4)$$

Subtracting equation (ii) from equation (i), we get

$$B - B_I = I_I$$

Similarly, $B - B_2 = I_2$ when cylinder number 2 is cut out,

$B - B_3 = I_3$ when cylinder number 3 is cut out,

$B - B_4 = I_4$ when cylinder number 4 is cut out

Then, for the engine, .

$$I = I_1 + I_2 + I_3 + I_4$$

Measurement of frictional power (F.P.):

The frictional power of an engine can be determined by the following methods:

1. Willan's line method (used for C.I. engines only)
2. Morse test
3. Motoring test
4. Difference between I.P. and B.P.

1. Willan's line method

At a constant engine speed the load is reduced in increments and the corresponding B.P. and gross fuel consumption readings are taken. A graph is then drawn of fuel consumption against B.P. as in Fig. 23.71. The graph drawn in called the Willan's line (analogous to Willan's line for a steam engine), and is extrapolated back to cut the B.P. axis at the point L. The reading OL is taken as the power loss of the engine at that speed. The fuel consumption at zero B.P. is given by OM; and if the relationship between fuel consumption and B.P. is assumed to be linear, then a fuel consumption OM is equivalent, to a power loss of OL.

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2. Morse test

In 'Morse test' (already discussed), frictional power can be found by subtracting (B.P')n from $(I,P)_n$

i.e., $F.P. = (I.P)_n - (B.P)_n$

Where n is the number of cylinders.

3. Motoring test

In this test the engine is first run up to the desired speed by its own power and allowed to remain under the given speed and load conditions for sometime so that oil, water and engine component temperatures reach stable conditions. The power of the engine during this period is absorbed by a dynamometer (usually of electrical type). The fuel supply is then cut off and by suitable electric switching devices the dynamometer is converted to run as a motor to drive or 'motor' the engine at the same speed at which it was previously running. The power supply to the motor is measured which is a measure of F.P of the engine.

4. Difference between I.P. and B.P.

The method of finding the F.P. by finding the difference between J.P. as obtained from an indicator diagram, and B.P. as obtained by a dynamometer is the ideal method. However, due to difficulties in obtaining accurate indicator diagrams, especially at high engine speeds, this method is usually only used in research laboratories and its use at commercial level is very limited.

1. *A single cylinder 4-stroke cycle oil engine works on diesel cycle. The following readings were taken when the engine was running at full load :*

Area of indicator = 3 cm², Length of the diagram = 4 cm., Spring constant = 10 bar/cm² cm., Speed of the engine = 400 r.p.m., Load on the brake = 380 N., Spring reading = 50 N, Diameter of the brake drum = 120 cm., Fuel consumption = 2.8 kg/hr., Calorific value of fuel = 42,000 kJ/kg. Diameter of the cylinder = 16 cm., Stroke of the piston = 20 cm.

From the above given data, find

(a) F.P. of the engine, and *(b) Mechanical efficiency.*

(c) Brake thermal efficiency, and *(d) Brake mean effective pressure.*

Solution. (a) I.P. of the engine $= \frac{100 \times p_m LA \cdot n}{1}$ kW

$$p_m = \frac{\text{Area of indicator diagram}}{\text{Length of indicator diagram}} \times \text{Spring constant} = \frac{3}{4} \times 10 = 7.5 \text{ bar}$$

$$\therefore \quad \text{I.P.} = \frac{100 \times 7.5 \times \frac{\pi}{4}(0.16)^2 \times \frac{20}{100} \times \frac{400}{60}}{1} = 10 \text{ kW}$$

$$\text{B.P.} = \frac{2\pi NT}{1000} = \frac{2\pi N(W-w)R}{1000} \text{ kW}$$

where $(W - w)$ is the net load on the brake and R is the radius of brake drum.

$$\therefore \quad \text{B.P.} = \frac{2\pi \times \frac{400}{60} \times (380-50) \times 0.6}{1000} = \mathbf{8.29\ kW}$$

$$\therefore \quad \text{F.P.} = 10 - 8.29 = \mathbf{1.71\ kW}$$

(b) Mechanical $\eta = \frac{\text{B.P.}}{\text{I.P.}} = \frac{8.29}{10} = 0.83$ or 83%.

(c) The brake thermal efficiency is given by, $\eta_{bt} = \frac{3600}{C_b \times \text{C.V.}}$

Specific fuel consumption $C_b = \frac{2.8}{8.29} = \mathbf{0.34\ kg/kWh}$

$$\therefore \quad \eta_{bt} = \frac{3600}{0.34 \times 42000} = 0.25 \text{ or } 25\%.$$

(d) Brake mean effective pressure is given by

$$BMEP = IMEP \times \eta_m = 7.5 \times 0.82 = \mathbf{6.15\ bar.}$$

2. *The following readings are taken on a single cylinder, 4-stroke gas engine 30 cm in diameter and 40 cm in stroke.*

R.P.M. = 200, No. of explosions = 80 per minute, Brake diameter = 150 cm, Net load on the brake = 1200 N, Net mean effective pressure = 6 bar, Gas used = 11.5 cu. m/hr, Pressure of the gas = 15 cm of water above atmospheric pressure, Barometer reading = 755 mm of Hg, Ambient temperature = 20°C, C.V. of the gas used = 21000 kJ/m³ at N.T.P.

Find out (a) the Mechanical efficiency and (b) the Brake thermal efficiency.

Solution. $\text{I.P.} = \frac{100 \times p_m LAn}{1} = \frac{100 \times 6 \times \frac{\pi}{4}(0.3)^2 \times \frac{40}{100} \times \frac{80}{60}}{1} = \mathbf{22.62\ kW}$

$$\text{B.P.} = \frac{2\pi NT}{1000} = \frac{2\pi N(W-w)R}{1000} = \frac{2\pi \times \frac{200}{60} \times 1200 \times 0.75}{1000} = \mathbf{18.85\ kW}$$

Mechanical $\eta_m = \frac{B.P.}{I.P.} = \frac{18.85}{22.62} = 0.832$ or 83.2%

Gas pressure $= 75.5 + \frac{15}{13.6} = 76.6$ cm of Hg

The pressure and temperature of the gas supplied to the engine are 76.6 cm Hg and 20°C.

The calorific value of the gas is given at N.T.P., so that the volume of gas used at N.T.P. should be found

$$V = \frac{11.5}{1} \times \frac{273}{293} \times \frac{766}{76} = 10.8 \text{ cu m/hr at N.T.P.}$$

∴ Specific gas consumption on B.P. basis

$$= \frac{10.8}{18.85} = 0.573 \text{ m}^3\text{/kWh at N.T.P.}$$

Brake thermal efficiency is given by

$$\eta_{bt} = \frac{3600}{C_b \times C.V.} = \frac{3600}{0.573 \times 21000} = 0.299 \text{ or } 29.9\%.$$

3. *A 4-cylinder, 4-stroke petrol engine 6 cm bore and 9 cm stroke was tested at constant speed. The fuel supply was fixed to 0.13 kg/min and plugs of 4-cylinders were successively short-circuited without change of speed :*

The power-measurements were as follows :

With all cylinders working = 16.25 kW, With No. 1st-cylinder cut-off = 11.55 kWh, With No. 2nd-cylinder cut-off = 11.65 kW (B.P.), With No. 3rd-cylinder cut-off = 11.70 kW (B.P.), With No. 4th-cylinder cut-off = 11.50 kW (B.P.)

Find (a) the I.P. of engine (b) the Mechanical efficiency,

(c) Indicated thermal efficiency if C.V. of fuel used is 42,000 kJ/kg

and (d) Also find the relative efficiency on I.P. basis assuming clearance volume 65 cu cm.

Solution. $(I.P.)_{1,2,3,4} = (B.P.)_{1,2,3,4} - F.P.$

$(I.P.)_{2,3,4} = (B.P.)_{2,3,4} - F.P.$

It is assumed that the F.P. is independent on load and it is dependent only on speed

From the above two equations

$$(I.P.)_1 = (B.P.)_{1,2,3,4} - (B.P.)_{2,3,4} = 16.25 - 11.55 = 4.70 \text{ kW}$$

Similarly,

$$(I.P.)_2 = (B.P.)_{1,2,3,4} - (B.P.)_{1,3,4} = 16.25 - 11.65 = 4.60 \text{ kW}$$

$$(I.P.)_3 = (B.P.)_{1,2,3,4} - (B.P.)_{1,2,4} = 16.25 - 11.70 = 4.55 \text{ kW}$$

$$(I.P.)_4 = (B.P.)_{1,2,3,4} - (B.P.)_{1,2,3} = 16.25 - 11.50 = 4.75 \text{ kW}$$

∴ I.P. of the engine $= (I.P.)_1 + (I.P.)_2 + (I.P.)_3 + (I.P.)_4$

$$= 4.70 + 4.60 + 4.55 + 4.75 = 18.50 \text{ kW}$$

Mechanical efficiency is given by $\eta_m = \frac{B.P.}{I.P.} \times 100 = \frac{16.25}{18.50} \times 100 = 88\%$

Indicated thermal efficiency is given by $\eta_{it} = \frac{3600}{C_i \times C.V.}$

where C_i is the specific fuel consumption on I.P. basis.

$$C_i = \frac{0.13 \times 60}{18.5} = 0.42 \text{ kg/kWh}$$

$$\therefore \quad \eta_{it} = \frac{3600}{0.42 \times 42000} = 0.204 = 20.4\%$$

Stroke volume $(v_s) = \frac{\pi}{4} d^2 \cdot L = \frac{\pi}{4} (6)^2 \times 9 = 254$ cu. cm

Compression ratio $= \frac{v_s + v_c}{v_c}$

$\therefore \quad R_c = \frac{254 + 60}{60} = \frac{314}{60} = 5.24$

Air standard efficiency is given by

$$\eta_a = 1 - \frac{1}{R_c^{\gamma - 1}} = 1 - \frac{1}{(5.24)^{0.4}} = 1 - \frac{1}{1.94} = 0.485 \text{ or } \mathbf{48.5\%}.$$

Relative efficiency on I.P. basis is given by

$$\eta_r = \frac{\eta_{it}}{\eta_a} = \frac{0.204}{0.485} = 0.42 \text{ or } 42\%.$$

4. *During the test on 4-cylinder, 4-stroke petrol engine, the following readings are taken Diameter of the cylinder = 8 cm, Stroke of the piston = 10 cm, Speed of the engine = 3000 r.p.m., Load on the hydraulic dynamometer = 160 N, Dynamometer constant = 20420 when the speed is in RPM, Fuel consumption = 8 kg/hr., C.V. of the fuel used = 43000 kJ/kg.*

The temperature and pressure of the charge at the end of suction stroke = 15°C and 1 bar.

A : F ratio = 13 : 1.

For the determination of the mechanical efficiency of the engine, a Morse test was carried out by shorting the spark-plugs of each cylinder successively without change of speed. The corresponding B.P. of the engine are 16.5, 16, 15.6 and 17.6 kW respectively. Determine :

(a) the B.P., Brake mean effective pressure and Brake thermal efficiency of the engine.

(b) Also find the mechanical efficiency and volumetric efficiency of the engine at suction condition. Take R (fuel-air mixture) = 287 Nm/kg - K.

Solution. B.P. (for hydraulic dynamometer $= \frac{WN}{K} = \frac{160 \times 3000}{20420} = \mathbf{23.4\ kW}$

B.P. of each cylinder $= \frac{23.4}{4} = 5.85$ kW

$$\text{B.P.} = \frac{100 \times p_{mb}\, LAn}{1}$$

$$\therefore \quad 5.85 = \frac{100 \times p_{mb} \times \frac{\pi}{4}\left(\frac{6}{100}\right)^2 \times \frac{9}{100} \times \frac{3000}{2 \times 60}}{1}$$

$\therefore \quad p_{mb} = \mathbf{6.9\ bar}$

Specific fuel consumption on B.P. basis

$$= \frac{8}{23.4} = \mathbf{0.342\ kg/kWh}$$

Brake thermal efficiency $= \frac{3600}{C_b \times \text{C.V}} = \frac{3600}{0.342 \times 43000} = 0.245 = \mathbf{24.5\%}$

(b) I.P. of the engine $= 23.4 \times 4 - (16.5 + 16 + 15.6 + 17.6)$
$= 93.6 - 65.7 = \mathbf{27.9\ kW}$

Mechanical efficiency $= \frac{\text{B.P.}}{\text{I.P.}} = \frac{23.4}{27.4} = 0.838$ or **83.8%**

$$\text{I.M.E.P.} = \frac{\text{B.M.E.P.}}{\eta_m} = \frac{6.9}{0.838} = \mathbf{8.23\ bar}$$

Mass of air used = 8 × 13 = 104 kg/hr.

Mass of mixture used = 8 + 104 = 112 kg/hr

Mass of mixture per cylinder per cycle

$$= \frac{112}{60} \times \frac{1}{4} \times \frac{1}{1500} = 3.12 \times 10^{-4} \text{ kg}$$

Volume of mixture per stroke at the given suction condition is calculated as

$$v_m = \frac{mRT}{p} = \left(\frac{3.12 \times 10^{-4} \times 287 \times 288}{1 \times 10^5} \right) \times 10^6 \text{ cm}^3 = 258 \text{ cm}^3$$

Stroke volume $= \frac{\pi}{4} d^2 L = \frac{\pi}{4} \times 64 \times 10 = 502.6$ cu. cm

$$\text{Volumetric efficiency} = \frac{\text{Actual volume at suction condition}}{\text{Stroke volume}} = \frac{258}{502.6} = 0.514 = \mathbf{51.4\%}.$$

. The following data is given for a 4-stroke, 4-cylinder diesel engine :

Diameter of the cylinder = 35 cm, Piston stroke = 40 cm, Speed of the engine = 315 r.p.m., Indicated mean effective pressure = 7 bar, B.P. of the engine = 260 kW, Fuel consumption = 80 kg/hr, C.V. of fuel used = 43000 kJ/kg, Hydrogen content in fuel = 13% and remaining is carbon, Air-consumption = 30 kg/min, Cooling water circulated = 90 kg/min, Rise in temperature of cooling water = 38°C, Piston cooling oil used = 45 kg/min, Rise in temperature of cooling oil = 23°C, C_p for cooling oil = 2.2 kJ/kg-K, Exhaust gas temperature = 322°C, C_p for exhaust gases = 1.1 kJ/kg-K, Ambient temperature = 22°C, C_p of superheated steam = 2 kJ/kg-K, Latent heat of steam = 2520 kJ/kg.

Find (a) The mechanical and indicated thermal efficiency.

(b) Draw up heat balance sheet on minute basis and percentage basis.

(c) Find the specific fuel consumption on B.P. basis when the load on the engine is 50% of full load assuming same indicated thermal efficiency.

Solution. I.P. of the engine

$$= \frac{100 \times p_m LAn}{1} \times 4 = \frac{100 \times 7 \times \frac{\pi}{4}(0.35)^2 \times \frac{40}{100} \times \frac{315}{2 \times 60}}{1} \times 4 = \mathbf{282.8 \text{ kW}}$$

$$\text{Mechanical } \eta = \frac{\text{B.P.}}{\text{I.P.}} = \frac{260}{282.8} = 0.92 = \mathbf{92\%}$$

Specific fuel consumption on I.P. basis

$$C_i = \frac{80}{282.8} = \mathbf{0.283 \text{ kg/kWh}}$$

The indicated thermal efficiency is given by

$$\eta_{it} = \frac{3600}{C_i \times C.V.} = \frac{3600}{0.283 \times 43000} = 0.29.6 = \mathbf{29.6\%}$$

$$\text{Heat supplied per minute} = \frac{80}{60} \times 43000 = \mathbf{57333 \text{ kJ}}$$

1. Heat in B.P. = 260 *kJ* × 60 = **15600 kJ/min**
2. Heat in cooling water = 90 × 4.2 × 38 = **14364 kJ/min**
3. Heat carried away by cooling oil = 45 × 2.2 × 23 = **2277 kJ/min.**

Total exhaust gases formed per minute

$$= \text{wt. of fuel/min.} + \text{weight of air/min} = \frac{80}{60} + 30 = 1.33 + 30 = \mathbf{31.33 \text{ kg/min.}}$$

31.33 kg of exhaust gases contain

$= \left(1.33 \times \frac{13}{100} \times 9 \right) =$ **1.455 kg of steam/min.** as the hydrogen in the fuel after burning forms the steam.

∴ Mass of dry exhaust gases per min.

$= 31.33 - 1.455 =$ **29.875 kg/min.**

4. Heat carried away by dry exhaust gases per min

$= 29.875 \times 1.1 \times (322 - 22) = 9858$ kJ/min

5. Heat carried by the steam formed per minute

$= 1.455\ [2520 + 2 \times 300] = 4540$ kJ/min

Heat unaccounted for $= a - (1 + 2 + 3 + 4 + 5)$

$= 57333 - [15600 + 14364 + 2277 + 9858 + 4540] = 10694$ kJ/min

Heat balance on minute basis

Heat supplied	*kJ*	*%*	*Heat Distributed*	*kJ*	*%*
Heat supplied by fuel	57333	100	(1) Heat in B.P.	15600	27.21
			(2) heat in cooling water	14364	25.00
			(3) Heat in cooling oil	2277	3.97
			(4) Heat in dry exhaust gases	9858	17.20
			(5) Heat in steam formed	4540	7.92
			(6) Heat unaccounted for	10694	18.70
	57333	100%		57333	100.00

At half load condition :

SFC (Specific fuel consumption) on I.P. basis = 0.283 kg/kWh

As indicated thermal efficiency is constant.

The Mech. $\eta = \frac{\text{B.P.}}{\text{B.P.} + \text{F.P.}}$

F.P. = 282.8 − 260 = 22.8 kW

$\therefore \quad \eta_m = \frac{(260/2)}{(260/2) + 22.8} = \frac{130}{152.8} = 0.85 = 85\%$

S.F.C. on B.P. basis $= \frac{0.283}{\text{Mech. } \eta} = \frac{0.283}{0.92} = 0.308$ kg/kWh.

The following data was collected during the trial on single cylinder, 4-stroke oil engine. Diameter of the cylinder = 250 mm, Length of the piston stroke = 600 mm, Area of indicator diagram = 4.5 cm², Length of indicator diagram = 7.1 cm, Spring constant = 8.5 bar/cm²-cm of compression, Speed of engine = 350 r.p.m., Load on hydraulic dynamometer = 980 N, Dynamometer constant = 11800 when load is in Newtons and speed is in RPM. Fuel used = 11.1 kg/hr, C.V. of fuel used = 42,000 kJ/kg. Cooling water circulated = 18.3 kg/min. Rise in temperature of cooling water = 25°C.

The mass analysis of oil used is given as C − 85%, H_2 − 13.5%, incombustible − 1.5%.

The volume analysis of exhaust gases is given as CO_2 − 8%, O_2 − 11%, N_2 − 81% by difference.

Temperature of exhaust gases = 400°C, Specific heat of exhaust gases = 1 kJ/kg-K, Ambient temperature = 25°C, Partial pressure of steam in exhaust gases = 0.035 bar, Specific heat of superheated steam = 2 kJ/kg-K.

Draw up heat balance sheet on minute basis and percentage basis.

Solution. $p_m = \frac{\text{Area of indicator diagram}}{\text{Length of indicator diagram}} \times \text{Spring strength} = \frac{4.5}{7.1} \times 8.5 = 5.38$ bar

$$\text{I.P.} = \frac{100 \times p_m LAn}{1} = \frac{100 \times 5.38 \times \frac{\pi}{4}(0.25)^2 \times \frac{60}{100} \times \frac{350}{2 \times 60}}{1} = 46.2 \text{ kW}$$

$$\text{B.P.} = \frac{WN}{K} = \frac{980 \times 350}{11800} = 29 \text{ kW}$$

Heat supplied per minute $= \frac{11.1}{60} \times 42{,}000 = $ **7770 kJ**

Heat in B.P. $= 29 \times 60 = $ **1740 kJ/min**

Heat carried away by cooling water per minute $= 81.3 \times 4.2 \times 25 = $ **1921 kJ/min**

The mass of air supplied per kg of fuel $= \frac{N \times C}{33\,(C_1 + C_2)} = \frac{81 \times 85}{33\,(0 + 8)} = $ **26.1 kg**

Mass of exhaust gases formed per kg of fuel $= 1 + 26.1 = $ **27.1 kg**

Mass of exhaust gases formed per minute $= \frac{11.1}{60} \times 27.1 = $ **5 kg**

Mass of steam formed and carried with the exhaust gases per minute due to the combustion of hydrogen in the fuel

$$= 0.135 \times \frac{11.1}{60} \times 9 = \mathbf{0.225\ kg}$$

Mass of dry exhaust gases formed per minute $= 5 - 0.225 = $ **4.775 kg**

Heat carried away by exhaust gases per minute $= 4.775 \times 1\,(400 - 25) = $ **1790 kJ/min**

Heat carried by the steam with exhaust gases per minute

$$= 0.225\,[h_f + h_{fg} + C_p\,(T_{sup} - T_s) - h']$$

the values of h_f, h_{fg} and T_s at pressure of 0.36 bar are taken from steam table.

$\therefore$ Heat carried by steam per minute $= 0.225\,[2550 + 2\,(400 - 27) - 25] = $ **736 kJ/min**

Heat unaccounted for is given by $= a - (1 + 2 + 3 + 4)$

$$= 7770 - [1740 + 1920 + 1790 + 736] = \mathbf{1583\ kJ/min.}$$

Heat Balance Sheet on Minute Basic

Heat supplied	*kJ*	*%*	*Heat Distributed*	*kJ*	*%*
Heat supplied by the fuel	7770	100	(1) Heat in B.P.	1740	22.40
			(2) Heat in cooling water	1921	24.70
			(3) Heat in dry exhaust gases	1790	23.00
			(4) Heat in steam carried with exhaust gases	736	9.50
			(5) Heat unaccounted for	1583	20.4
	7770	100		7770	100

The following observations were made during the test on an oil engine :

B.P. of the engine = 31.5 kW, Fuel used = 10.5 kg/hr., C.V. of fuel = 43000 kJ/kg., Jacket circulating water = 540 kg/hr., Rise in temperature of cooling water = 56°C.

Exhaust gases are passed through the exhaust gas calorimeter for finding the heat carried away by exhaust gases.

Water circulated through exhaust gas calorimeter = 454 kg/hr.

Rise in temperature of water passing through exhaust gas calorimeter = 36°C.

Temperature of exhaust gas leaving the exhaust gas calorimeter = 82°C.

A : F ratio = 19 : 1, Ambient temperature = 17°C., C_p for exhaust gases = 1 kJ/kg-K.

Draw up the heat balance sheet on minute and percentage basis.

Solution. Heat supplied by the fuel $= \frac{10.5}{60} \times 43000 = 7525$ kJ/min.

Heat in B.P. $= 31.5 \times 60 = 1890$ kJ/min.

Heat carried away by the jacket cooling water

$$= \frac{540}{60} \times 4.2 \times 56 = 2117 \text{ kJ/min.}$$

Mass of exhaust gases formed per minute

$$= \frac{10.5}{60} \times 20 \text{ as one kg of fuel supplied with 19 kg of air}$$

$$= 3.5 \text{ kg/min.}$$

Heat carried away by exhaust gases,

$$= m_w (T_{wo} - T_{wi}) + m_g C_{pg} (T_{go} - T_a)$$

$$= \frac{545}{60} \times 4.2\,(36) + 3.5 \times 1\,(82 - 17) = 1373 + 227 = 1600 \text{ kJ/min}$$

Heat unaccounted for $= a - (1 + 2 + 3) = 7525 - [1890 + 2117 + 1600] =$ **1918 kJ/min.**

Heat Balance Sheet on Minute Basis

Heat supplied	*kJ*	*%*	*Heat Distributed*	*kJ*	*%*
Heat supplied by the fuel	7525	100	(1) Heat in B.P.	1890	25.12
			(2) Heat in jacket cooling water	2117	28.13
			(3) Heat in exhaust gases	1600	21.25
			(4) Heat unaccounted for	1918	25.50
Total	7525	100		7525	100

Brake thermal efficiency $= \frac{\text{Heat in B.P.}}{\text{Heat supplied}} \times 100 = \frac{1890}{7525} \times 100 =$ **25.12%.**

UNIT-V

Belt, Rope and Chain Drives

Introduction:

The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds. The amount of power transmitted depends upon the following factors:

1. The velocity of the belt.
2. The tension under which the belt is placed on the pulleys.
3. The arc of contact between the belt and the smaller pulley.
4. The conditions under which the belt is used. It may be noted that

(a) The shafts should be properly in line to insure uniform tension across the belt section.
(b) The pulleys should not be too close together, in order that the arc of contact on the smaller pulley may be as large as possible.
(c) The pulleys should not be so far apar t as to cause the belt to weigh heavily on the shafts, thus increasing
the friction load on the bearings.
(*d*) A long belt tends to swing from side to side, causing the belt to run out of the pulleys, which in turn develops crooked spots in the belt.
(*e*) The tight side of the belt should be at the bottom, so that whatever sag is present on the loose side will increase the arc of contact at the pulleys.
(*f*) In order to obtain good results with flat belts, the maximum distance between the shafts should not exceed 10 metres and the minimum should not be less than 3.5 times the diameter of the larger pulley.

Selection of a Belt Drive

Following are the various important factors upon which the selection of a belt drive depends:

1. Speed of the driving and driven shafts,
2. Speed reduction ratio,
3. Power to be transmitted,
4. Centre distance between the shafts,
5. Positive drive requirements,
6. Shafts layout,
7. Space available, and
8. Service conditions.

Types of Belt Drives

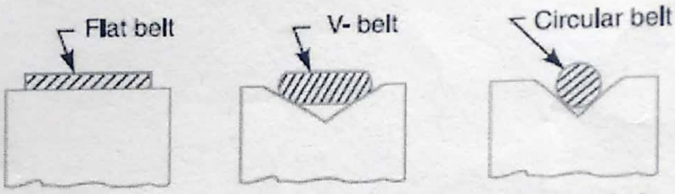
The belt drives are usually classified into the following three groups :

1. *Light drives*. These are used to transmit small powers at belt speeds upto about 10 m/s, as in agricultural machines and small machine tools.
2. *Medium drives*. These are used to transmit medium power at belt spe eds over 10 m/s but up to 22 m/s, as in machine tools.
3. *Heavy drives*. These are used to transmit large powers at belt speeds above 22 m/s, as in compressors and generators.

Types of Belts:
(*a*) Flat belt.
(*b*) V-belt.
(*c*) Circular belt.

Though there are many types of belts used these days, yet the following are important from the subject point of view :

1. ***Flat belt.*** The flat belt, as shown in Fig. 1(*a*), is mostly used in the factories and workshops, where a moderate amount of power is to be tr ansmitted, from one pulley to another when the two pulleys are not more than 8 metres apart.
2. ***V-belt.*** The V-belt, as shown in Fig. 1 (*b*), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.
3. ***Circular belt or rope***. The circular belt or rope, as shown in Fig. 1 (*c*), is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to ano ther, when the two pulleys are more than 8 meters apart.



(*a*) Flat belt. (*b*) V-belt. (*c*) Circular belt.

1. If a huge amount of power is to be transmitted, then a single belt may not be sufficient. In such a case, wide pulleys (for V -belts or circular belts) with a number of grooves are used. Then a belt in each groove is provided to transmit the required amount of power from one pulley to another.

Material used for Belts

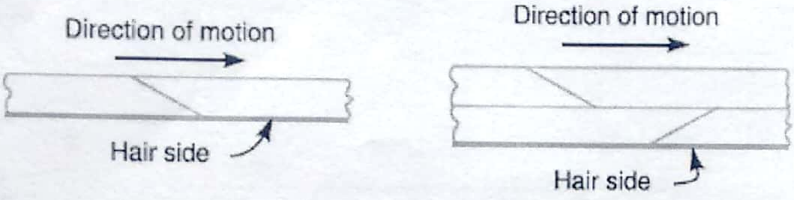
The material used for belts and ropes must be strong, flexible, and durable. It must have a high coefficient of frictio n. The belts, according to the material used, are classified as follows :

Leather belts. The most important material for the belt is leather. The best leather belts are made from 1.2 meters to 1.5 metres long strips cut from either side of the back bone of the top grade steer hides. The hair side of the leather is smoother and harder than the flesh side, but the flesh side is stronger. The fibers on the hair side are perpendicular to the surface, while those on the flesh side are interwoven and parallel to the surface. Therefore for these reasons, the hair side of a belt should be in contact with the pulley surface, as shown in Fig. . This gives a more intimate contact between the belt and the pulley and places the greatest tensile strength of the belt secti on on the outside, where the tension is maximum as the belt passes over the pulley.

The leather may be either oak-tanned or mineral salt tanned *e.g.* chrome tanned. In order to increase the thickness of belt, the strips are cemented together. The belts are specified according to the number of layers *e.g.* single, double or triple ply and according to the thickness of hides used *e.g.* light, medium or heavy.

The leather belts must be periodically cleaned and dressed or treated with a compound or
dressing containing neats foot or other suitable oils so that the belt will remain soft and flexible.

Cotton or fabric belts. Most of the fabric belts are made by folding canvass or cotton duck to three or more layers (depending upon the thickness desired) and



(*a*) Single layer belt. (*b*) Double layer belt.

stitching together. These belts are woven also into a strip of the desired width and thickness. They are impregnated with some filler like linseed oil in order to make the belts water proof and to prevent injury to the fibres. The cotton belts are cheaper and suitable in warm climates, in damp atmospheres and in exposed positions. Since the cotton belts require little attention, therefore these belts are mostly used in farm machinery, belt conveyor etc.

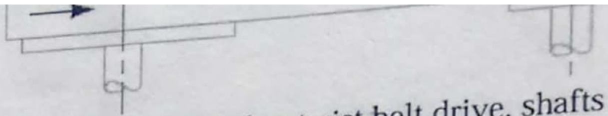
Rubber belt. The rubber belts are made of layers of fabric im pregnated with rubber composition and have a thin layer of rubber on the faces. These belts are very flexible but are quickly destroyed if allowed to come into contact with heat, oil or grease. One of the principal advantages of these belts is that they may be easily made endless. These belts are found suitable for saw mills, paper mills where they are exposed to moisture.

Balata belts. These belts are similar to rubber belts except that balata gum is used in place of rubber. These belts are acid proof and water proof and it is not affected by animal oils or alkalies. The balata belts should not be at temperatures above 40° C because at this temperature the balata begins to soften and becomes sticky. The strength of balata belts is 25 per cent higher than ru bber belts.

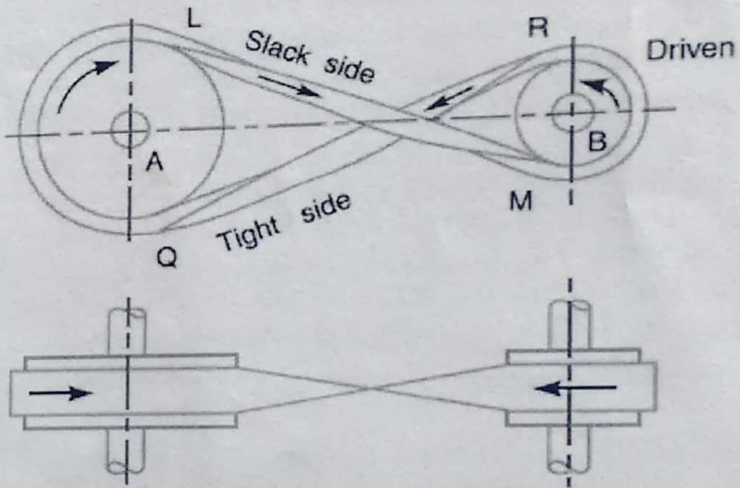
Types of Flat Belt Drives

The power from one pulley to another may be transmitted by any of the following types of belt drives:

Open belt drive. The open belt drive, as shown in Fig. is used with shafts arranged parallel and rotating in the same direction. In this case, the driver *A* pulls the belt from one side (*i.e.*lower side *RQ*) and delivers it to the other side (*i.e.* upper side *LM*). Thus the tension in the lower side belt will be more than that in the upper side belt. The lower side belt (because of more tension) is known as ***tight side*** whereas the upper side belt (because of less tension) is known as ***slack side***, as shown in Fig.

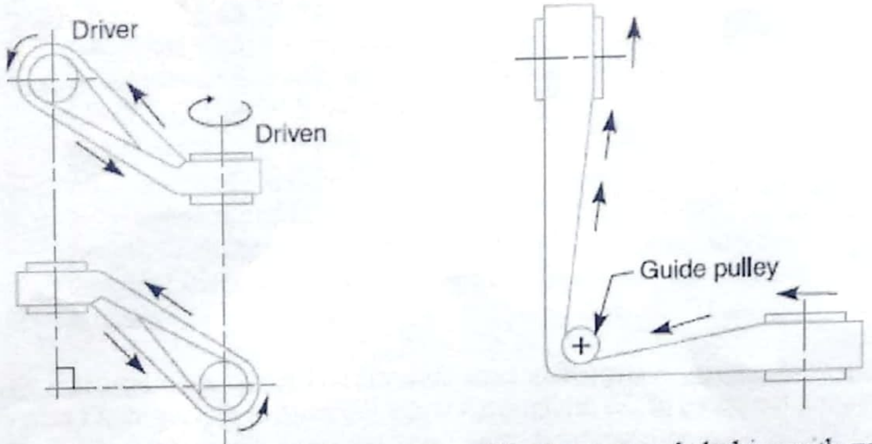


Crossed or twist belt drive. The crossed or twist belt drive, shafts arranged parallel and rotating in the opposite directions.
In this case, the driver pulls the belt from one side (*i.e.* *RQ*) and delivers it to the other side (*i.e.* *LM*). Thus the tension in the belt *RQ* will be more than that in the belt *LM*. The belt *RQ* (because of more tension) is known as ***tight side***, whereas the belt *LM* (because of less tension) is known as ***slack side***, as shown in Fig



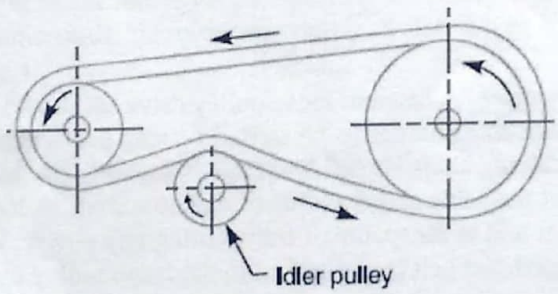
A little consideration will show that at a point where the belt crosses, it rubs against each other and there will be excessive wear and tear. In order to avoid th is, the shafts should be placed at a maximum distance of 20 *b*, where *b* is the width of belt and the speed of the belt should be less than 15 m/s.
Quarter turn belt drive. The quarter turn belt drive also known as right angle belt drive, as shown in Fig. (*a*), is used with shafts arranged at right angles and rotating in one definite direction. In order to prevent the belt from leaving the pulley, the width of the face of the pulley should be greater or equal to *b*, where *b* is the width of belt.
In case the pulleys cannot be arranged, as shown in Fig. (*a*), or when the reversible

motion is desired, then a ***quarter turn belt drive with guide pulley***, as shown in Fig. (*b*), may be used.



(*a*) Quarter turn belt drive. (*b*) Quarter turn belt drive with guide pulley.

Belt drive with idler pulleys. A belt drive with an idler pulley, as shown in Fig. (*a*), is used with shafts arranged parallel and when an open belt drive cannot be used due to small angle of contact on the smaller pulley. This type of drive is provided to obtain high velocity ratio and when the required belt tension cannot be obtained by other means.



When it is desired to transmit motion from one shaft to several shafts, all arranged in parallel,a belt drive with many idler pulleys, as shown in Fig. (*b*), may be employed.
Compound belt drive. A compound belt drive, as shown in Fig. is used when power is transmitted from one shaft to another through a number of pulleys.

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad ...(\because N_2 = N_3, \text{ being keyed to the same shaft})$$

A little consideration will show, that if there are six pulleys, then

$$\frac{N_6}{N_1} = \frac{d_1 \times d_3 \times d_5}{d_2 \times d_4 \times d_6}$$

$$\frac{\text{Speed of last driven}}{\text{Speed of first driver}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameters of drivens}}$$

Slip of Belt

In the previous articles, we have discussed the motion of belts and shafts assuming a firm frictional grip between the belts and the shafts. But sometimes, the fricti onal grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This may also cause some forward motion of the belt without carrying the driven pulley with it. This is called ***slip of the belt*** and is generally expressed as a percentage.The result of the belt slipping is to reduce the velocity ratio of the system. As the slipping of the belt is a common phenomenon, thus the belt should never be used where a definite velocity ratio is of importance (as in the case of hour,minute and second arms in a watch).

Let $s_1\,\%$ = Slip between the driver and the belt, and

$s_2\,\%$ = Slip between the belt and the follower.

∴ Velocity of the belt passing over the driver per second

$$v = \frac{\pi d_1 . N_1}{60} - \frac{\pi d_1 . N_1}{60} \times \frac{s_1}{100} = \frac{\pi d_1 . N_1}{60}\left(1 - \frac{s_1}{100}\right)$$

velocity of the belt passing over the follower per second,

$$\frac{\pi d_2 . N_2}{60} = v - v \times \frac{s_2}{100} = v\left(1 - \frac{s_2}{100}\right)$$

Substituting the value of v from equation (i),

$$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1}{100} - \frac{s_2}{100}\right)$$

$$= \frac{d_1}{d_2}\left(1 - \frac{s_1 + s_2}{100}\right) = \frac{d_1}{d_2}\left(1 - \frac{s}{100}\right)$$

If thickness of the belt (t) is considered, then

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{s}{100}\right)$$

Creep of Belt:

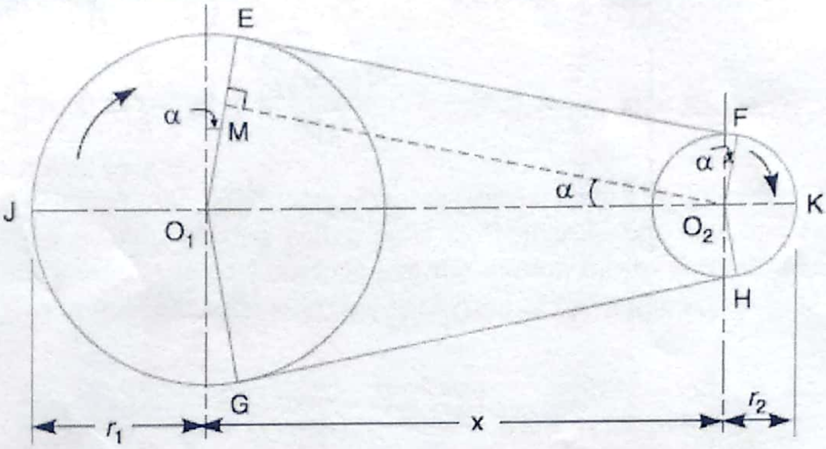
When the belt passes from the slack side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as **creep.** The total effect of creep is to reduce slightly the speed of the driven pulley or follower. Considering creep, the velocity ratio is given by

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

σ_1 and σ_2 = Stress in the belt on the tight and slack side respectively, and

E = Young's modulus for the material of the belt.

Length of an Open Belt Drive



Let r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* $O_1 O_2$), and

L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H as shown

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$
$$= 2\,(\text{Arc } JE + EF + \text{Arc } FK)$$

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - EM}{O_1 O_2} = \frac{r_1 - r_2}{x}$$

Since α is very small, therefore putting

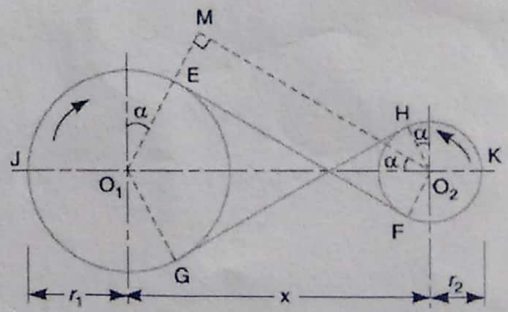
$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 - r_2}{x}$$

Similarly Arc $FK = r_2\left(\frac{\pi}{2} - \alpha\right)$

and
$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 - r_2)^2}$$
$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem,
$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + ...\right] = x - \frac{(r_1 - r_2)^2}{2x}$$
$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 - r_2)^2}{2x} + r_2\left(\frac{\pi}{2} - \alpha\right)\right]$$
$$= 2\left[r_1 \times \frac{\pi}{2} + r_1.\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \times \frac{\pi}{2} - r_2.\alpha\right]$$
$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$$
$$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$
$$L = \pi(r_1 + r_2) + 2 \times \frac{(r_1 - r_2)}{x} \times (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$
$$= \pi(r_1 + r_2) + \frac{2(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$
$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$
$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x}$$

Length of a Cross Belt Drive:



Let r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* $O_1 O_2$), and

L = Total length of the belt.

$$L = \pi(r_1 + r_2) + \frac{2(r_1 + r_2)}{x} \times (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

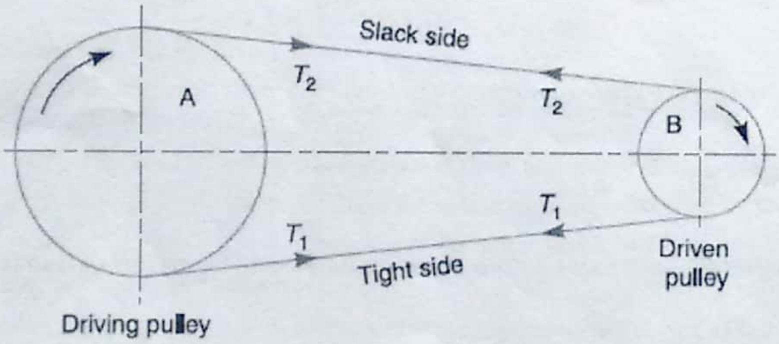
$$= \pi(r_1 + r_2) + \frac{2(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \quad \text{...(In terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \quad \text{...(In terms of pulley diameters)}$$

Power Transmitted by a Belt

Fig. shows the driving pulley (or driver) A and the driven pulley (or follower) B. We have already discussed that the driving pulley pulls the belt from one side and delivers the same to the other side. It is thus obvious that the tension on the former side (*i.e.* tight side) will be greater than the latter side (*i.e.* slack side) as shown in Fig.



Let

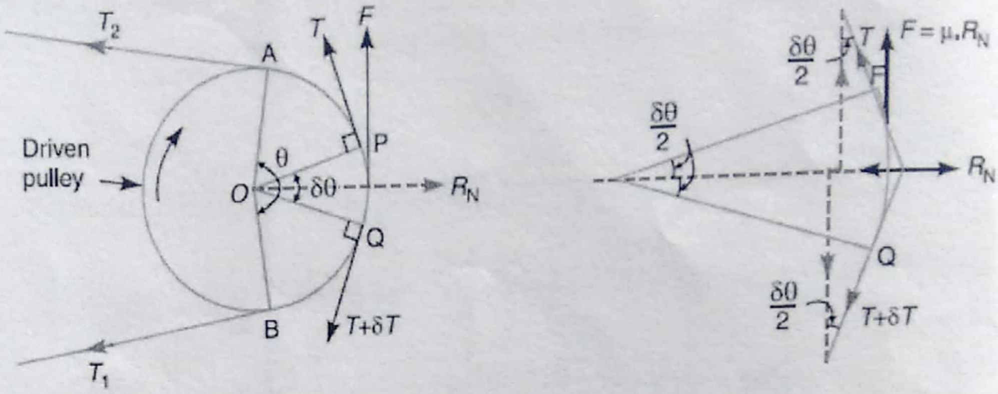
T_1 and T_2 = Tensions in the tight and slack side of the belt respectively in Newton

The effective turning (driving) force at the circumference of the follower is the difference between the two tensions (*i.e.* $T_1 - T_2$).

Work done per second = $(T_1 - T_2)\ v$ N-m/s

and power transmitted, $P = (T_1 - T_2)\ v$ W

Ratio of Driving Tensions For Flat Belt Drive:



T_1 = Tension in the belt on the tight side,
T_2 = Tension in the belt on the slack side, and
θ = Angle of contact in radians

Resolving all the forces horizontally and equating the same,

$$R_N = (T+\delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2}$$

Now resolving the forces vertically, we have

$$\mu \times R_N = (T+\delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2}$$

$$\mu \times R_N = T + \delta T - T = \delta T \quad \text{or} \quad R_N = \frac{\delta T}{\mu}$$

$$T.\delta\theta = \frac{\delta T}{\mu} \quad \text{or} \quad \frac{\delta T}{T} = \mu.\delta\theta$$

Integrating both sides between the limits T_2 and T_1 and from 0 to θ respectively,

i.e. $$\int_{T_2}^{T_1}\frac{\delta T}{T} = \mu\int_0^{\theta}\delta\theta \quad \text{or} \quad \log_e\left(\frac{T_1}{T_2}\right) = \mu.\theta \quad \text{or} \quad \frac{T_1}{T_2} = e^{\mu.\theta}$$

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

Centrifugal Tension

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as the slack sides. The tension caused by centrifugal force is called ***centrifugal tension***. At lower belt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s), its effect is considerable and thus should be taken into account.

Consider a small portion PQ of the belt subtending an angle $d\theta$ the centre of the pulley as shown in Fig.

Let m = Mass of the belt per unit length in kg,

v = Linear velocity of the belt in m/s,

r = Radius of the pulley over which the belt runs in metres, and

T_C = Centrifugal tension acting tangentially at P and Q in newtons.

We know that length of the belt PQ

$$= r.\, d\theta$$

mass of the belt PQ $= m.\, r.\, d\theta$

$\therefore$ Centrifugal force acting on the belt PQ,

$$F_C = (m.r.d\theta)\frac{v^2}{r} = m.d\theta.v^2$$

$$T_C \sin\left(\frac{d\theta}{2}\right) + T_C \sin\left(\frac{d\theta}{2}\right) = F_C = m.d\theta.v^2$$

$$2T_C\left(\frac{d\theta}{2}\right) = m.d\theta.v^2$$

$$T_C = m\,.\,v^2$$

Maximum Tension in the Belt

A little consideration will show that the maximum tension in the belt (T) is equal to the total tension in the tight side of the belt (T_{t1}).

Let σ = Maximum safe stress in N/mm^2,

b = Width of the belt in mm, and

t = Thickness of the belt in mm.

We know that maximum tension in the belt,

T = Maximum stress × cross-sectional area of belt = $\sigma.\, b.\, t$

When centrifugal tension is neglected, then

T (or T_{t1}) = T_1, *i.e.* Tension in the tight side of the belt

and when centrifugal tension is considered, then

T (or T_{t1}) = $T_1 + T_C$

Condition For the Transmission of Maximum Power

We know that power transmitted by a belt,

$$P = (T_1 - T_2)\, v$$

T_1 = Tension in the tight side of the belt in newtons,

T_2 = Tension in the slack side of the belt in newtons, and

v = Velocity of the belt in m/s.

$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \text{or} \quad T_2 = \frac{T_1}{e^{\mu.\theta}}$$

$$P = \left(T_1 - \frac{T_1}{e^{\mu.\theta}}\right)v = T_1\left(1 - \frac{1}{e^{\mu.\theta}}\right)v = T_1 .v.C$$

where $C = 1 - \dfrac{1}{e^{\mu.\theta}}$

We know that $T_1 = T - T_C$

where T = Maximum tension to which the belt can be subjected in newtons, and

T_C = Centrifugal tension in newtons.

$$P = (T - T_C)\,v.C$$

$$= (T - m.v^2)\,v.C = (T.v - m\,v^3)\,C$$

$$\frac{dP}{dv} = 0 \quad \text{or} \quad \frac{d}{dv}(T.v - mv^3)\,C = 0$$

$$\therefore \quad T - 3\,m\,.\,v^2 = 0$$

$$T - 3\,T_C = 0 \text{ or } T = 3\,T_C$$

$$v = \sqrt{\frac{T}{3m}}$$

Initial Tension in the Belt

When a belt is wound round the two pulleys (*i.e.* driver and follower), its two ends are joined together ; so that the belt may continuously move over the pulleys, since the motion of the belt from the driver and the follower is governed by a firm grip, due to friction between the belt and the pulleys. In order to increase this grip, the belt is tightened up. At this stage, even when the pulleys are stationary, the belt is subjected to some tension, called ***initial tension***.

When the driver starts rotating, it pulls the belt from one side (increasing tension in the belt on this side) and delivers it to the other side (decreasing the tension in the belt on that side). The increased tension in one side of the belt is called tension in tight side and the decreased tension in the other side of the belt is called tension in the slack side.

Let T_0 = Initial tension in the belt,

T_1 = Tension in the tight side of the belt,

T_2 = Tension in the slack side of the belt, and

α = Coefficient of increase of the belt length per unit force.

A little consideration will show that the increase of tension in the tight side

$$= T_1 - T_0$$

and increase in the length of the belt on the tight side

$$= \alpha (T_1 - T_0)$$

Similarly, decrease in tension in the slack side

$$= T_0 - T_2$$

and decrease in the length of the belt on the slack side

$$= \alpha (T_0 - T_2)$$

Assuming that the belt material is perfectly elastic such that the length of the belt remains constant, when it is at rest or in motion, therefore increase in length on the tight side is equal to decrease in the length on the slack side.

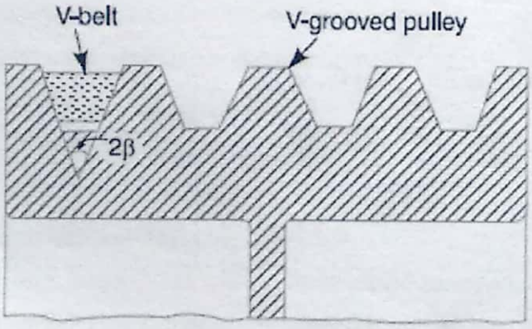
$$\alpha (T_1 - T_0) = \alpha (T_0 - T_2) \quad \text{or} \quad T_1 - T_0 = T_0 - T_2$$

$$\therefore \quad T_0 = \frac{T_1 + T_2}{2} \quad \text{...(Neglecting centrifugal tension)}$$

$$= \frac{T_1 + T_2 + 2T_C}{2} \quad \text{...(Considering centrifugal tension)}$$

V-belt drive

We have already discussed that a V-belt is mostly used in factories and workshops where a great amount of power is to be transmitted from one pulley to another when the two pulleys are very near to each other.



Advantages and Disadvantages of V-belt Drive Over Flat Belt Drive

Following are the advantages and disadvantages of the V-belt drive over flat belt drive.

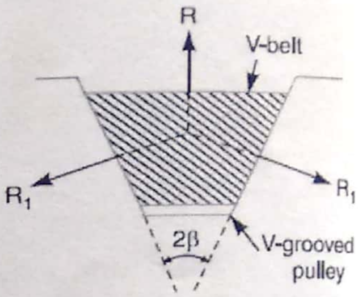
Advantages

1. The V-belt drive gives compactness due to the small distance between the centres of pulleys.
2. The drive is positive, because the slip between the belt and the pulley groove is negligible.
3. Since the V-belts are made endless and there is no joint trouble, therefore the drive is smooth.
4. It provides longer life, 3 to 5 years.

Disadvantages

1. The V-belt drive cannot be used with large centre distances.
2. The V-belts are not so durable as flat belts.
3. The construction of pulleys for V-belts is more complicated than pulleys for flat belts.
4. Since the V-belts are subjected to certain amount of creep, therefore these are not suitable for constant speed application such as synchronous machines, and timing devices.
5. The belt life is greatly influenced with temperature changes, improper belt tension and mismatching of belt lengths.
6. The centrifugal tension prevents the use of V-belts at speeds below 5 m/s and above 50m/s.

Ratio of Driving Tensions for V-belt



A V-belt with a grooved pulley is shown in Fig.

Let R_1 = Normal reaction between the belt and sides of the groove.

R = Total reaction in the plane of the groove.

2β = Angle of the groove.

μ = Coefficient of friction between the belt and sides of the groove.

Resolving the reactions vertically to the groove,

$$R = R_1 \sin\beta + R_1 \sin\beta = 2 R_1 \sin\beta$$

$$R_1 = \frac{R}{2\sin\beta}$$

We know that the frictional force

$$= 2\mu . R_1 = 2\mu \times \frac{R}{2\sin\beta} = \frac{\mu . R}{\sin\beta} = \mu . R \operatorname{cosec}\beta$$

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu . \theta \operatorname{cosec}\beta$$

Rope Drive

The rope drives are widely used where a large amount of power is to be transmitted, from one pulley to another, over a considerable distance. It may be noted that the use of flat belts is limited for the transmission of moderate power from one pulley to another when the two pulleys are not more than 8 metres apart. If large amounts of power are to be transmitted by the flat belt, then it would result in excessive belt cross-section. It may be noted that frictional grip in case of rope drives is more than that in V-drive. One of the main advantage of rope drives is that a number of separate drives may be taken from the one driving pulley. For example, in many spinning mills, the line shaft on each floor is driven by ropes passing directly from the main engine pulley on the ground floor.

The rope drives use the following two types of ropes :

1. Fibre ropes, and 2. Wire ropes.

The fibre ropes operate successfully when the pulleys are about 60 metres apart, while the wire ropes are used when the pulleys are upto 150 metres apart.

Advantages of Fibre Rope Drives

The fibre rope drives have the following advantages :

1. They give smooth, steady and quiet service.
2. They are little affected by out door conditions.
3. The shafts may be out of strict alignment.
4. The power may be taken off in any direction and in fractional parts of the whole amount.
5. They give high mechanical efficiency.

Ratio of Driving Tensions for Rope Drive

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta\ \text{cosec}\,\beta$$

Chain Drives

Advantages and Disadvantages of Chain Drive Over Belt or Rope Drive

Advantages

1. As no slip takes place during chain drive, hence perfect velocity ratio is obtained.
2. Since the chains are made of metal, therefore they occupy less space in width than a belt or rope drive.
3. The chain drives may be used when the distance between the shafts is less.
4. The chain drive gives a high transmission efficiency (upto 98 per cent).
5. The chain drive gives less load on the shafts.
6. The chain drive has the ability of transmitting motion to several shafts by one chain only.

Disadvantages

1. The production cost of chains is relatively high.
2. The chain drive needs accurate mounting and careful maintenance.
3. The chain drive has velocity fluctuations especially when unduly stretched.

Relation Between Pitch and Pitch Circle Diameter

Let d = Diameter of the pitch circle, and

T = Number of teeth on the sprocket.

$$p = AB = 2\,AO\sin\left(\frac{\theta}{2}\right) = 2\times\frac{d}{2}\sin\left(\frac{\theta}{2}\right) = d\sin\left(\frac{\theta}{2}\right)$$

$$p = d\sin\left(\frac{360^\circ}{2T}\right) = d\sin\left(\frac{180^\circ}{T}\right) \quad d = p\,\text{cosec}\left(\frac{180^\circ}{T}\right)$$

Let d = Diameter of the pitch circle, and

T = Number of teeth on the sprocket.

$$p = AB = 2\,AO\sin\left(\frac{\theta}{2}\right) = 2 \times \frac{d}{2}\sin\left(\frac{\theta}{2}\right) = d\sin\left(\frac{\theta}{2}\right)$$

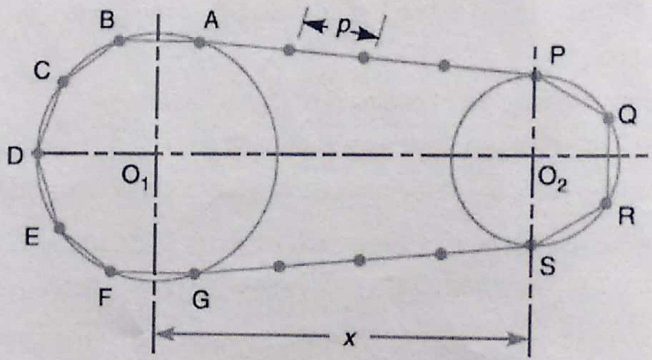
We know that $\theta = \dfrac{360^\circ}{T}$

$$\therefore \quad p = d\sin\left(\frac{360^\circ}{2T}\right) = d\sin\left(\frac{180^\circ}{T}\right)$$

$$d = p\,\text{cosec}\left(\frac{180^\circ}{T}\right)$$

Length of Chain

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$



Let T_1 = Number of teeth on the larger sprocket,

T_2 = Number of teeth on the smaller sprocket, and

p = Pitch of the chain.

$$d = p\,\text{cosec}\left(\frac{180^\circ}{T}\right) \quad \text{or} \quad r = \frac{p}{2}\,\text{cosec}\left(\frac{180^\circ}{T}\right)$$

$\therefore$ For larger sprocket,

$$r_1 = \frac{p}{2}\,\text{cosec}\left(\frac{180^\circ}{T_1}\right)$$

and for smaller sprocket, $r_2 = \frac{p}{2}\,\text{cosec}\left(\frac{180^\circ}{T_2}\right)$

Since the term $\pi(r_1 + r_2)$ is equal to half the sum of the circumferences of the pitch circles, therefore the length of chain corresponding to

$$\pi(r_1 + r_2) = \frac{p}{2}(T_1 + T_2)$$

$$L = \frac{p}{2}(T_1 + T_2) + 2x + \frac{\left[\frac{p}{2}\,\text{cosec}\left(\frac{180^\circ}{T_1}\right) - \frac{p}{2}\,\text{cosec}\left(\frac{180^\circ}{T_2}\right)\right]^2}{x}$$

If $x = m.p$, then

$$L = p\left[\frac{(T_1 + T_2)}{2} + 2m + \frac{\left[\text{cosec}\left(\frac{180^\circ}{T_1}\right) - \text{cosec}\left(\frac{180^\circ}{T_2}\right)\right]^2}{4\,m}\right] = p.K$$

where K = Multiplying factor

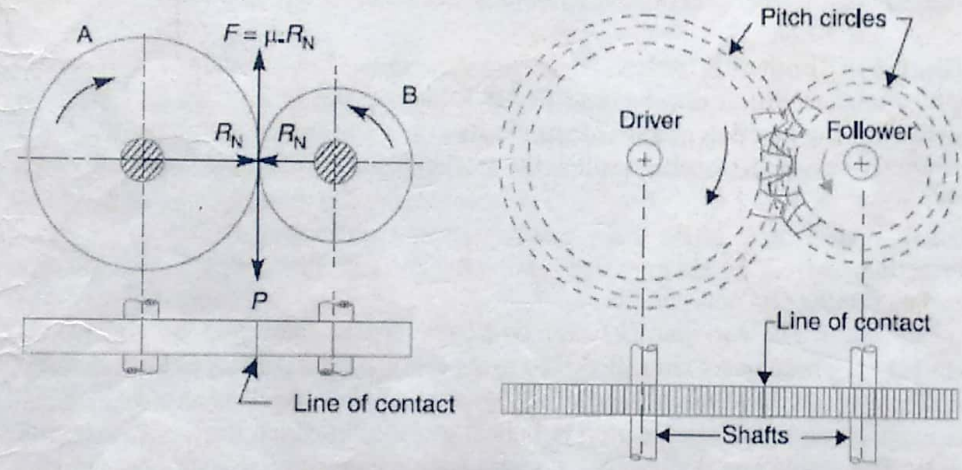
$$= \frac{(T_1 + T_2)}{2} + 2m + \frac{\left[\text{cosec}\left(\frac{180^\circ}{T_1}\right) - \text{cosec}\left(\frac{180^\circ}{T_2}\right)\right]^2}{4m}$$

Toothed Gears

The slipping of a belt or rope is a common phenomenon, in the transmission of motion or power between two shafts. The effect of slipping is to reduce the velocity ratio of the system. In precision machines, in which a definite velocity ratio is of importance (as in watch mechanism), the only positive drive is by means of gears or toothed wheels. A gear drive is also provided, when the distance between the driver and the follower is very small.

Friction Wheels:

The motion and power transmitted by gears is kinematically equivalent to that transmitted by friction wheels or discs. In order to understand how the motion can be transmitted by two toothed wheels, consi der two plain circular wheels A and B mounted on shafts, having sufficient rough surfaces and pressing against each other as shown in Fig. 1(a).



(a) Friction wheels. (b) Toothed wheels.

FIG: 1

Let the wheel A be keyed to the rotating shaft and the wheel B to the shaft, to be rotated. A little consideration will show, that when the wheel A is rotated by a rotating shaft, it will rotate the wheel B in the opposite direction as shown in Fig. 1(a). The wheel B will be rotated (by the wheel A) so long as the tangential force exerted by the wheel A does not exceed the maximum frictional resistance between the two wheels. But when the tangential force (P) exceeds the frictional resistance (F), slipping will take place between the two wheels. Thus the friction drive is not a positive drive.

In order to avoid the slipping, a number of projections (called teeth) as shown in Fig. 1 (b), are provided on the periphery of the wheel A, which will fit into the corresponding recesses on the periphery of the wheel B. A friction wheel with the teeth cut on it is known as toothed wheel or gear. The usual connection to show the toothed wheels is by their pitch circles

Advantages and Disadvantages of Gear Drive :
The following are the advantages and disadvantages of the gear drive as compared to belt, rope and chain drives :

Advantages:
1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

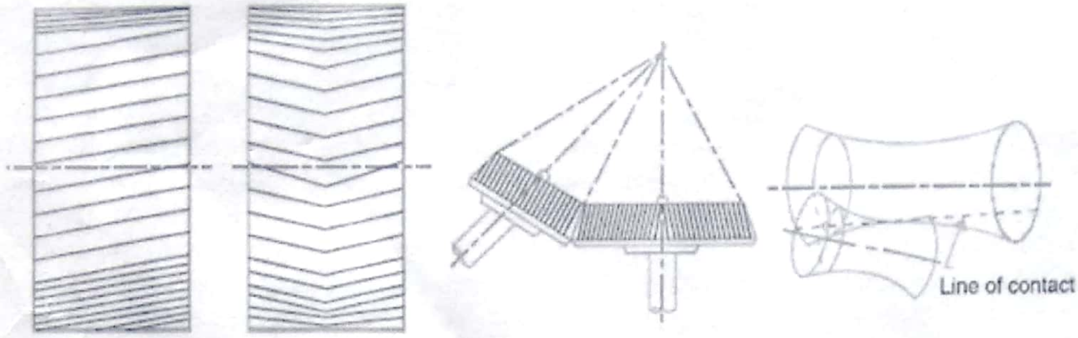
Disadvantages:
1. The manufacture of gears require special tools and equipm ent.
2. The error in cutting teeth may cause vibrations and noise during operation.

Classification of Toothed Wheels :
The gears or toothed wheels may be classified as follows :

1. According to the position of axes of the shafts .
The axes of the two shafts between which the motion is to be transmitted, may be

(a) Parallel,
(b) Intersecting, and
(c) Non-intersecting and non-parallel.

The two parallel and co-planar shafts connected by the gears is shown in Fig. 1. These gears are called spur gears and the arrangement is known as spur gearing. These gears have teeth parallel to the axis of the wheel as shown in Fig. 1. Another name given to the spur gearing is helical gearing, in which the teeth are inclined to the axis. The single and double helical gears connecting parallel shafts are shown in Fig. 2 (a) and (b) respectively. The double helical gears are known as herringbone gears. A pair of spur gears are kinematically equivalent to a pair of cylindrical discs, keyed to parallel shafts and having a line contact.



(*a*) Single helical gear. (*b*) Double helical gear. (*c*) Bevel gear. (*d*) Spiral gear.

FIG: 2

The two non-parallel or intersecting, but coplanar shafts connected by gears is shown in Fig. 2 (c). These gears are called bevel gears and the arrangement is known as bevel gearing. The bevel gears, like spur gears, may also have their teeth inclined to the face of the bevel, in which case they are known as helical bevel gears.

The two non-intersecting and non-parallel i.e. non-coplanar shaft connected by gears is shown in Fig. 2 (d). These gears are called skew bevel gears or spiral gears and the arrangement is known as skew bevel gearing or spiral gearing. This type of gearing also have a line contact, the rotation of which about the axes generates the two pitch surfaces known as hyperboloids.

When equal bevel gears (having equal teeth) connect two shafts whose axes are mutually perpendicular, then the bevel gears are known as *mitres.*

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2. According to the peripheral velocity of the gears.

The gears, according to the peripheral velocity of the gears may be classified as :

(a) Low velocity,

(b) Medium velocity, and

(c) High velocity.

The gears having velocity less than 3 m/s are termed as low velocity gears

Gears having velocity between 3 and 15 m/s are known as medium velocity gears.

If the velocity of gears is more than 15 m/s, then these are called high speed gears.

3. According to the type of gearing:

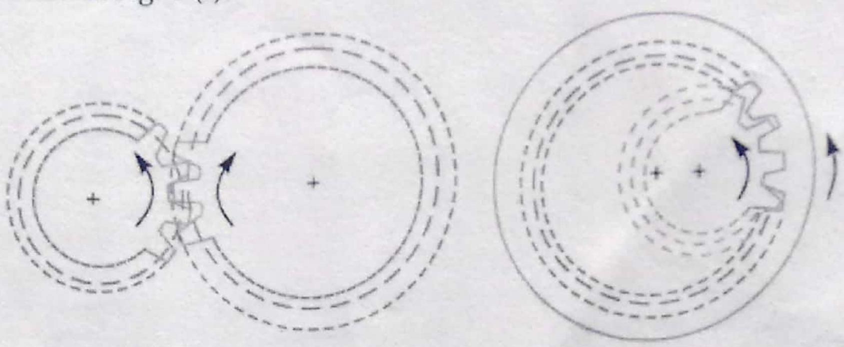
The gears, according to the type of gearing may be classified as :

(a) External gearing,

(b) Internal gearing, and

(c) Rack and pinion.

In external gearing, the gears of the two shafts mesh externally with each other as shown in Fig. 3 (a). The larger of these two wheels is called spur wheel and the smaller wheel is called pinion. In an external gearing, the motion of the two wheels is always unlike, as shown in Fig. 3 (a).

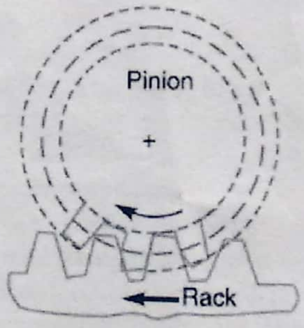


(a) External gearing. (b) Internal gearing.

FIG: 3

In internal gearing, the gears of the two shafts mesh internally with each other as shown in 3 (b). The larger of these two wheels is called annular wheel and the smaller wheel is called pinion. In an internal gearing, the motion of the two wheels is always like, as shown in Fig. 3 (b).

Sometimes, the gear of a shaft meshes externally and internally with the gears in a straight line, as shown in Fig. 4. Such type of gear is called rack and pinion. *The straight line gear is called rack and the circular wheel is called pinion.* A little consideration will show that with the help of a rack and pinion, we can convert linear motion into rotary motion and vice-versa as shown in Fig. .4.



Rack and pinion.

FIG: 4

4. According to position of teeth on the gear surface :

The teeth on the gear surface may be

(a) straight,

(b) inclined, and

(c) curved.

We have discussed earlier that the spur gears have straight teeth where as helical gears have their teeth inclined to the wheel rim. In case of spiral gears, the teeth are curved over the rim surface.

Terms Used in Gears:

The following terms, which will be mostly used in this chapter, should be clearly understood at this stage. These terms are illustrated in Fig.

1. Pitch circle. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

2. Pitch circle diameter. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as **pitch diameter**.

3. Pitch point. It is a common point of contact between two pitch circles.

4. Pitch surface. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. Pressure angle or angle of obliquity. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by φ. The standard pressure angles are $14\frac{1}{2}°$ and 20°.

6. Addendum. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. Dedendum. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. Addendum circle. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. Dedendum circle. It is the circle drawn through the bottom of the teeth. It is also called root circle.

Root circle diameter = Pitch circle dia meter × cos φ,
where φ is the pressure angle.

10. Circular pitch. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by pc.

Mathematically,

Circular pitch, $p_c = \pi D/T$

where D = Diameter of the pitch circle, and
T = Number of teeth on the wheel.

11. Diametral pitch. It is the ratio of number of teeth to the pitch circle diameter in milimetres.It is denoted by P_d.

Mathematically,

$$p_d = \frac{T}{D} = \frac{\pi}{p_c}$$

12. Module. It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m. Mathematically,

Module, m = D /T

13. Clearance. It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as **clearance circle**.

14. Total depth. It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.
15. Working depth. It is the radial distance from the addendum circle to the clearance circle.
It is equal to the sum of the addendum of the two meshing gears.
16. Tooth thickness. It is the width of the tooth measured along the pitch circle.
17. Tooth space . It is the width of space between the two adjacent teeth measured along the pitch
circle.
18. Backlash. It is the difference between the tooth space and the tooth thickness, as measured
along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some
backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.
19. Face of tooth. It is the surface of the gear tooth above the pitch surf ace.
20. Flank of tooth. It is the surface of the gear tooth below the pitch surface.
21. Top land. It is the surface of the top of the tooth.
22. Face width. It is the width of the gear tooth measured parallel to its axis.
23. Profile. It is the curve formed by the face and flank of the tooth.
24. Fillet radius. It is the radius that connects the root circle to the profile of the tooth.
25. Path of contact. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.
26. Length of the path of contact. It is the length of the common normal cut -off by the addendum circles of the wheel and pinion.
27. Arc of contact. It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, i.e.
(a) Arc of approach. It is the portion of the path of contact from the beginning of the engagement to the pitch point.
(b) Arc of recess. It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

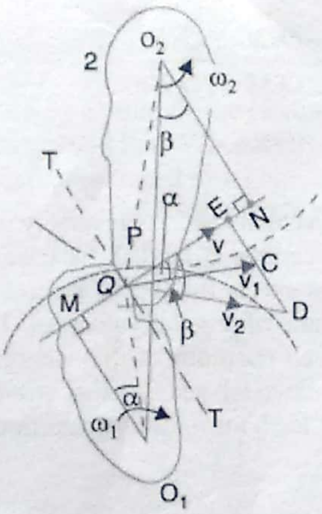
The ratio of the length of arc of contact to the circular pitch is known as **contact ratio** i.e. number of pairs of teeth in contact.

Gear Materials
The material used for the manufacture of gears depends upon the strength and service conditions like wear, noise etc. The gears may be manufactured from metallic or non - metallic materials.The metallic gears with cut teeth are commercially obtainable in cast iron, steel and bronze. The nonmetallic materials like wood, raw hide, compressed paper and synthetic resins like nylon are used for gears, especially for reducing noise.The cast iron is widely used for the manufacture of gears due to its good wearing properties, excellent machinability and ease of producing complicated shapes by casting method. The cast iron gears with cut teeth may be employed, where smooth action is not important.The steel is used for high strength gears and steel may be plain carbon steel or

alloy steel. The steel gears are usually heat treated in order to combine properly the toughness and tooth hardness. The phosphor bronze is widely used for worm gears in order to reduce wear of the worms which will be excessive with cast iron or steel.

Condition for Constant Velocity Ratio of Toothed Wheels–Law of Gearing

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig. Let the two teeth come in contact at point Q, and the wheels rotate in the directions as shown in the figure. Let TT be the common tangent and MN be the common normal to the curves at the point of contact Q. From the centres O_1 and O_2, draw O_1M and O_2N perpendicular to MN. A little consideration will show that the point Q moves in the direction QC, when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel 2. Let v_1 and v_2 be the velocities of the point Q on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal.



Law of gearing.

$$v_1 \cos\alpha = v_2 \cos\beta$$

$$(\omega_1 \times O_1Q)\cos\alpha = (\omega_2 \times O_2Q)\cos\beta$$

$$(\omega_1 \times O_1Q)\frac{O_1M}{O_1Q} = (\omega_2 \times O_2Q)\frac{O_2N}{O_2Q} \quad \text{or} \quad \omega_1 \times O_1M = \omega_2 \times O_2N$$

$$\therefore \quad \frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}$$

Also from similar triangles O_1MP and O_2NP,

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

$$\therefore \quad \frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

From above, we see that the angular velocity ratio is inversely proportional to the ratio of the distances of the point P from the centres O_1 and O_2, or the common normal to the two surfaces at the point of contact Q intersects the line of centres at point P which divides the centre distance inversely as the ratio of angular velocities. Therefore in order to have a constant angular velocity ratio for all positions of the wheels, the point P must be the fixed point (called pitch point) for the two wheels. In other words, the common normal at the point of contact between a pair of teeth must always pass through the pitch point .

Velocity of Sliding of Teeth:

The sliding between a pair of teeth in contact at Q occurs along the common tangent T T to the tooth curves as shown in Fig. The velocity of sliding is the velocity of one tooth relative to its mating tooth along the common tangent at the point of contact .

The velocity of point Q, considered as a point on wheel 1, along the common tangent T T is represented by EC. From similar triangles QEC and O_1MQ,

$$\frac{EC}{MQ} = \frac{v}{O_1Q} = \omega_1 \quad \text{or} \quad EC = \omega_1.MQ$$

the velocity of point Q, considered as a point on wheel 2, along the common tangent T T is represented by ED. From similar triangles QCD and O_2 NQ,

$$\frac{ED}{QN} = \frac{v_2}{O_2Q} = \omega_2 \quad \text{or} \quad ED = \omega_2.QN$$

Let v_S = Velocity of sliding at Q.

$$\therefore \quad v_S = ED - EC = \omega_2 . QN - \omega_1 . MQ$$

$$= \omega_2 (QP + PN) - \omega_1 (MP - QP)$$

$$= (\omega_1 + \omega_2) QP + \omega_2 . PN - \omega_1 . MP$$

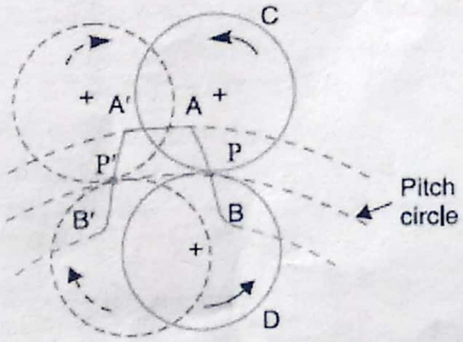
Since $\dfrac{\omega_1}{\omega_2} = \dfrac{O_2 P}{O_1 P} = \dfrac{PN}{MP}$ or $\omega_1 . MP = \omega_2 . PN$,

$$v_S = (\omega_1 + \omega_2) QP$$

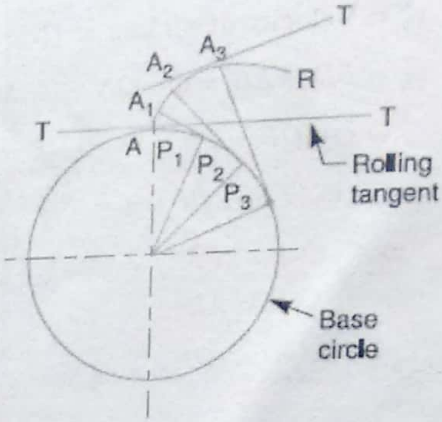
Forms of teeth:
1. *Cycloidal* **teeth**
2. **Involute teeth.**

Cycloidal Teeth

A *cycloid* is the curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line. When a circle rolls without slipping on the outside of a fixed circle, the curve traced by a point on the circumference of a circle is known as ***epi-cycloid***. On the other hand, if a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a point on the circumference of a circle is called ***hypo-cycloid***.

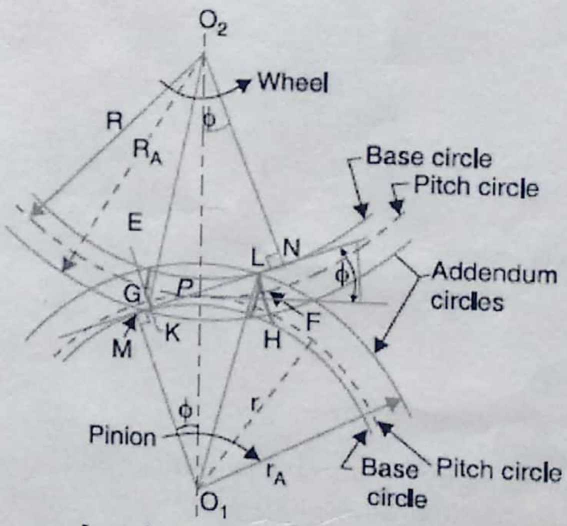


Involute Teeth:
An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which in unwrapped fr om a reel as shown in Fig. . In connection with toothed wheels, the circle is known as base circle. The involute is traced as follows.



Length of Path of Contact:

Consider a pinion driving the wheel as shown in Fig. . When the pinion rotates in clockwise direction, the contact between a pair of involute teeth begins at K (on the flank near the base circle of pinion or the outer end of the tooth face on the wheel) and ends at L (outer end of the tooth face on the pinion or on the flank near the base circle of wheel). MN is the common normal at the point of contacts and the common tangent to the base circles. The point K is the intersection of the addendum circle of wheel and the common tangent. The point L is the intersection of the addendum circle of pinion and common tangent.



Length of path of contact.

That the length of path of contact is the length of common normal cutoff by the addendum circles of the wheel and the pinion. Thus the length of path of contact is KL

which is the sum of the parts of the path of contacts KP and PL. The part of the path of contact KP is known as ***path of approach*** and the part of the path of contact PL is known as ***path of recess***.

Let $r_A = O_1L$ = Radius of addendum circle of pinion,

$R_A = O_2K$ = Radius of addendum circle of wheel,

$r = O_1P$ = Radius of pitch circle of pinion, and

$R = O_2P$ = Radius of pitch circle of wheel.

From Fig. we find that radius of the base circle of pinion,

$$O_1M = O_1P \cos\phi = r\cos\phi$$

and radius of the base circle of wheel,

$$O_2N = O_2P\cos\phi = R\cos\phi$$

Now from right angled triangle O_2KN,

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

and $$PN = O_2P\sin\phi = R\sin\phi$$

$\therefore$ Length of the part of the path of contact, or the path of approach,

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

Similarly from right angled triangle O_1ML,

and $$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2\cos^2\phi}$$

$$MP = O_1P\sin\phi = r\sin\phi$$

$\therefore$ Length of the part of the path of contact, or path of recess,

$$PL = ML - MP = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$\therefore$ Length of the path of contact,

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R + r)\sin\phi$$

Length of Arc of Contact:

Length of the arc of contact

$$= \text{arc } GP + \text{arc } PH = \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi}$$

$$= \frac{\text{Length of path of contact}}{\cos\phi}$$

Contact Ratio (or Number of Pairs of Teeth in Contact):

The contact ratio or the number of pairs of teeth in contact is defined as the ratio of the length of the arc of contact to the circular pitch.

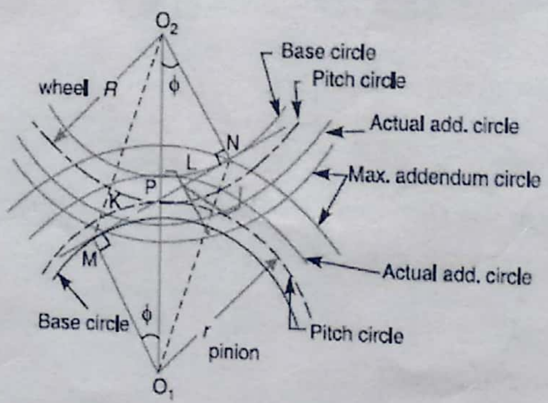
Contact ratio $= \dfrac{\text{Length of the arc of contact}}{p_c}$

p_c = Circular pitch = πm, and

m = Module.

Interference in Involute Gears

Fig. shows a pinion with centre O_1, in mesh with wheel or gear with centre O_2. MN is the common tangent to the base circles and KL is the path of contact between the two mating teeth.



Interference in involute gears.

A little consideration will show, that if the radius of the addendum circle of pinion is increased to O_1N, the point of contact L will move from L to N. When this radius is further increased, the point of contact L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel. The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as *interference*, and occurs when the teeth are being cut. In brief, *the phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.*

Similarly, if the radius of the addendum circle of the wheel increases beyond O_2M, then the tip of tooth on wheel will cause interference with the tooth on pinion. The points M and N are called *interference points*. Obviously, interference may be avoided if the path of contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is O_1N and of the wheel is O_2M.

From the above discussion, we conclude that the interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both the teeth. In other

When interference is just avoided, the maximum length of path of contact is MN when the maximum addendum circles for pinion and wheel pass through the points of tangency N and M respectively as shown in Fig.

Maximum length of path of approach,

$$MP = r \sin \phi$$

and maximum length of path of recess,

$$PN = R \sin \phi$$

∴ Maximum length of path of contact,

$$MN = MP + PN = r \sin \phi + R \sin \phi = (r + R) \sin \phi$$

and maximum length of arc of contact

$$= \frac{(r + R) \sin \phi}{\cos \phi} = (r + R) \tan \phi$$

Minimum Number of Teeth on the Pinion in Order to Avoid Interference

We have already discussed in the previous article that in order to avoid interference, the addendum circles for the two mating gears must cut the common tangent to the base circles between the points of tangency. The limiting condition reaches, when the addendum circles of pinion and wheel pass through points N and M (see Fig.

Let t = Number of teeth on the pinion,,

T = Number of teeth on the wheel,

m = Module of the teeth,

r = Pitch circle radius of pinion $= m.t / 2$

G = Gear ratio $= T / t = R / r$

ϕ = Pressure angle or angle of obliquity.

From triangle O_1NP,

$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \cos O_1PN$$

$$= r^2 + R^2 \sin^2 \phi - 2r.R \sin\phi \cos (90^\circ + \phi)$$

$$\ldots(\because\ PN = O_2P \sin \phi = R \sin \phi)$$

$$= r^2 + R^2 \sin^2 \phi + 2r.R \sin^2 \phi$$

$$= r^2 \left[1 + \frac{R^2 \sin^2 \phi}{r^2} + \frac{2R \sin^2 \phi}{r}\right] = r^2 \left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2 \phi\right]$$

∴ Limiting radius of the pinion addendum circle,

$$O_1N = r\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi} = \frac{m.t}{2}\sqrt{1 + \frac{T}{t}\left[\frac{T}{t} + 2\right]\sin^2\phi}$$

Let $A_P m$ = Addendum of the pinion, where A_P is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

We know that the addendum of the pinion

$$= O_1N - O_1P$$

$$\therefore \quad A_P.m = \frac{m.t}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - \frac{m.t}{2} \quad \ldots(\because O_1P = r = m.t/2)$$

$$= \frac{m.t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$A_P = \frac{t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$\therefore \quad t = \frac{2A_P}{\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1} = \frac{2A_P}{\sqrt{1 + G(G + 2)\sin^2\phi} - 1}$$

Minimum Number of Teeth on the Wheel in Order to Avoid Interference

Let T = Minimum number of teeth required on the wheel in order to avoid interference,

$A_W m$ = Addendum of the wheel, where A_W is a fraction by which the standard addendum for the wheel should be multiplied.

we have from triangle O_2MP

$$(O_2M)^2 = (O_2P)^2 + (PM)^2 - 2 \times O_2P \times PM \cos O_2PM$$

$$= R^2 + r^2 \sin^2\phi - 2\, Rr \sin\phi \cos(90° + \phi)$$

$$...(\because PM = O_1P \sin\phi = r)$$

$$= R^2 + r^2 \sin^2\phi + 2R\,r \sin^2\phi$$

$$= R^2\left[1 + \frac{r^2\sin^2\phi}{R^2} + \frac{2r\sin^2\phi}{R}\right] = R^2\left[1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi\right]$$

∴ Limiting radius of wheel addendum circle,

$$O_2M = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi} = \frac{mT}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi}$$

We know that the addendum of the wheel

$$= O_2M - O_2P$$

$$\therefore \quad A_W m = \frac{mT}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - \frac{mT}{2} \qquad ...(\because O_2P = R = mT/2)$$

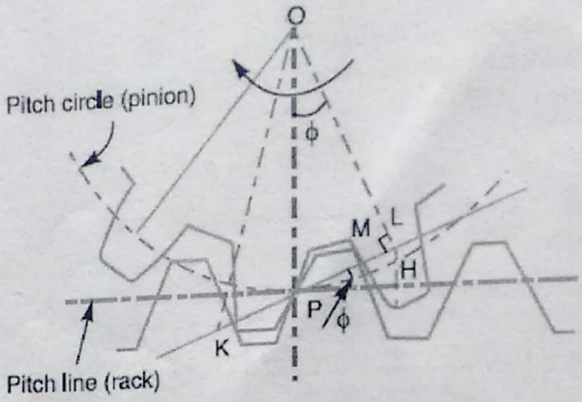
$$= \frac{mT}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$A_W = \frac{T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$\therefore \quad T = \frac{2A_W}{\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1} = \frac{2A_W}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

Minimum Number of Teeth on a Pinion for Involute Rack in Order to Avoid Interference

A rack and pinion in mesh is shown in Fig



Rack and pinion in mesh.

Let t = Minimum number of teeth on the pinion,

r = Pitch circle radius of the pinion = $m.t / 2$, and

ϕ = Pressure angle or angle of obliquity, and

$A_R.m$ = Addendum for rack, where A_R is the fraction by which the standard addendum of one module for the rack is to be multiplied.

We know that a rack is a part of toothed wheel of infinite diameter. Therefore its base circle diameter and the profiles of the involute teeth are straight lines. Since these straight profiles are tangential to the pinion profiles at the point of contact, therefore they are perpendicular to the tangent *PM*. The point *M* is the interference point.

Addendum for rack,

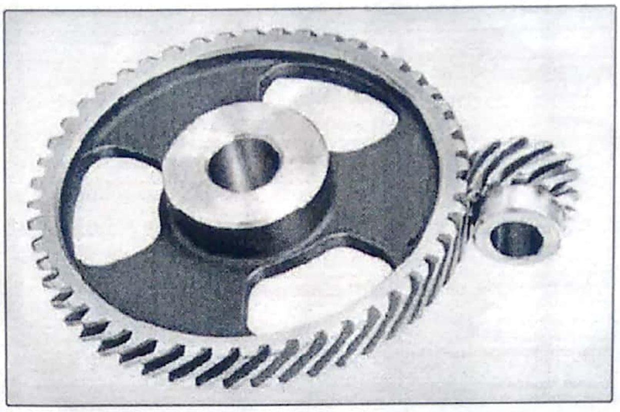
$$A_R.m = LH = PL\sin\phi$$

$$= (OP\sin\phi)\sin\phi = OP\sin^2\phi \qquad ...(\because PL = OP\sin\phi)$$

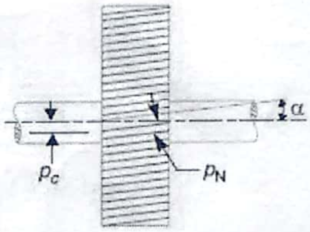
$$= r\sin^2\phi = \frac{m.t}{2}\times\sin^2\phi$$

$$\therefore \qquad t = \frac{2A_R}{\sin^2\phi}$$

Helical Gears



A helical gear has teeth in the form of helix around the gear. Two such gears may be used to connect two parallel shafts in place of spur gear. The helixes may be right handed on one wheel and left handed on the other. The pitch surfaces are cylindrical as in spur gearing, but the teeth instead of being parallel to the axis, wind around the cylinders helically like screw threads. The teeth of helical gears with parallel axis have line contact, as in spur gearing. This provides gradual engagement and continuous contact of the engaging teeth. Hence helical gears give smooth drive with a high efficiency of transmission.



Helical gear.

We have already discussed that the helical gears may be of single helical type or double helical type. In case of single helical gears, there is some axial thrust between the teeth, which is a disadvantage. In order to eliminate this axial thrust, double helical gears are used. It is equivalent to two single helical gears, in which equal and opposite thrusts are produced on each gear and the resulting axial thrust is zero.

The following definitions may be clearly understood in connection with a helical gear as shown in Fig.

1. *Normal pitch*. It is the distance between similar faces of adjacent teeth, along a helix on the pitch cylinder normal to the teeth. It is denoted by p_N.

2. *Axial pitch*. It is the distance measured parallel to the axis, between similar faces of adjacent teeth. It is the same as circular pitch and is therefore denoted by p_c. If α is the helix angle, then circular pitch,

$$p_c = \frac{p_N}{\cos\alpha}$$

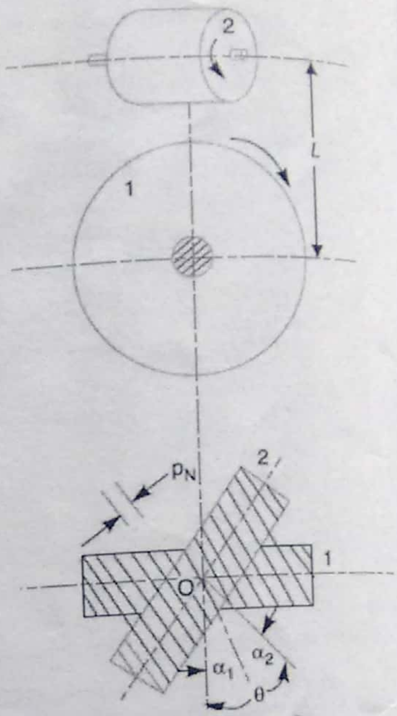
The helix angle is also known as spiral angle of the teeth.

Spiral Gears

We have already discussed that spiral gears (also known as skew gears or screw gears) are used to connect and transmit motion between two non-parallel and non-intersecting shafts. The pitch surfaces of the spiral gears are cylindrical and the teeth have point contact. These gears are only suitable for transmitting small power. We have seen that helical gears, connected on parallel shafts, are of opposite hand. But spiral gears may be of the same hand or of opposite hand.

Centre Distance for a Pair of Spiral Gears

The centre distance, for a pair of spiral gears, is the shortest distance between the two shafts making any angle between them. A pair of spiral gears 1 and 2, both having left hand helixes (*i.e.* the gears are of the same hand) is shown in Fig. The shaft angle θ is the angle through which one of the shafts must be rotated so that it is parallel to the other shaft, also the two shafts be rotating in opposite directions.



Centre distance for a pair of spiral gears.

Let α_1 and α_2 = Spiral angles of gear teeth for gears 1 and 2 respectively,

p_{c1} and p_{c2} = Circular pitches of gears 1 and 2,

T_1 and T_2 = Number of teeth on gears 1 and 2,

d_1 and d_2 = Pitch circle diameters of gears 1 and 2,

N_1 and N_2 = Speed of gears 1 and 2,

G = Gear ratio = $\frac{T_2}{T_1} = \frac{N_1}{N_2}$,

p_N = Normal pitch, and

L = Least centre distance between the axes of shafts.

Since the normal pitch is same for both the spiral gears, therefore

$$p_{c1} = \frac{p_N}{\cos\alpha_1}, \quad \text{and} \quad p_{c2} = \frac{p_N}{\cos\alpha_2}$$

We know that

$$p_{c1} = \frac{\pi d_1}{T_1}, \quad \text{or} \quad d_1 = \frac{p_{c1} \times T_1}{\pi}$$

$$p_{c2} = \frac{\pi d_2}{T_2}, \quad \text{or} \quad d_2 = \frac{p_{c2} \times T_2}{\pi}$$

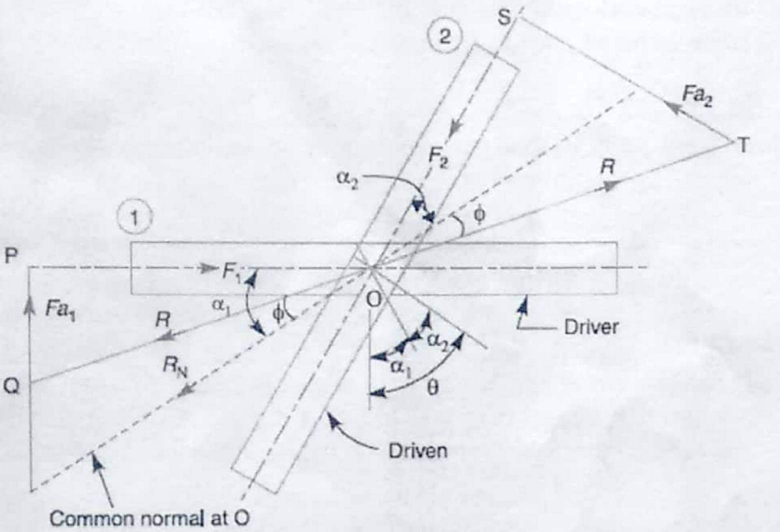
$$\therefore \quad L = \frac{d_1 + d_2}{2} = \frac{1}{2}\left(\frac{p_{c1} \times T_1}{\pi} + \frac{p_{c2} \times T_2}{\pi}\right)$$

$$= \frac{T_1}{2\pi}\left(p_{c1} + p_{c2} \times \frac{T_2}{T_1}\right) = \frac{T_1}{2\pi}\left(\frac{p_N}{\cos\alpha_1} + \frac{p_N}{\cos\alpha_2} \times G\right)$$

$$= \frac{p_N \times T_1}{2\pi}\left(\frac{1}{\cos\alpha_1} + \frac{G}{\cos\alpha_2}\right)$$

Efficiency of Spiral Gears

A pair of spiral gears 1 and 2 in mesh is shown in Fig. Let the gear 1 be the driver and the gear 2 the driven. The forces acting on each of a pair of teeth in contact are shown in Fig. The forces are assumed to act at the centre of the width of each teeth and in the plane tangential to the pitch cylinders.



Let F_1 = Force applied tangentially on the driver,

F_2 = Resisting force acting tangentially on the driven,

F_{a1} = Axial or end thrust on the driver,

F_{a2} = Axial or end thrust on the driven,

R_N = Normal reaction at the point of contact,

ϕ = Angle of friction,

R = Resultant reaction at the point of contact, and

θ = Shaft angle = $\alpha_1 + \alpha_2$

...($\because$ Both gears are of the same hand)

From triangle OPQ, $F_1 = R \cos (\alpha_1 - \phi)$

$\therefore$ Work input to the driver

$$= F_1 \times \pi\, d_1 . N_1 = R \cos (\alpha_1 - \phi)\, \pi\, d_1 . N_1$$

From triangle OST, $F_2 = R \cos (\alpha_2 + \phi)$

$\therefore$ Work output of the driven

$$= F_2 \times \pi\, d_2 . N_2 = R \cos (\alpha_2 + \phi)\, \pi\, d_2 . N_2$$

∴ Efficiency of spiral gears,

$$\eta = \frac{\text{Work output}}{\text{Work input}} = \frac{R\cos(\alpha_2 + \phi)\pi d_2 . N_2}{R\cos(\alpha_1 - \phi)\pi d_1 . N_1}$$

$$= \frac{\cos(\alpha_2 + \phi) d_2 . N_2}{\cos(\alpha_1 - \phi) d_1 . N_1}$$

$$d_1 = \frac{p_{c1} \times T_1}{\pi} = \frac{p_N}{\cos\alpha_1} \times \frac{T_1}{\pi} \qquad d_2 = \frac{p_{c2} \times T_2}{\pi} = \frac{p_N}{\cos\alpha_2} \times \frac{T_2}{\pi}$$

$$\therefore \quad \frac{d_2}{d_1} = \frac{T_2 \cos\alpha_1}{T_1 \cos\alpha_2}$$

We know that $\frac{N_2}{N_1} = \frac{T_1}{T_2}$

$$\frac{d_2 . N_2}{d_1 . N_1} = \frac{\cos\alpha_1}{\cos\alpha_2}$$

$$\therefore \quad \eta = \frac{\cos(\alpha_2 + \phi)\cos\alpha_1}{\cos(\alpha_1 - \phi)\cos\alpha_2}$$

$$= \frac{\cos(\alpha_1 + \alpha_2 + \phi) + \cos(\alpha_1 - \alpha_2 - \phi)}{\cos(\alpha_2 + \alpha_1 - \phi) + \cos(\alpha_2 - \alpha_1 + \phi)}$$

$$= \frac{\cos(\theta + \phi) + \cos(\alpha_1 - \alpha_2 - \phi)}{\cos(\theta - \phi) + \cos(\alpha_2 - \alpha_1 + \phi)}$$

Since the angles θ and ϕ are constants, therefore the efficiency will be maximum, when $\cos(\alpha_1 - \alpha_2 - \phi)$ is maximum, *i.e.*

$$\cos(\alpha_1 - \alpha_2 - \phi) = 1 \quad \text{or} \quad \alpha_1 - \alpha_2 - \phi = 0$$

$$\therefore \quad \alpha_1 = \alpha_2 + \phi \quad \text{and} \quad \alpha_2 = \alpha_1 - \phi$$

Since $\alpha_1 + \alpha_2 = \theta$, therefore

$$\alpha_1 = \theta - \alpha_2 = \theta - \alpha_1 + \phi \quad \text{or} \quad \alpha_1 = \frac{\theta + \phi}{2}$$

Similarly, $\alpha_2 = \frac{\theta - \phi}{2}$

Substituting $\alpha_1 = \alpha_2 + \phi$ and $\alpha_2 = \alpha_1 - \phi$,

$$\therefore \quad \eta_{max} = \frac{\cos(\theta + \phi) + 1}{\cos(\theta - \phi) + 1}$$

UINIT-VIII
Gear Trains

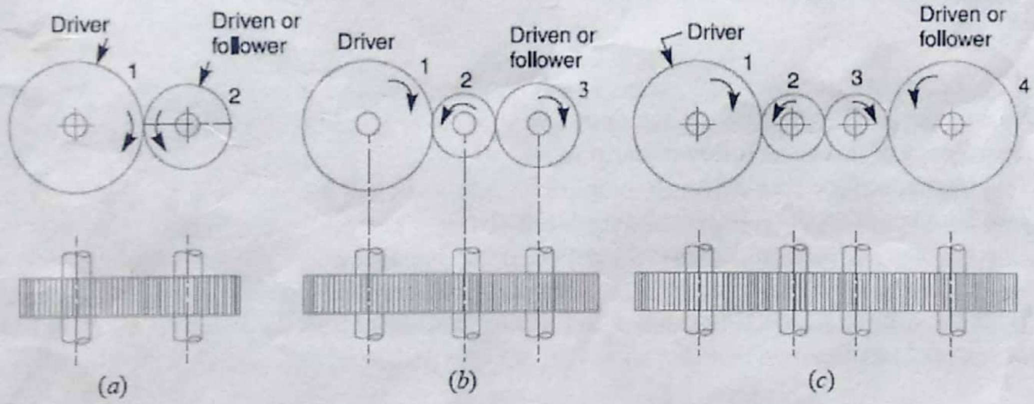
Introduction:
Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called ***gear train*** or ***train of toothed wheels.*** The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

Types of Gear Trains
Following are the different types of gear trains, depending upon the arrangement of wheels :
1. Simple gear train,
2. Compound gear train,
3. Reverted gear train, and
4. Epicyclic gear train.

In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other. But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

Simple Gear Train:

When there is only one gear on each shaft, as shown in **Fig.**, it is known as ***simple gear train***. The gears are represented by their pitch circles. When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in **Fig**. (*a*). Since the gear 1 drives the gear 2, therefore gear 1 is called the ***driver*** and the gear 2 is called the ***driven*** or ***follower***. It may be noted that the motion of the driven gear is opposite to the motion of driving gear.



Let

N_1 = Speed of gear 1(or driver) in r.p.m.,
N_2 = Speed of gear 2 (or driven or follower) in r.p.m.,
T_1 = Number of teeth on gear 1, and

T_2 = Number of teeth on gear 2.

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as ***train value*** of the gear train. Mathematically,

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

From above, we see that the train value is the reciprocal of speed ratio.
Sometimes, the distance between the two gears is large. The motion from one gear to another, in such a case, may be transmitted by either of the following two methods :

1. By providing the large sized gear, or
2. By providing one or more intermediate gears.

A little consideration will show that the former method (*i.e.* providing large sized gears) is very inconvenient and uneconomical method ; whereas the latter method (*i.e.* providing one or more intermediate gear) is very convenient and economical. It may be noted that when the number of intermediate gears are ***odd***, the motion of both the gears (*i.e.* driver and driven or follower) is ***like*** as shown in **Fig**. (*b*). But if the number of intermediate gears are ***even***, the motion of the driven or follower will be in the opposite direction of the driver as shown in **Fig**. (*c*).

Now consider a simple train of gears with one intermediate gear as shown in **Fig** (*b*).

Let

N_1 = Speed of driver in r.p.m.,
N_2 = Speed of intermediate gear in r.p.m.,
N_3 = Speed of driven or follower in r.p.m.,
T_1 = Number of teeth on driver,
T_2 = Number of teeth on intermediate gear, and
T_3 = Number of teeth on driven or follower.

Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1}$$

Similarly, as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is

$$\frac{N_2}{N_3} = \frac{T_3}{T_2}$$

$$\frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2}$$

$$\frac{N_1}{N_3} = \frac{T_3}{T_1}$$

$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

Similarly, it can be proved that the above equation holds good even if there are any umber of intermediate gears. From above, we see that the speed ratio and the train value, in a simple train of gears, is independent of the size and number of intermediate gears. These intermediate gears are called ***idle gears***, as they do not effect the speed ratio or train value of the system. The idle gears are used for the following two purposes
:

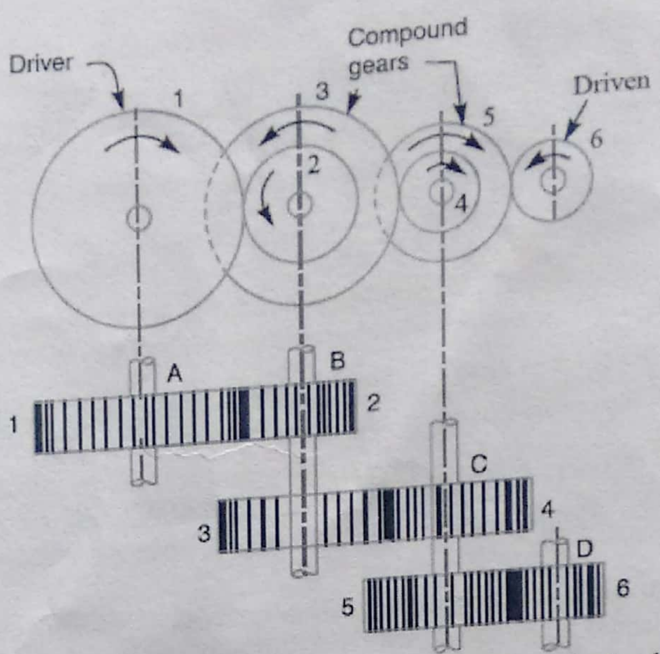
1. To connect gears where a large centre distance is required, and
2. To obtain the desired direction of motion of the driven gear (*i.e.* clockwise or anticlockwise).

Compound Gear Train:

When there are more than one gear on a shaft, as shown in **Fig**., it is called a compound train of gear.

The idle gears, in a simple train of gears do not effect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven.

But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts. In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in **Fig**.



In a compound train of gears, as shown in **Fig.**, the gear 1 is the driving gear mounted on shaft *A*, gears 2 and 3 are compound gears which are mounted on shaft *B*. The gears 4 and 5 are also compound gears which are mounted on shaft *C* and the gear 6 is the driven gear mounted on shaft *D*.

Let

$N1$ = Speed of driving gear 1,
$T1$ = Number of teeth on driving gear 1,
$N2, N3 \ldots, N6$ = Speed of respective gears in r.p.m., and
$T2, T3 \ldots, T6$ = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1}$$

Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3}$$

and for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5}$$

The speed ratio of compound gear train is obtained by multiplying the equations

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

$$\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}}$$

$$= \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

$$\text{Train value} = \frac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}}$$

$$= \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}}$$

Design of Spur Gears

Sometimes, the spur gears (*i.e.* driver and driven) are to be designed for the given velocity ratio and distance between the centres of their shafts.

Let

x = Distance between the centres of two shafts,
N_1 = Speed of the driver,
T_1 = Number of teeth on the driver,
d_1 = Pitch circle diameter of the driver,
N_2 , T_2 and d_2 = Corresponding values for the driven or follower, and
pc = Circular pitch.

We know that the distance between the centres of two shafts,

$$x = \frac{d_1 + d_2}{2}$$

and speed ratio or velocity ratio,

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{T_2}{T_1}$$

Reverted Gear Train:

When the axes of the first gear (*i.e.* first driver) and the last gear (*i.e.* last driven or follower) are co-axial, then the gear train is known as ***reverted gear* train** as shown in **Fig**. We see that gear 1 (*i.e.* first driver) drives the gear 2 (*i.e.* first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (*i.e.* the last driven or follower) in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is ***like.***

Let

$T1$ = Number of teeth on gear 1,
$r1$ = Pitch circle radius of gear 1, and
$N1$ = Speed of gear 1 in r.p.m.

Similarly,

T_2, T_3, T_4 = Number of teeth on respective gears,
r_2, r_3, r_4 = Pitch circle radii of respective gears, and
N_2, N_3, N_4 = Speed of respective gears in r.p.m.

Since the distance between the centres of the shafts of gears 1 and 2 as well as gears 3 and 4 is same, therefore

$$r_1 + r_2 = r_3 + r_4$$

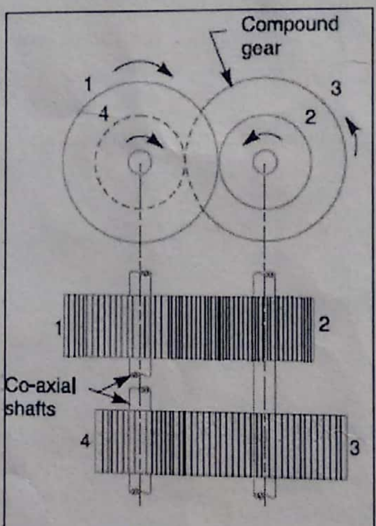
Also, the circular pitch or module of all the gears is assumed to be same, therefore number of teeth on each gear is directly proportional to its circumference or radius.

$$T_1 + T_2 = T_3 + T_4$$

$$\text{Speed ratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$

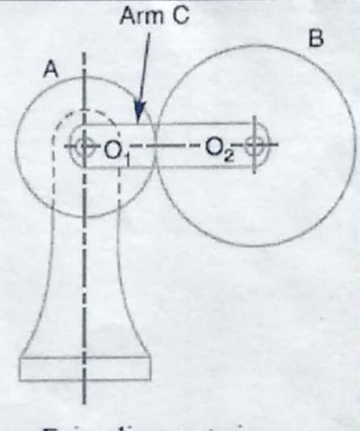
$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}$$

The reverted gear trains are used in automotive transmissions, lathe back gears, industrial speed reducers, and in clocks (where the minute and hour hand shafts are co-axial).



Reverted gear train.

arm is fixed, the gear train is simple and gear *A* can drive gear *B* or ***vice- versa***, but if gear *A* is fixed and the arm is rotated about the axis of gear *A* (*i.e.* *O*1), then the gear *B* is forced to rotate ***upon*** and ***around*** gear *A*. Such a motion is called **epicyclic** and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as ***epicyclic gear trains*** (***epi.*** means upon and ***cyclic*** means around). The epicyclic gear trains may be ***simple*** or ***compound***. The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.



Epicyclic gear train.

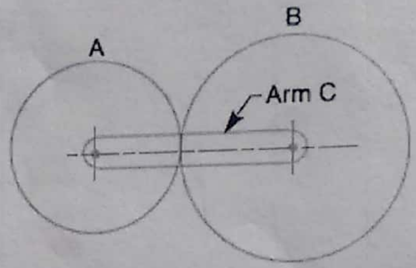
Velocity Ratioz of Epicyclic Gear Train:
The following two methods may be used for finding out the velocity ratio of an epicyclic gear train.
1. *Tabular method.* Consider an epicyclic gear train as shown in **Fig**.

Let

TA = Number of teeth on gear *A*, and
TB = Number of teeth on gear *B*.

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution i.e. 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	+ x	$-x \times \frac{T_A}{T_B}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y
4.	Total motion	+ y	x + y	$y - x \times \frac{T_A}{T_B}$

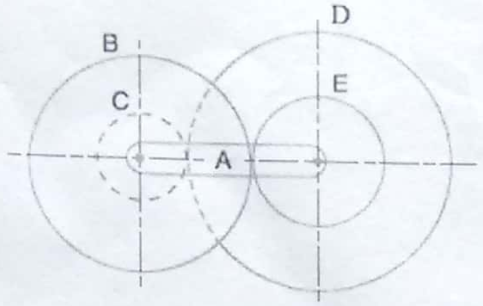
Example:



:

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution (i.e. 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	+ x	$-x \times \frac{T_A}{T_B}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y
4.	Total motion	+ y	x + y	$y - x \times \frac{T_A}{T_B}$

Example:

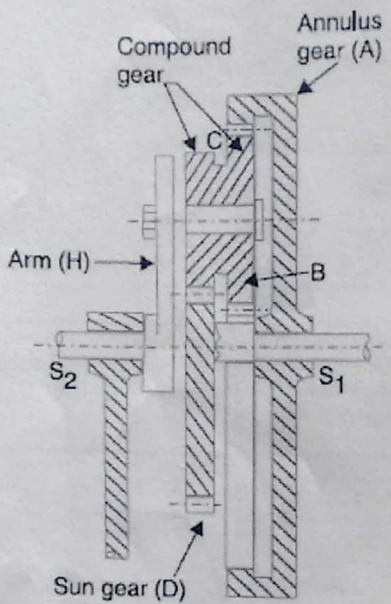


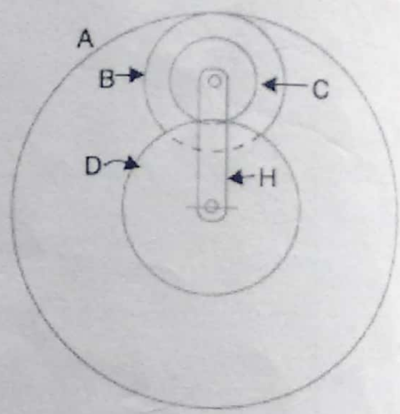
Step No.	Conditions of motion	Revolutions of elements			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed-compound gear D-E rotated through + 1 revolution (i.e. 1 rev. anticlockwise)	0	+ 1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed-compound gear D-E rotated through + x revolutions	0	$+x$	$-x\times\frac{T_E}{T_B}$	$-x\times\frac{T_D}{T_C}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\times\frac{T_E}{T_B}$	$y-x\times\frac{T_D}{T_C}$

Compound Epicyclic Gear Train—Sun and Planet Gear:

A compound epicyclic gear train is shown in **Fig**. It consists of two co-axial shafts $S1$ and $S2$, an annulus gear A which is fixed, the compound gear (or planet gear) B-C, the sun gear D and the arm H. The annulus gear has internal teeth and the compound gear is carried by the arm and revolves freely on a pin of the arm H. The sun gear is co-axial with the annulus gear and the arm but independent of them. The annulus gear A meshes with the gear B and the sun gear D meshes with the gear C. It may be noted that when the annulus gear is fixed, the sun gear provides the drive and when the sun gear is fixed, the annulus gear provides the drive. In both cases, the arm acts as a follower.

The gear at the centre is called the ***sun gear*** and the gears whose axes move are called ***planet gears***.

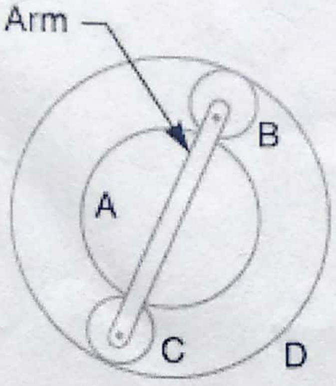




Compound epicyclic gear train.

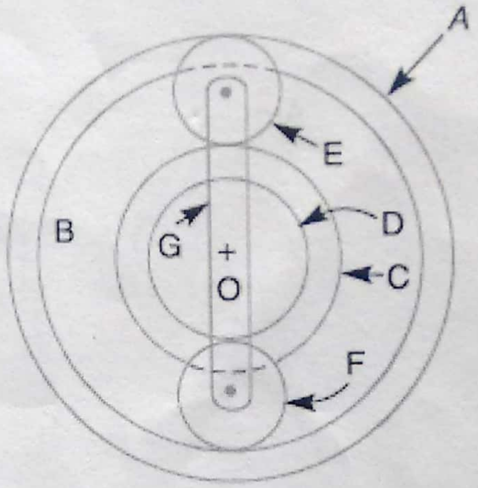
Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear D	Compound gear B-C	Gear A
1.	Arm fixed-gear D rotates through + 1 revolution	0	+ 1	$-\frac{T_D}{T_C}$	$-\frac{T_D}{T_C}\times\frac{T_B}{T_A}$
2.	Arm fixed-gear D rotates through + x revolutions	0	$+x$	$-x\times\frac{T_D}{T_C}$	$-x\times\frac{T_D}{T_C}\times\frac{T_B}{T_A}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\times\frac{T_D}{T_C}$	$y-x\times\frac{T_D}{T_C}\times\frac{T_B}{T_A}$

EXAMPLE



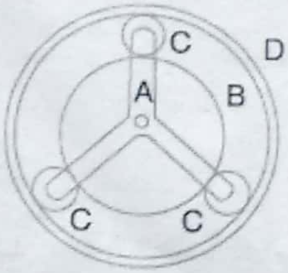
Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound gear B-C	Gear D
1.	Arm fixed, gear A rotates through – 1 revolution (i.e. 1 rev. clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_D}=+\frac{T_A}{T_D}$
2.	Arm fixed, gear A rotates through – x revolutions	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_D}$
3.	Add – y revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x\times\frac{T_A}{T_B}-y$	$x\times\frac{T_A}{T_D}-y$

EXAMPLE



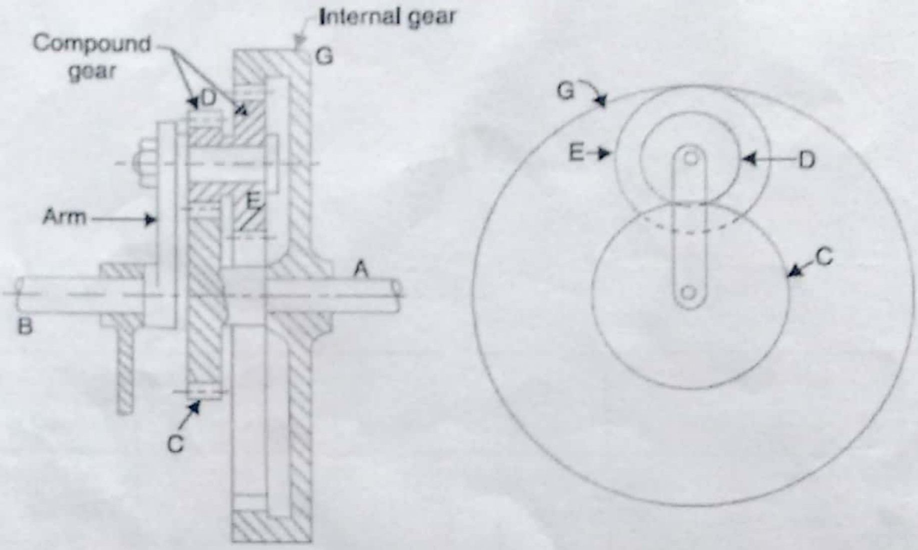
Step No.	Conditions of motion	Revolutions of elements					
		Arm G	Wheel A	Wheel E	Compound wheel C-D	Wheel F	Wheel B
1.	Arm fixed- wheel A rotates through + 1 revolution (i.e. 1 rev. anticlockwise)	0	+1	$+\frac{T_A}{T_E}$	$-\frac{T_A}{T_E}\times\frac{T_E}{T_C}$ $=-\frac{T_A}{T_C}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}\times\frac{T_F}{T_B}$ $=+\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
2.	Arm fixed-wheel A rotates through + x revolutions	0	$+x$	$+x\times\frac{T_A}{T_E}$	$-x\times\frac{T_A}{T_C}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y+x\times\frac{T_A}{T_E}$	$y-x\times\frac{T_A}{T_C}$	$y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$

EXAMPLE



Step No.	Conditions of motion	Revolutions of elements			
		Spider A	Sun wheel B	Planet wheel C	Internal gear D
1.	Spider A fixed, sun wheel B rotates through + 1 revolution (i.e. 1 rev. anticlockwise)	0	+ 1	$-\frac{T_B}{T_C}$	$-\frac{T_B}{T_C}\times\frac{T_C}{T_D}=-\frac{T_B}{T_D}$
2.	Spider A fixed, sun wheel B rotates through + x revolutions	0	+ x	$-x\times\frac{T_B}{T_C}$	$-x\times\frac{T_B}{T_D}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y	+ y
4.	Total motion	+ y	x + y	$y-x\times\frac{T_B}{T_C}$	$y-x\times\frac{T_B}{T_D}$

EXAMPLE



Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear C (or shaft A)	Compound gear D-E	Gear G
1.	Arm fixed - gear C rotates through + 1 revolution	0	+ 1	$-\frac{T_C}{T_D}$	$-\frac{T_C}{T_D} \times \frac{T_E}{T_G}$
2.	Arm fixed - gear C rotates through + x revolutions	0	+ x	$-x \times \frac{T_C}{T_D}$	$-x \times \frac{T_C}{T_D} \times \frac{T_E}{T_G}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y	+ y
4.	Total motion	+ y	x + y	$y - x \times \frac{T_C}{T_D}$	$y - x \times \frac{T_C}{T_D} \times \frac{T_E}{T_G}$

Differential gear of an automobile. The differential gear used in the rear drive of an automobile is shown in Fig.

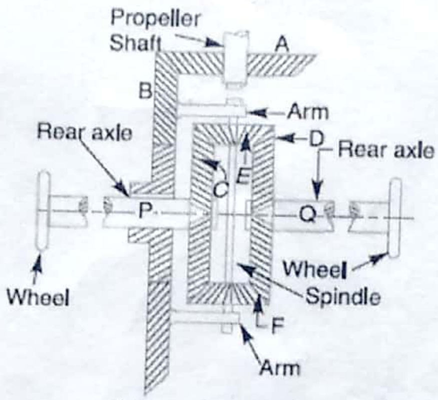
Its function is

(*a*) to transmit motion from the engine shaft to the rear driving wheels, and

(*b*) to rotate the rear wheels at different speeds while the automobile is taking a turn

As long as the automobile is running on a straight path, the rear wheels are driven directly by the engine and speed of both the wheels is same. But when the automobile is taking a turn, the outer wheel will run faster than the inner wheel because at that time the outer rear wheel has to cover more distance than the inner rear wheel. This is achieved by epicyclic gear train with bevel gears as shown in Fig.

The bevel gear *A* (known as pinion) is keyed to the propeller shaft driven from the engine shaft through universal coupling. This gear *A* drives the gear *B* (known as crown gear) which rotates freely on the axle *P*. Two equal gears *C* and *D* are mounted on two separate parts *P* and *Q* of the rear axles respectively. These gears, in turn, mesh with equal pinions *E* and *F* which can rotate freely on the spindle provided on the arm attached to gear *B*.



Differential gear of an automobile.

When the automobile runs on a straight path, the gears *C* and *D* must rotate together. These gears are rotated through the spindle on the gear *B*. The gears *E* and *F* do not rotate on the spindle. But when the automobile is taking a turn, the inner rear wheel should have lesser speed than the outer rear wheel and due to relative speed of the inner and outer gears *D* and *C*, the gears *E* and *F* start rotating about the spindle axis and at the same time revolve about the axle axis.

Due to this epicyclic effect, the speed of the inner rear wheel decreases by a certain amount and the speed of the outer rear wheel increases, by the same amount. This may be well understood by drawing the table of motions as follows :

Step No.	*Conditions of motion*	*Revolutions of elements*			
		Gear B	*Gear C*	*Gear E*	*Gear D*
1.	Gear *B* fixed-Gear *C* rotated through + 1 revolution (*i.e.* 1 revolution anticlockwise)	0	+ 1	$+\frac{T_C}{T_E}$	$-\frac{T_C}{T_E}\times\frac{T_E}{T_D}=-1$ ($\because T_C=T_D$)
2.	Gear *B* fixed-Gear *C* rotated through + *x* revolutions	0	$+x$	$+x\times\frac{T_C}{T_E}$	$-x$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y+x\times\frac{T_C}{T_E}$	$y-x$

From the table, we see that when the gear B, which derives motion from the engine shaft, rotates at y revolutions, then the speed of inner gear D (or the rear axle Q) is less than y by x revolutions and the speed of the outer gear C (or the rear axle P) is greater than y by x revolutions. In other words, the two parts of the rear axle and thus the two wheels rotate at two different speeds. We also see from the table that the speed of gear B is the mean of speeds of the gears C and D.