The state of rest and state of motion of the bodies under the action of different forces has engaged the attention of philosophers, mathematicians & scientists for many centuries. The branch of physical science that deal with the state of rest or the state of motion is termed as Mechanics. Starting from the analysis of rigid bodies under gravitational force and simple applied forces the mechanics has grown to the analysis of robotics, aircrafts, space crafts under dynamic forces, atmospheric forces, temperature forces etc.

Archemedes, Galileo, Sir Issac Newton & Einstein have contributed a lot to the development of mechanics. Contributions by Varignon, Euler, D. Alembert are also substantial.

The mechanics developed by these researchers may be grouped as

1. Classical mechanics/Newtonian mechanics
2. Relativistic mechanics
3. Quantum mechanics / Wave mechanics

Sir Issac Newton, the principal architect of mechanics, consolidated the philosophy & experimental findings developed around the state of rest and state of motion of the bodies and put forth them in the form of three laws of motion as well as the law of gravitation. The mechanics based on these laws is called classical mechanics.

Albert Einstein proved that Newtonian mechanics fails to explain the behaviour of high speed (speed of light) bodies. He put forth the theory of Relativistic Mechanics.

Schrodinge & Broglie showed that Newtonian mechanics fails to explain the behaviour of particles when atomic distances are concerned. They put forth the theory of Quantum Mechanics.

Engineers are keen to use the laws of Mechanics to actual field problems. Application of laws of mechanics to field problem is termed as Engineering Mechanics. For all the problems between atomic distances to high speed distances class-ical or Newtonian mechanics has stood the test of time and hence that is the mechanics used by engineers.

Science may be defined as the growth of ideas through observation & experimentation.

The branch of science, which co-ordinates the research work, for practical utility & services of the man-kind, is known as Applied science.

The subject of Engineering Mechanics is that branch of science, which deals with the laws & principles of Mechanics, along with their applications to engineering problems. As a matter of fact, knowledge of Engineering Mechanics is very essential for an engineer in planning, designing and construction of his various types of structures & machines. In order to take up his job more skilfully, an engineer must persue the study of Engineering Mechanics in a most systematic and scientific manner.

Basic Terminologies in Mechanics

Mass : The quantity of the matter possessed by a body is called mass. The mass of a body will not change unless the body is damaged and part of it is physically separated.

When a body is taken out in a space craft, the mass will not change but its weight may change due to change in gravitational force. Even the body may become weightless when gravitional force vanishes but the mass remain the same.

Time : Time is the measure of succession of events. The successive event selected is the rotation of earth about its own axis and this is called a day.

To have convenient units for various activities, a day is divided into 24 hours, an hour into 60 minutes & a minute into 60 seconds. clocks are the instruments developed to measure time.

Space : The geometric region in which study of body is involved is called space. At point in the space may be referred with respect to a predetermined point by a set of linear & angular measurements. The reference point is called the origin and set of measurements as coordinates.

Length : It is a concept to measure linear distances.

Displacement : It is defined as the distance moved by a body/ particle in the specified direction.

Velocity : The rate of change of displacement with respect to time is defined as velocity.

Acceleration : Acceleration is the rate of change of velocity with respect to time. Thus

$$a = \frac{dv}{dt}$$ where v is velocity

Momentum : The product of mass & velocity is called momentum.

Momentum = Mass * Velocity

Continuum : A body consists of several matters. It is a well known fact that each particle can be subdivided into molecules, atoms & electrons. It is not possible to solve any engineering problem by treating a body as a conglomeration of such discrete particles. The body is assumed to consist of a continuous distribution of matter. In other words, the body is treated as continuum.

Rigid Body : A body is said to be rigid, if the relative positions of any two particles do not change under the action of the forces.

Many engineering problems can be solved satisfactorily by assuming bodies rigid.

Particle : A particle may be defined as an object which has only mass & no size. Such a body cannot exist theoretically.

However in dealing with problems involving distances considerably larger compared to the size of the body, the body may be treated as particle, without sacrificing accuracy.

Laws of Mechanics

1. Newton's first law
2. Newton's second law
3. Newton's third law
4. Newton's law of gravitation
5. Law of transmissibility of forces
6. Parallelogram law of forces

Derived Laws

1. Triangle Law of forces
2. Polygon law of forces

Newton's First Law

It states that everybody continues in its state of rest or of uniform motion in a straight line unless it is compelled by an external agency acting on it. This leads to the definition of force as the external agency which changes or tends to change the state of rest or uniform linear motion of the body.

Newton's Second Law

It states that the rate of change of momentum of a body is directly proportional to the impressed force & it takes place in the direction of the force acting on it. Thus according to this law

Force $\propto$ rate of change of momentum

But momentum = mass * Velocity

As mass do not change,

Force $\propto$ mass * rate of change of velocity

Force $\propto$ mass * acceleration

$F \propto ma$

Newton's Third Law

It states that for every action there is an equal and opposite reaction. Consider the two bodies in contact with each other. Let one body apply a force F on another. According to this law the second body develops a reactive force R which is equal in magnitude to force F & acts in the line same as F but in the opposite direction.

(4)

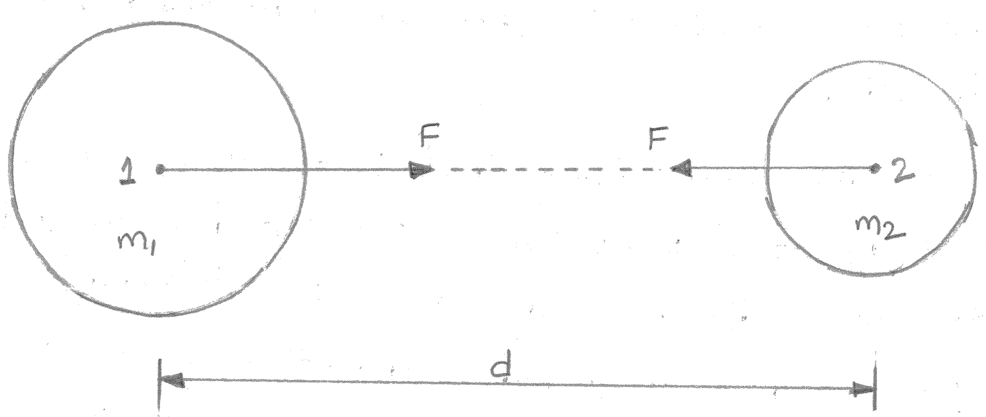
Newton's Law of Gravitation

Every body attracts the other body. The force of attraction between any two bodies is directly proportional to their masses and inversely proportional to the square of the distance between them.

According to this law the force of attraction between the bodies of mass m_1 & mass m_2 at distance 'd' as shown in Fig. is

$$F = G \frac{m_1 m_2}{d^2}$$

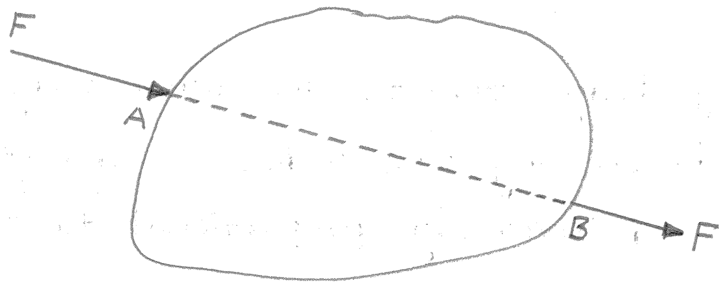
where G is the constant of proportionality and is known as constant of gravitation.



Law of Transmissibility of Force

According to this law the state of rest or motion of the rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the replaced force.

Let F be the force acting on a rigid body at point A as shown in Fig. According to the law of transmissibility, this force has the same effect on the state of body as the force F applied at point B.



In using law of transmissibility of forces it should be carefully noted that is applicable only if the body can be treated as rigid.

Parallelogram Law of Forces

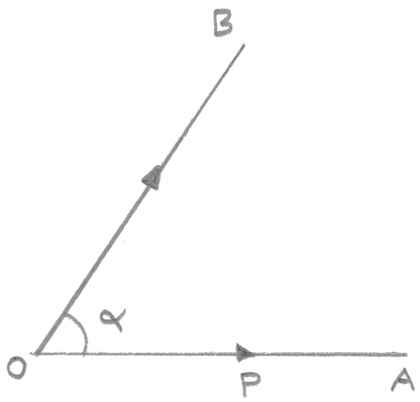
The parallelogram law of forces enables us to determine the single force called resultant which can replace the two forces acting at a point with the same effect as that of the two forces. This law was formulated based on experimental results.

This law states that if two forces acting simultaneously on a body at a point are represented in magnitude & direction by the two adjacent sides of a parallelogram, their resultant is represented in magnitude & direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces.

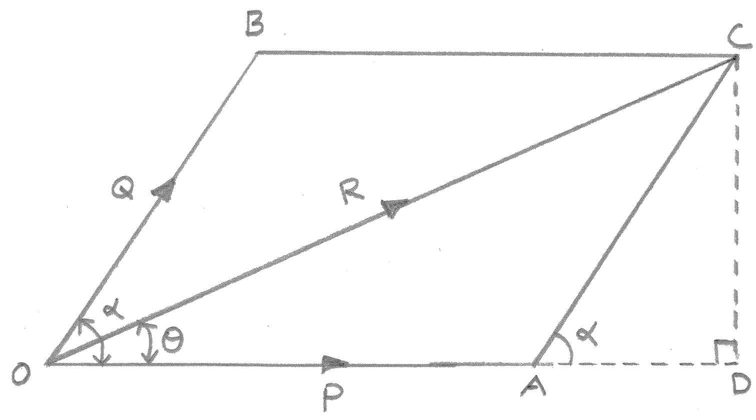
5 Law of Parallelogram of Forces

The law of parallelogram of forces is used to determine the resultant of two forces acting at a point in a plane.

It states, "If two forces, acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point."



(a)



(b)

Let two forces P and Q act at a point O as shown in Fig. The force P is represented in magnitude & direction by OA whereas the force Q is represented in magnitude & direction by OB. Let the angle between the two forces be α. The resultant of these two forces will be obtained in magnitude & direction by the diagonal (passing through O) of the parallelogram of which OA & OB are two adjacent sides. Hence draw the parallelogram with OA & OB as adjacent sides as shown in Fig (b). The resultant R is represented by OC in magnitude & direction.

Magnitude of Resultant

From C draw CD perpendicular to OA produced.

Let α = Angle between two forces P & Q = $\angle AOB$

Now $\angle DAC = \angle AOB = \alpha$

In parallelogram OACB, AC is parallel & equal to OB.

$\therefore AC = Q$

In triangle ACD,

$$AD = AC\cos\alpha = Q\cos\alpha$$

$$CD = AC\sin\alpha = Q\sin\alpha$$

In triangle OCD,

$$OC^2 = OD^2 + DC^2$$

But $OC = R$, $OD = OA + AD = P + Q\cos\alpha$

and $DC = Q\sin\alpha$

$(\because \cos^2\alpha + \sin^2\alpha = 1)$

$$\therefore\ R^2 = (P + Q\cos\alpha)^2 + (Q\sin\alpha)^2$$

$$= P^2 + Q^2\cos^2\alpha + 2PQ\cos\alpha + Q^2\sin^2\alpha$$

$$= P^2 + Q^2(\cos^2\alpha + \sin^2\alpha) + 2PQ\cos\alpha$$

$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

The above equation gives the magnitude of resultant force R.

Direction of Resultant

Let θ = Angle made by resultant with OA.

Then from triangle OCD,

$$\tan\theta = \frac{CD}{OD} = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)$$

The above equation gives the direction of resultant (R).

Note:

1. If the two forces P & Q act at right angles, then $\alpha = 90°$

magnitude of resultant, $R = \sqrt{P^2 + Q^2 + 2PQ\cos 90°}$

$$R = \sqrt{P^2 + Q^2}$$

direction of resultant, $\theta = \tan^{-1}\left(\frac{Q\sin 90°}{P + Q\cos 90°}\right)$

$\because \cos 90° = 0$ $\qquad \theta = \tan^{-1}\left(\frac{Q}{P}\right)$

$\because \sin 90° = 1$

(6) 2. The two forces P and Q are equal and are acting at an angle α between them. Then the magnitude & direction of resultant is given as

$$R = \sqrt{P^2+Q^2+2PQ\cos\alpha} = \sqrt{P^2+P^2+2P*P\cos\alpha}$$

$(\because P = Q)$

$$R = \sqrt{2P^2+2P^2\cos\alpha} = \sqrt{2P^2(1+\cos\alpha)}$$

$\because 1+\cos\alpha = 2\cos^2\frac{\alpha}{2}$

$$= \sqrt{2P^2 * 2\cos^2\frac{\alpha}{2}}$$

$$\boxed{R = 2P\cos\frac{\alpha}{2}}$$

$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P+Q\cos\alpha}\right)$$

$$= \tan^{-1}\left(\frac{P\sin\alpha}{P+P\cos\alpha}\right) = \tan^{-1}\left(\frac{\sin\alpha}{1+\cos\alpha}\right)$$

$$= \tan^{-1}\left(\frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}}\right)$$

$$\theta = \tan^{-1}\left(\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\right) = \tan^{-1}\left(\tan\frac{\alpha}{2}\right)$$

$$\boxed{\theta = \frac{\alpha}{2}}$$

3. If $\alpha = 0°$ i.e., when the forces act along the same line, then

$$R = P+Q$$

4. If $\alpha = 180°$ i.e., when the forces act along the same straight line but in opposite directions, then

$$R = P-Q$$

In this case, the resultant force will act in the direction of the greater force.

It is not necessary that one of two forces, should be along x-axis. The forces P & Q may be in any direction as shown in Fig.

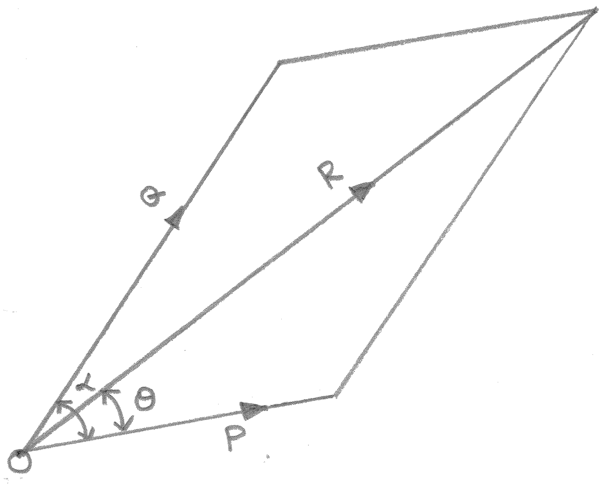
If the angle between the two forces is α, then their resultant will be given by equation

$$R = \sqrt{P^2+Q^2+2PQ\cos\alpha}$$

The direction of the resultant would be obtained from equation

$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P+Q\cos\alpha}\right)$$

But angle θ will be the angle made by resultant with the direction of P.



System of Forces

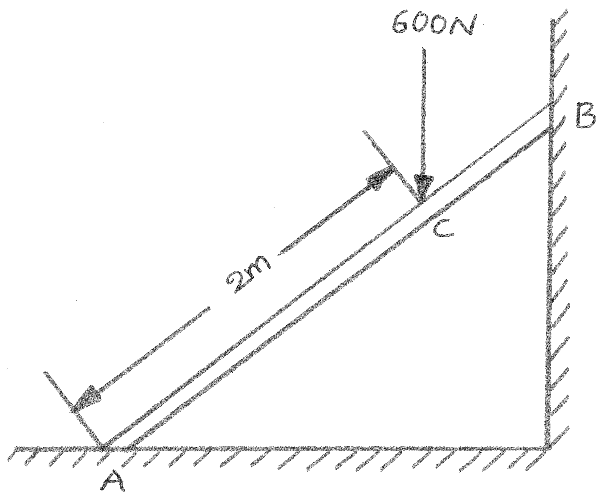
Force: It is defined as an agent which produces or tends to produce, destroys or tends to destroy motion.

Characteristics of a Force

From Newton's first law, we defined the force as the agency which tries to change state of stress or state of uniform motion of the body. From Newton's second law of motion we arrived at practical definition of unit force as the force required to produce unit acceleration in a body of unit mass. Thus 1 Newton is the force required to produce an acceleration of $1\, m/sec^2$ in a body of 1 kg mass. It may be noted that a force is completely specified only when the following four characteristics are specified:

1. Magnitude
2. Point of application
3. Line of action
4. Direction

In Fig. AB is a ladder kept against a wall. At point C, a person weighing 600 N is standing. The force applied by the person on the ladder has the following characters:



- Magnitude is 600N
- The point of application is at C which is 2m from A along the ladder
- The line of action is vertical
- The direction is downward

Note that the magnitude of the force is written near the arrow. The line of the arrow shows the line of application & the arrow head represents the point of application and the direction of the force.

System of Forces

When several forces act simultaneously on a body, they constitute a system of forces. If all the forces in a system do not lie in a single plane they constitute the system of forces in space.

If all the forces in a system lie in a single plane, it is called a coplanar force system. If the line of action of all the forces in a system pass through a single point, it is called a concurrent force system. In a system of parallel forces all the forces are parallel to each other. If the line of action of all the forces lie along a single line then it is called a collinear force system.

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Force System	Characteristic	Examples
Collinear forces	Line of action of all the forces act along the same line	Forces on a rope in tug of war.
coplanar parallel forces	All forces are parallel to each other & lie in a single plane	System of forces acting on a beam subjected to vertical loads
coplanar like parallel forces	All forces are parallel to each other, lie in a single plane and are acting in the same direction.	weight of a stationary train on a rail when the track is straight
coplanar concurrent forces	Line of action of all forces pass through a single point and forces lie in the same plane.	Forces on a rod resting against a wall.
coplanar non-concurrent forces	All forces do not meet at a point, but lie in a single plane	Forces on a ladder resting against a wall when a person stands on a rung which is not at its centre of gravity.
Non-coplanar parallel forces	All the forces are parallel to each other, but not in same plane.	The weight of benches in a class room.

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Force System	Characteristic	Examples
Non-coplanar concurrent forces	All forces do not lie in the same plane, but their lines of action pass through a single point	A tripod carrying a camera.
Non-coplanar nonconcurrent forces	All forces do not lie in the same plane & their lines of action do not pass through a single point.	Forces acting on a moving bus.

Resultant of System of forces

It is possible to find a single force which will have the same effect as that of a number of forces acting on a body. Such a single force is called Resultant force & the process of finding the resultant force is called composition of forces.

Finding resultant for the following system of forces

1. coplanar concurrent force system
2. coplanar non-concurrent force system
3. concurrent system of forces in space

Resultant of coplanar concurrent Force system

one can find the resultant of coplanar concurrent force system using parallelogram law, triangle law or polygonal law of forces. The advantage of this method is it gives clear picture of the work being carried out. This method needs drawing aids like pencil, scale, drawing sheet, drawing board etc. Hence engineers prefer to go for analytical method of finding resultant

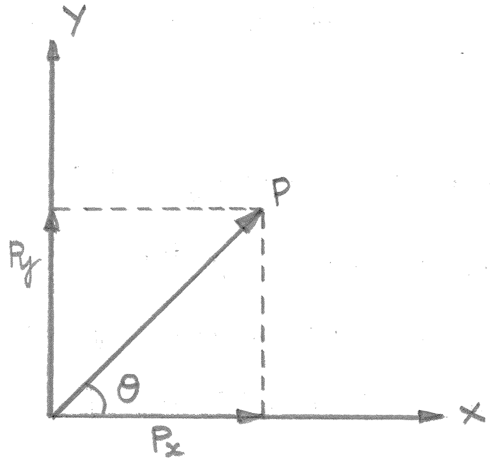
composition of forces by method of resolution is a general method used for finding the resultant. First let us see what is resolution and then will see how it helps in composition of a system of concurrent forces.

Resolution of Forces

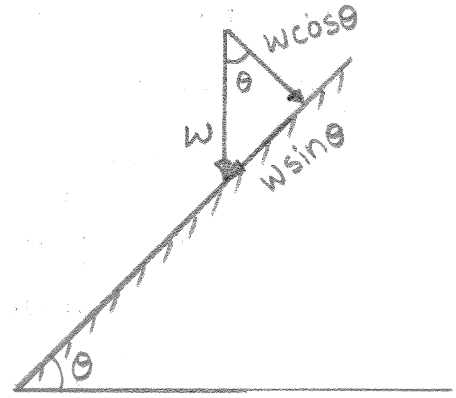
The resolution of forces is exactly the opposite process of composition of forces. It is the process of finding a number of component forces which will have the same effect as the given single force. If we apply the opposite process of polygon law, we can find a set of components of a single force. But this process is of no use in the further study.

Resolving the given force into its two components which are in mutually perpendicular directions is more useful in the further study.

In Fig (a) the given force P is resolved into its x component & y components while in Fig (b), they are resolved into components parallel to and perpendicular to a plane inclined at angle 'θ' to horizontal.



Fig(a): $P_x = P\cos\theta$, $P_y = P\sin\theta$

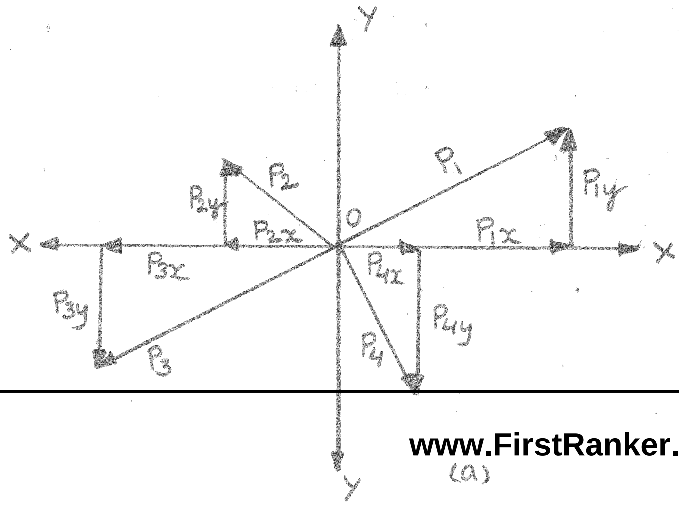


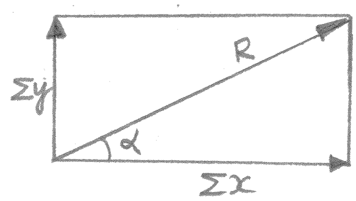
Fig(b): Parallel to plane $W\sin\theta$
perpendicular to plane $W\cos\theta$

Composition of concurrent forces by Method of Resolution

This is an analytical method of finding the resultant of multiple of forces. In this method the components of each force in the system are first found in two mutually perpendicular directions. Then the components in each direction are algebraically added to get the two components. These two component forces which are mutually perpendicular, are combined to get the resultant.

Let P_1, P_2, P_3 & P_4 shown in Fig. be the system of four forces the resultant of which is required.





(b)

(a)

The procedure to get the resultant is given below:

Step 1: Find the components of all the forces in X & Y directions. Thus P_{1x}, P_{2x}, P_{3x}, P_{4x}, P_{1y}, P_{2y}, P_{3y} & P_{4y} are obtained.

Step 2: Find the algebraic sum of the component forces in X & Y directions.

$$\Sigma X = P_{1x} + P_{2x} + P_{3x} + P_{4x}$$

$$\Sigma Y = P_{1y} + P_{2y} + P_{3y} + P_{4y}$$

Note: In the above case P_{2x}, P_{3x}, P_{3y} & P_{4y} have −ve values

Step 3: Now the system of forces is equal to two mutually perpendicular forces, namely, ΣX & ΣY as shown in Fig (6). Since these two forces are perpendicular, the parallelogram of forces becomes a rectangle. Hence the resultant R is given by

$$R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2}$$

Its inclination to X-axis is given by

$$\alpha = \tan^{-1}\left(\frac{\Sigma Y}{\Sigma X}\right)$$

Note:

$$R\cos\alpha = \Sigma X$$

$$R\sin\alpha = \Sigma Y$$

i.e., ΣX & ΣY are the X & Y components of the resultant.

(11) The following forces act at a point:

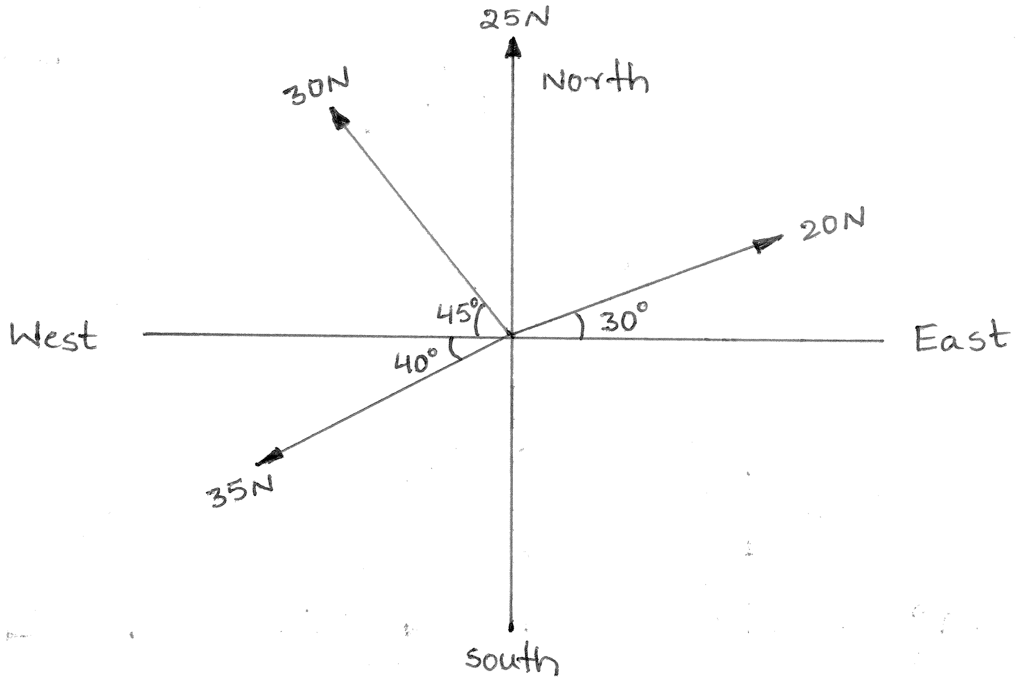
i) 20 N inclined at 30° towards North of East,

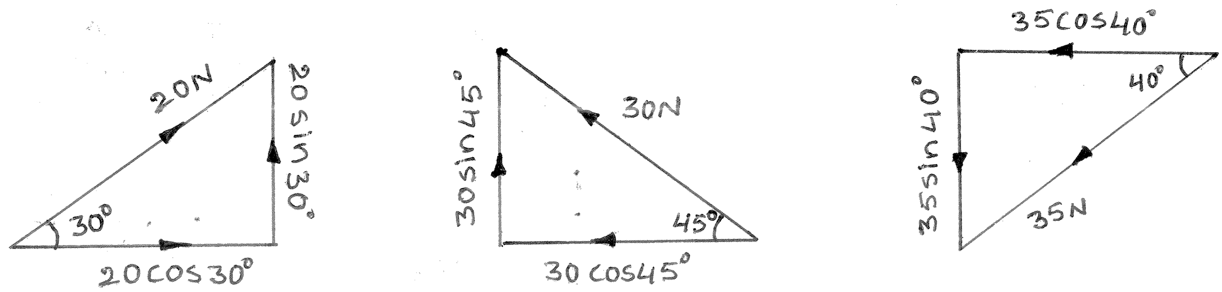
ii) 25 N towards North

iii) 30 N towards North west, and

iv) 35 N inclined at 40° towards South of west.

Find the magnitude and direction of the resultant force.





$$\Sigma F_x = 20\cos 30^\circ - 30\cos 45^\circ - 35\cos 40^\circ$$

$$= -30.7\ N$$

$$\Sigma F_y = 20\sin 30 + 30\sin 45^\circ - 35\sin 40 + 25$$

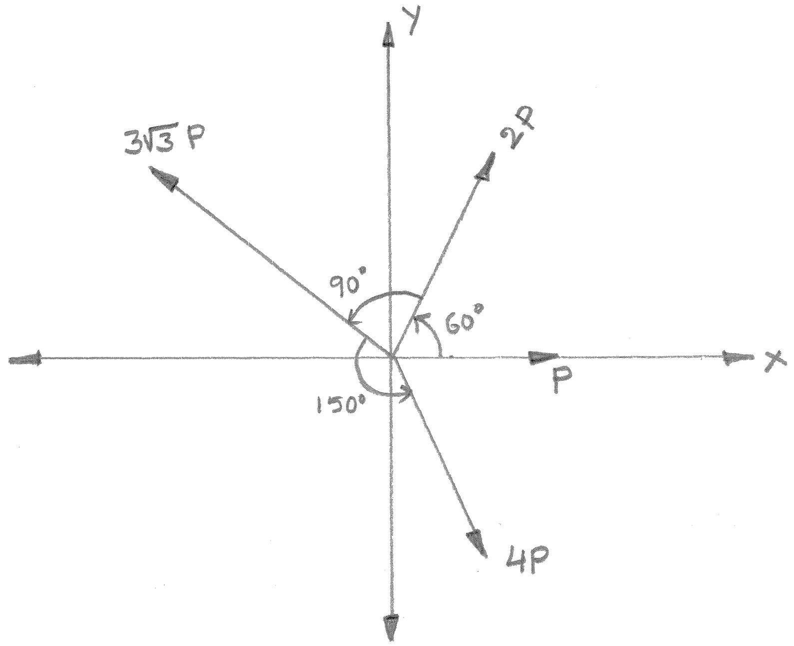
$$= 33.7\ N$$

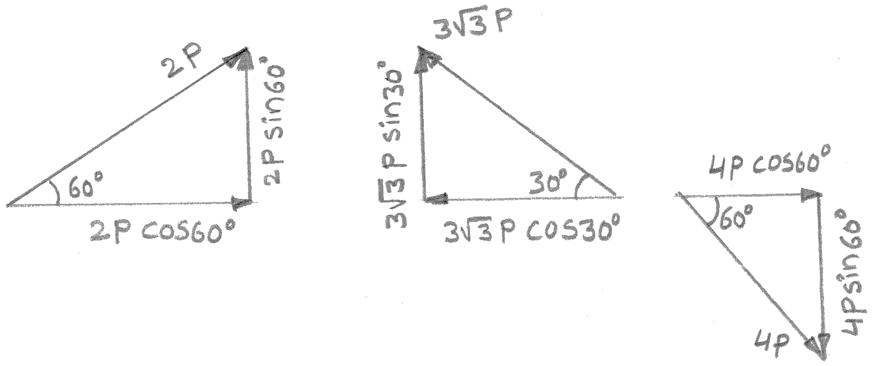
$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(-30.7)^2 + (33.7)^2} = 45.6\ N$$

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{33.7}{-30.7} = -1.098 \quad \text{or} \quad \theta = 47.7^\circ$$

Since ΣF_x is −ve & ΣF_y is +ve, therefore resultant lies between 90° & 180°. Thus actual angle of the resultant $= 180 - 47.7^\circ = 132.3^\circ$

(12) Find the magnitude and direction of the resultant R of four concurrent forces acting as shown in Fig.



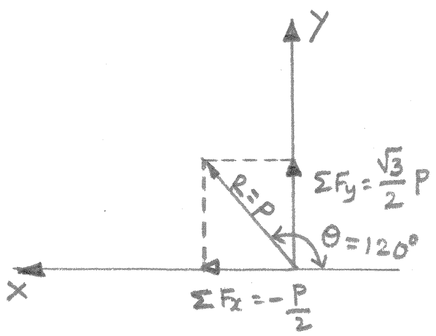


$$\Sigma F_x = 2P\cos 60^\circ - 3\sqrt{3}P\cos 30^\circ + 4P\cos 60^\circ + P$$

$$= 2P\left(\tfrac{1}{2}\right) - 3\sqrt{3}P\left(\tfrac{\sqrt{3}}{2}\right) + 4P\left(\tfrac{1}{2}\right) + P$$

$$= P - \frac{9P}{2} + 2P + P = 4P - \frac{9P}{2}$$

$$\Sigma F_x = -\frac{P}{2}$$



$$\Sigma F_y = 2P\sin 60^\circ + 3\sqrt{3}P\sin 30^\circ - 4P\sin 60^\circ$$

$$= 2P\left(\tfrac{\sqrt{3}}{2}\right) + 3\sqrt{3}P\left(\tfrac{1}{2}\right) - 4P\left(\tfrac{\sqrt{3}}{2}\right)$$

$$\Sigma F_y = 3\sqrt{3}P\left(\tfrac{1}{2}\right) - 2P\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}(3P - 2P) = \frac{\sqrt{3}}{2}P$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{\left(-\tfrac{P}{2}\right)^2 + \left(\tfrac{\sqrt{3}}{2}P\right)^2} = \sqrt{\frac{P^2}{4} + \frac{3P^2}{4}}$$

$$R = P$$

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{\frac{\sqrt{3}}{2}P}{-\frac{P}{2}} = -\sqrt{3}$$

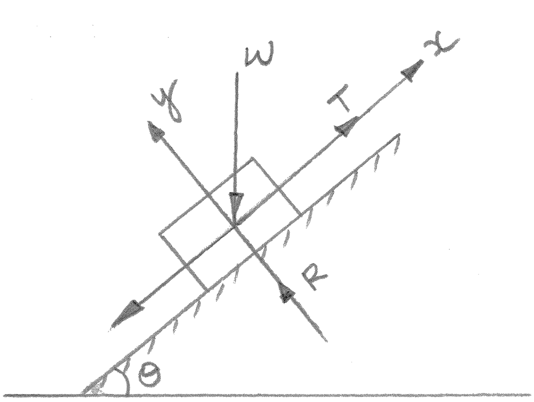
$$\theta = 120^\circ$$

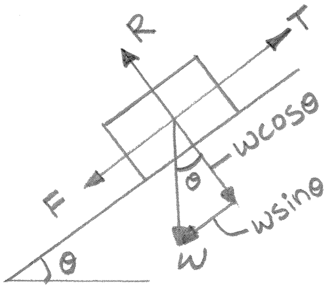
$$\tan\theta = -\sqrt{3}$$

$$\tan\theta = \tan 120^\circ$$

$$\theta = 120^\circ$$

(1) A system of forces acting on a body resting on an inclined plane is as shown in Fig. Determine the resultant force if $\theta = 60°$ and if $W = 1000\,N$, $R = 500\,N$, $F = 100\,N$ and $T = 1200\,N$.





In this problem, note that selecting x & y axes parallel to the plane and perpendicular to the plane is convenient.

$$\Sigma F_x = T - F - W\sin\theta$$

$$= 1200 - 100 - 1000\sin 60°$$

$$= 233.97\,N$$

$$\Sigma F_y = R - W\cos\theta$$

$$= 500 - 1000\cos 60°$$

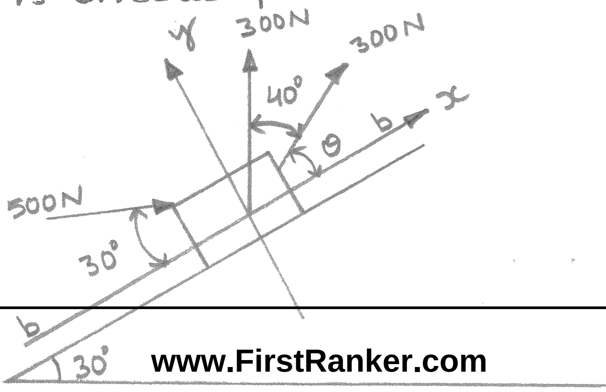
$$= 0$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(233.97)^2 + 0}$$

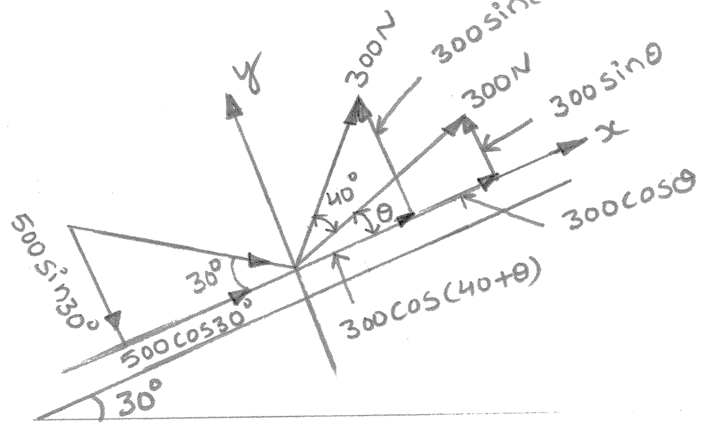
$$R = 233.97\,N$$

Resultant is a force of 233.97 N directed up the plane

(P) Three forces acting at a point are shown in Fig. The direction of the 300 N forces may vary, but the angle between them is always 40°. Determine the value of θ for which the resultant of the three forces is directed parallel to b-b.



(13)



Let the x & y axes be as shown in Fig. If the resultant is directed along the x-axis, its component in y direction is zero.

i.e., $\Sigma F_y = 0$

$$300 \sin\theta + 300 \sin(40+\theta) - 500 \sin 30^\circ = 0$$

$$\sin\theta + \sin(40+\theta) = \frac{500 \sin 30^\circ}{300}$$

$2\sin\theta\cos\theta = \sin 2\theta$

$$\sin\theta + \sin(40+\theta) = 0.8333$$

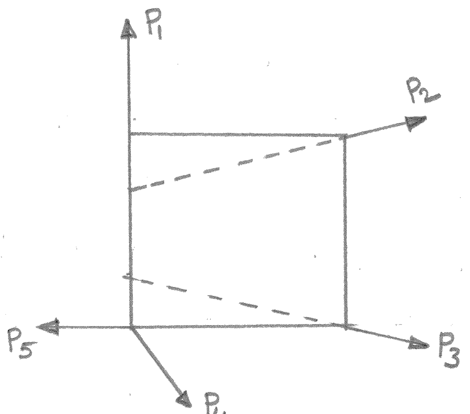
$$2\sin\left(\frac{40+\theta+\theta}{2}\right)\cos\left(\frac{40+\theta-\theta}{2}\right) = 0.8333$$

$$2\sin(20+\theta)\cos 20 = 0.8333$$

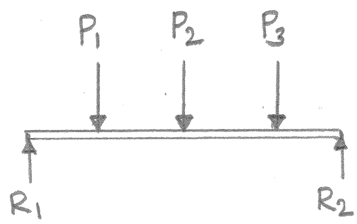
$$\theta = 6.32^\circ$$

(14) ## Resultant of coplanar non-concurrent system of force

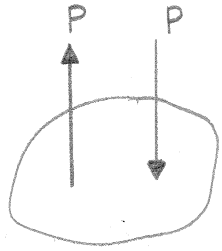
If all the forces in a system lie in the same plane and the lines of action of all the forces do not pass through a single point, the system is said to be coplanar nonconcurrent force system. Three such systems are shown in Fig.



(a)



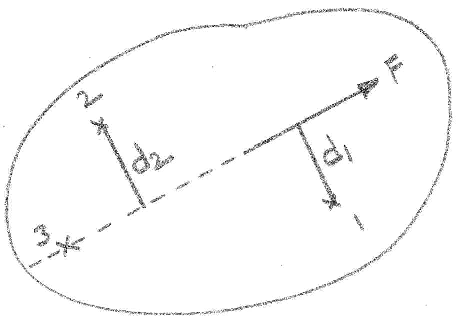
(b)



(c)

Moment of a Force

Moment of a force about a point is the measure of its rotational effect. Moment is defined as the product of the magnitude of the force and the perpendicular distance of the point from the line of action of the force. The point about which the moment is considered is called moment centre and the perpendicular distance of the point from the line of action of the force is called moment arm.



Refering to Fig. if d_1 is the perpendicular distance of point 1 from the line of action of force F, the moment of F about point 1 is given by

$$M_1 = Fd_1$$

Similarly, moment about point 2 is given by

$$M_2 = Fd_2$$

If the moment centre 3 lies on the line of action of the force F, the moment arm is zero and hence,

$$M_3 = F \times 0 = 0$$

Thus, it may be noted that if a point lie on the line of action of a force, the moment of the force about that point is zero.

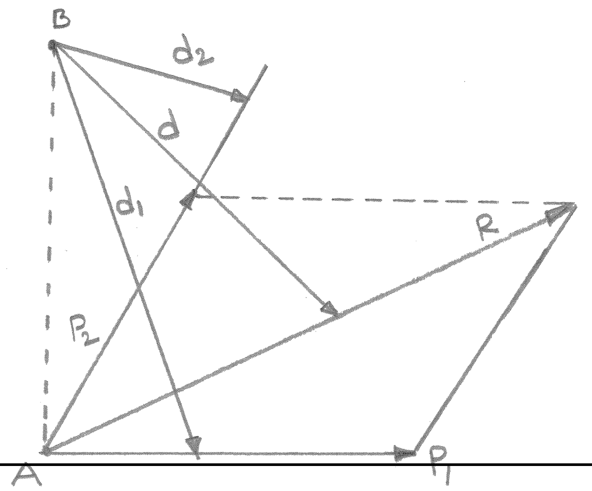
The moment of a force has got direction also. In Fig, it may be noted that M_1 is clockwise & M_2 is anticlockwise. To find the direction of the moment, imagine that the force is connected to the point by a rigid rod pinned at the point and is free to move around the point. The direction of the rotation indicates the direction of the moment.

If the force is taken in Newton unit and the distance in millimetre, the unit of moment will be N-mm. Commonly used units of moment in engineering are kN-m, N-m, kN-mm and N-mm.

Varignon's Theorem

French mathematician Varignon gave the following theorem which is also known as principle of moments:

The algebraic sum of the moments of a system of coplanar forces about a moment centre in their plane is equal to the moment of their resultant force about the same moment centre.



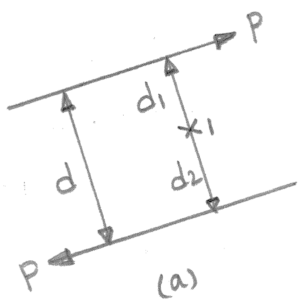
Referring to Fig. let R be the resultant of forces P_1 & P_2 and B the moment centre. Let d, d_1 & d_2 be the moment arms of the forces, R, P_1 & P_2 respectively from the moment centre B. Then in this case, we have to prove that :

$$Rd = P_1 d_1 + P_2 d_2$$

Couple

Two parallel forces equal in magnitude and opposite in direction and separated by a definite distance are said to form a couple. The sum of the forces forming a couple in any direction is zero, which means the translatory effect of the couple is zero.

An interesting property can be observed if we consider rotational effect of a couple about any point. Let the magnitude of the forces forming the couple be P and the perpendicular distance between the two forces be d. Consider the moment of the two forces constituting a couple about point 1 as shown in Fig(a).



(a)

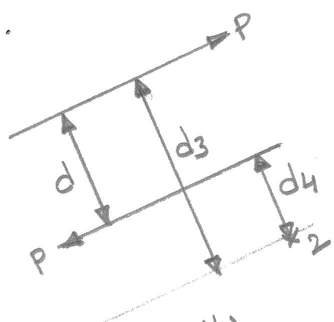
Let the moment be M_1 then,

$$M_1 = Pd_1 + Pd_2$$

$$= P(d_1 + d_2)$$

$$M_1 = Pd$$

Now, consider the moment of the forces about point 2 which is outside the two forces as shown in Fig(b).



(b)

Let M_2 be the moment,

$$M_2 = Pd_3 - Pd_4$$

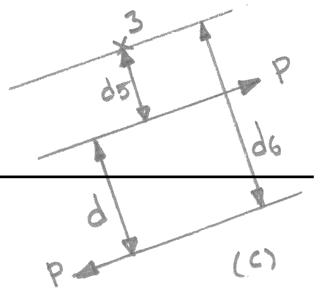
$$= P(d_3 - d_4)$$

$$M_2 = Pd$$

Similarly it can be seen that $M_3 = Pd$

$$M_3 = Pd_6 - Pd_5$$

$$= P(d_6 - d_5)$$



(c)

$$M_3 = Pd$$

Thus, moment of a couple about any point is the same. Now we can list the following characteristics of a couple:

- A couple consists of a pair of equal & opposite parallel forces which are separated by a definite distance.
- The translatory effect of a couple on the body is zero.
- The rotational effect (moment) of a couple about any point is a constant and it is equal to the product of the magnitude of the forces and the perpendicular distance between the two forces.

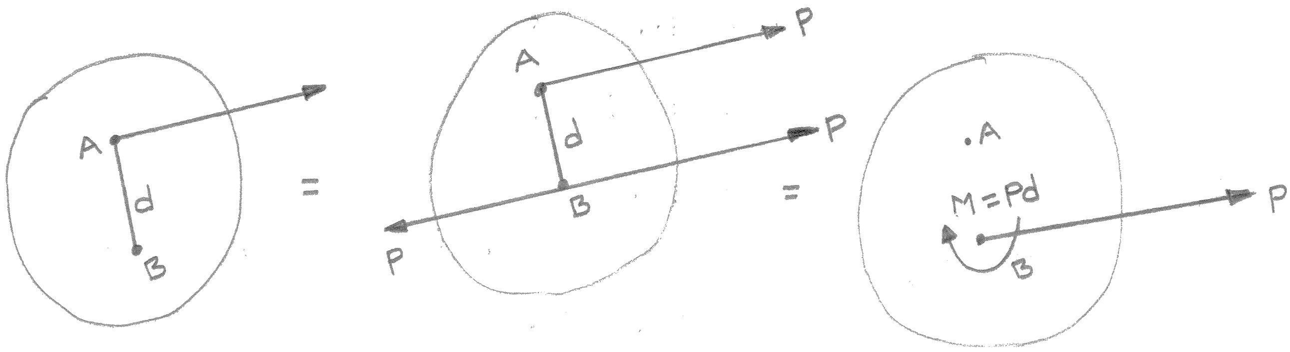
Since the only effect of a couple is a moment and this moment is the same about any point, the effect of a couple is unchanged if:

- The couple is rotated through any angle
- The couple is shifted to any other position
- The couple is replaced by another pair of forces whose rotational effect is the same.

Resolution of a force into a Force and a couple

It will be advantageous to resolve a force acting at a point on a body into a force acting at some other suitable point on the body and a couple.

In Fig (a), P is a force acting on a body at A. Now it can be shown that P at A may be resolved into a force P at B and a couple of magnitude $M = P \times d$, where d is the perpendicular distance of B from the line of action of P through A.



(a) (b) (c)

Let P_1, P_2 & P_3 constitute a system of forces acting on a body. Each force can be replaced by a force of the same magnitude and acting in the same direction at point O and a moment about O. Thus, the given system in Fig (a) is equal to the system shown in Fig (b) where ΣM_o is the algebraic sum of the moments of the given forces about O.

At O, the concurrent forces P_1, P_2 & P_3 can be combined as usual to get the resultant force R. Now the resultant of the given system is equal to a force R at O and a moment ΣM_o as shown in Fig (c).

The force R and moment ΣM_o shown in Fig (c) can be replaced by a single force R acting at a distance d from O such that the moment produced by this force R is equal to ΣM_o [Fig (d)]

Thus, we get a single force R acting at a distance d from the point O which gives the same effect as the constituent forces of the systems. Thus, the resultant of the given forces may be reduced to a single force.

Mathematically,

$$R = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$\tan\alpha = \frac{\Sigma y}{\Sigma x}$$

$$d = \frac{\Sigma M_o}{R}$$

where, Σx — algebraic sum of the components of all forces in x-direction

Σy — algebraic sum of the components of all forces in y-direction

α — Inclination of the resultant R to x-direction

ΣM_o — algebraic sum of the moments of all the forces about a point O.

d — is perpendicular distance of the resultant R from the point O

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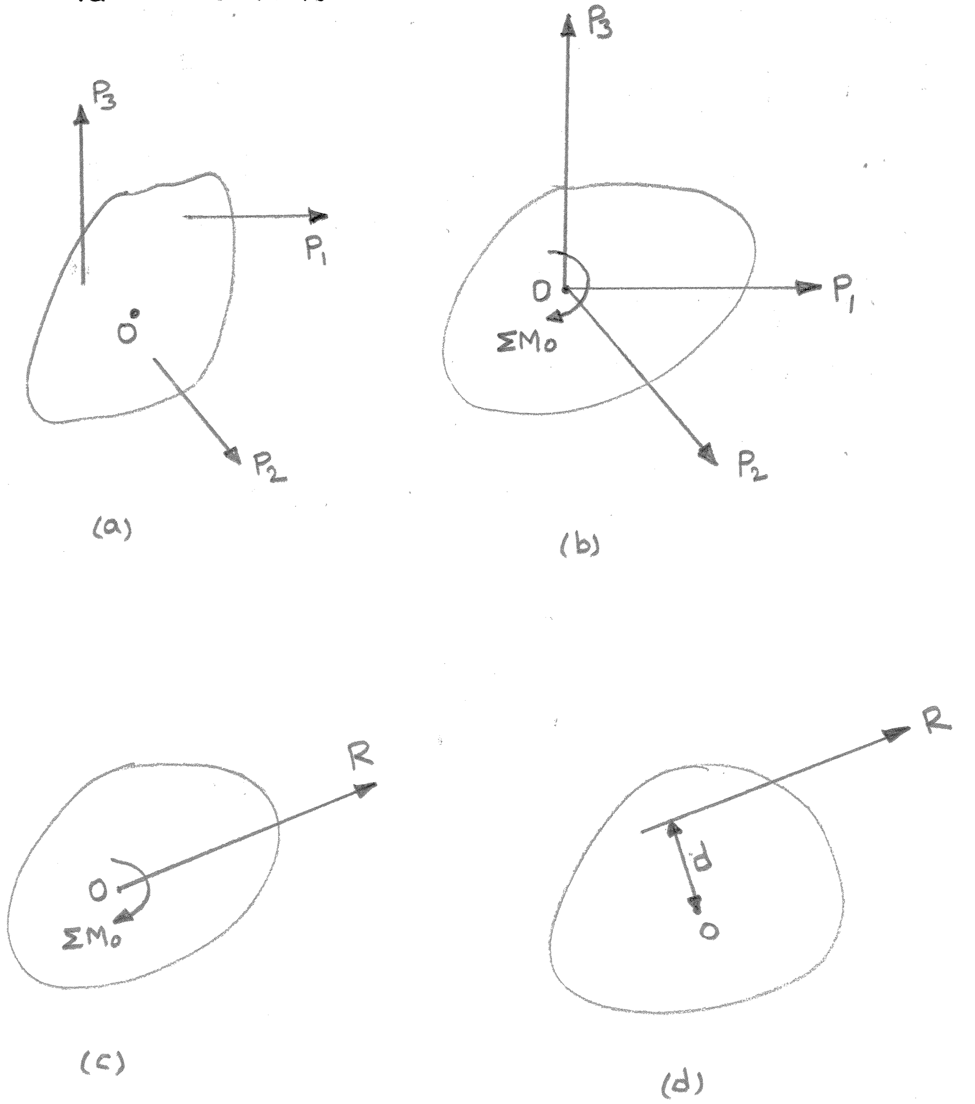
By applying equal and opposite forces P at B the system of forces is not disturbed. Hence the system of forces in Fig (b) is the same as the system given in Fig (a). Now the original force P at A and the opposite force P at B form a couple of magnitude Pd. The system in Fig (b) can be replaced by the system shown in Fig (c). Thus, the given force P at A is replaced by a force P at B and a moment Pd.

Resultant of Force systems

The resultant of force system is the one which will have the same rotational and translation effect as the given system of forces. It may be a single force, a pure moment or a force and a moment.



Note: R is marked at distance d such that it produces the same direction of moment about point O as ΣM_o.

Sometimes the values of Σx & Σy may come out to be zero, but ΣM_o may exist. This means that the resultant of the system gets reduced to a pure couple.

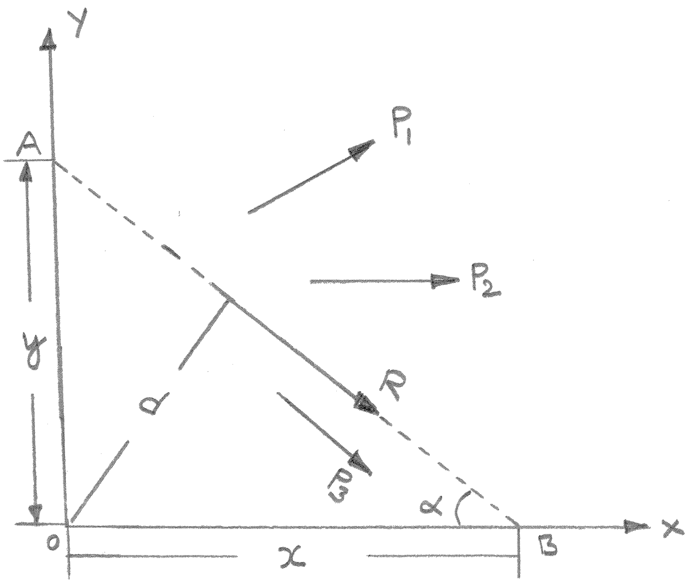
x and y intercepts of Resultant

In sometimes we may be interested in finding only the distance of R along x or y-axis, that is x & y intercepts.

Let d be the distance of the resultant from O and α be its inclination to x-axis. Then the intercepts are given by:

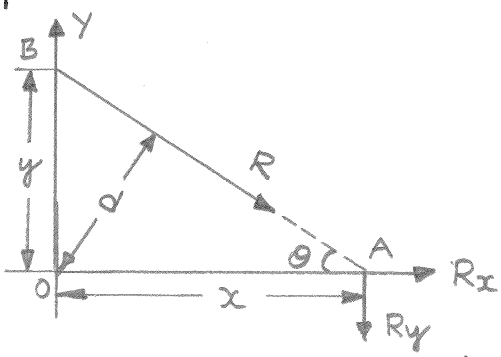
$$x = \frac{d}{\sin\alpha}$$

$$y = \frac{d}{\cos\alpha}$$



Another method of finding the intercepts is as follows:

Let $R_x = \Sigma x$ & $R_y = \Sigma y$ be the components of the resultant R in x & y directions. Considering the moment of R about O as the sum of moments of its components about A (Varignon's theorem) we get



$$Rd = \Sigma M_o$$

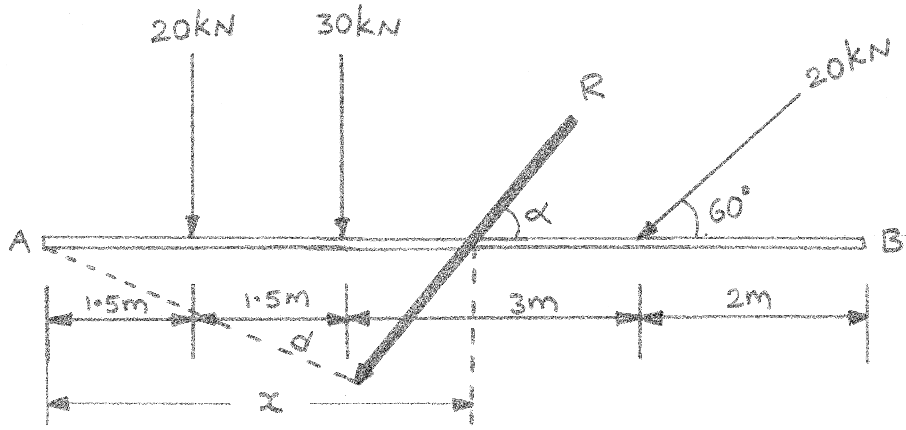
$$R_x * 0 + R_y * x = \Sigma M_o$$

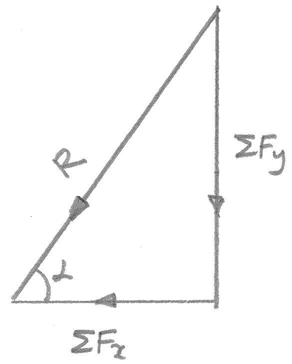
$$x = \frac{\Sigma M_o}{R_y} = \frac{\Sigma M_o}{\Sigma y}$$

similarly, resolving the resultant into its components at B, it can be shown that:

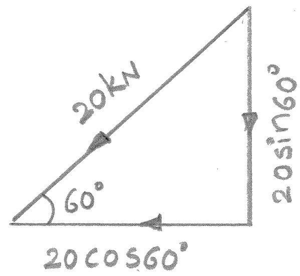
$$y = \frac{\Sigma M_o}{R_x} = \frac{\Sigma M_o}{\Sigma x}$$

(P) (18) A system of loads acting on a Beam is shown in Fig. Determine the resultant of the loads.





Taking horizontal direction as x-axis and the vertical direction as y-axis.



$$\Sigma F_x = -20\cos 60^\circ = -10\,kN = \overleftarrow{10\,kN}$$

$$\Sigma F_y = -20 - 30 - 20\sin 60^\circ = -67.32\,kN = 67.32 \downarrow$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(10)^2 + (67.32)^2} = 68.05\,kN$$

$$\tan\alpha = \frac{\Sigma F_y}{\Sigma F_x} = \frac{67.32}{10} = 6.73$$

$$\alpha = 81.55^\circ$$

Now taking moment about A

$$\Sigma M_A = 20 \times 1.5 + 30 \times 3 + 20\sin 60^\circ \times 6 = 223.92\ kN\text{-}m$$

The distance of the resultant from A is given by

$$d = \frac{\Sigma M_A}{R} = \frac{223.92}{68.05} = 3.29\,m$$

$$x = \frac{d}{\sin\alpha} = \frac{3.29}{\sin 81.55} = 3.32\ m$$

The value of x intercept may be obtained using

$$x = \frac{\Sigma M_A}{R_y} = \frac{\Sigma M_A}{\Sigma F_y}$$

$$= \frac{223.92}{67.32}$$

$$x = 3.32\ m$$

Resultant of Concurrent Force System in Space

Spatial Concurrent forces

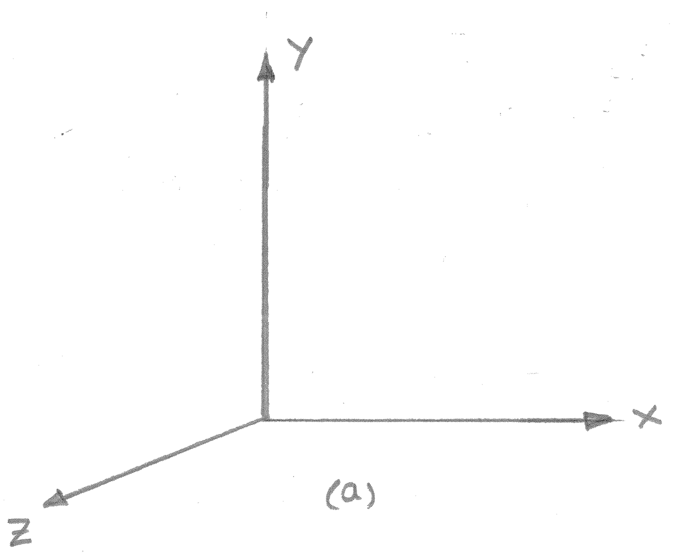
For the analysis of system of forces in space we need three dimensional cartesian coordinate system x, y, z. In literature every one has used the cartesian coordinate system represented by right hand rule that if thumb, index finger and middle finger are stretched to form three mutually perpendicular coordinate system,

x- coordinate is represented by thumb

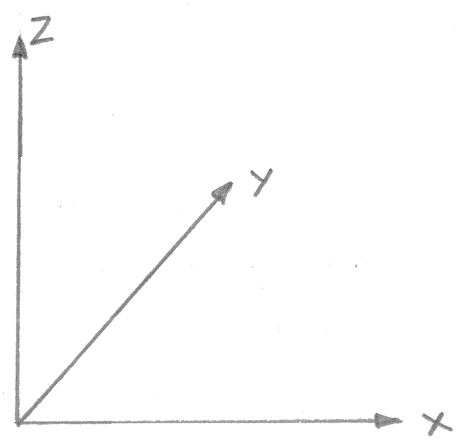
y- coordinate by index finger and

z- coordinate by middle finger

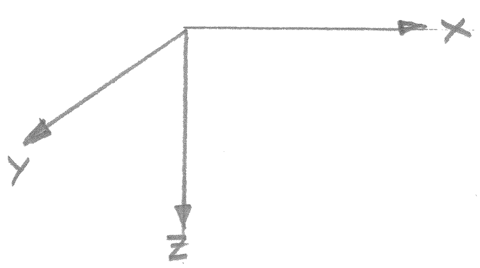
However different authors have used different orientations as shown in Fig. The expressions derived hold good for any orientation of right hand rule of cartesian coordinate system.



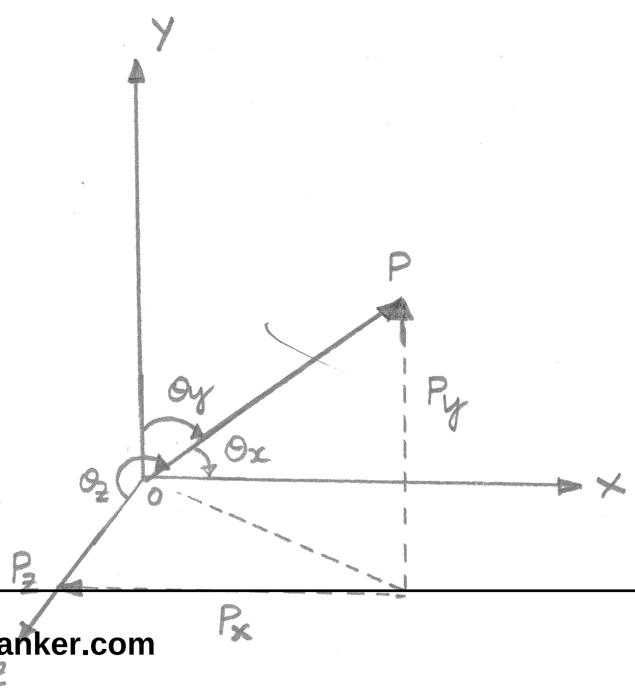
(a)



(b)



(c)



If P is a force in space making angle θ_x, θ_y & θ_z with the coordinate systems, then its components in x, y & z directions are given by,

$$P_x = P\cos\theta_x$$

$$P_y = P\cos\theta_y$$

$$P_z = P\cos\theta_z$$

$\cos\theta_x$, $\cos\theta_y$ & $\cos\theta_z$ are termed as direction cosines.

If i and j are the two points on the line of action of a force P, with coordinates (x_i, y_i, z_i) & (x_j, y_j, z_j) then the direction cosines from i to j are given by

$$\cos\theta_x = \frac{x_j - x_i}{L}$$

$$\cos\theta_y = \frac{y_j - y_i}{L}$$

$$\cos\theta_z = \frac{z_j - z_i}{L}$$

where $L = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}$

Using the above equations, the component of a force in three mutually perpendicular coordinate systems x, y, z can be found.

The procedure to find the resultant several concurrent forces in space

If P_1, P_2, P_3 ---- are the concurrent forces in space, the directions of which are known, then we can find $P_{1x}, P_{1y}, P_{1z}, P_{2x}, P_{2y}, P_{2z}, P_{3x}, P_{3y}, P_{3z}$, ----- Then

$$R_x = \Sigma P_x = P_{1x} + P_{2x} + P_{3x} + ----$$

$$R_y = \Sigma P_y = P_{1y} + P_{2y} + P_{3y} + ----$$

$$R_z = \Sigma P_z = P_{1z} + P_{2z} + P_{3z} + ----$$

Then the magnitude of resultant R is given by

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(\Sigma P_x)^2 + (\Sigma P_y)^2 + (\Sigma P_z)^2}$$

The direction of the resultant is given by

$$\theta_x = \cos^{-1}\frac{R_x}{R}, \quad \theta_y = \cos^{-1}\frac{R_y}{R}, \quad \theta_z = \cos^{-1}\frac{R_z}{R}$$

(P) (20) Forces P_1, P_2, P_3 and P_4 of magnitudes 10 kN, 20 kN, 25 kN and 40 kN are concurrent in space and are directed through the points A(3, 2, 5), B(1, 7, 4), C(4, −2, 4) and D(−2, 4, −3) respectively. Determine the resultant of the system of forces. Given the system forces are concurrent at the origin.

The length of OA, OB, OC and OD are given by

$O(0,0,0)$

$$OA = \sqrt{3^2+2^2+5^2} = 6.164$$

$$OB = \sqrt{1^2+7^2+4^2} = 8.124$$

$$OC = \sqrt{4^2+(-2)^2+4^2} = 6$$

$$OD = \sqrt{(-2)^2+4^2+(-3)^2} = 5.385$$

$$\cos\theta_{1x} = \frac{3-0}{6.164} = 0.4867 \qquad \cos\theta_{1y} = \frac{2-0}{6.164} = 0.3245 \qquad \cos\theta_{1z} = \frac{5-0}{6.164} = 0.8112$$

$$P_{1x} = 10 * 0.4867 = 4.867 \text{ kN} \qquad P_{1y} = 10 * 0.3245 = 3.245 \text{ kN} \qquad P_{1z} = 10 * 0.8112 = 8.112 \text{ kN}$$

Similarly,

$$\cos\theta_{2x} = \frac{1}{8.124} = 0.1231 \qquad \cos\theta_{2y} = \frac{7}{8.124} = 0.8616 \qquad \cos\theta_{2z} = \frac{4}{8.124} = 0.4924$$

$$P_{2x} = 20 * 0.1231 = 2.462 \text{ kN} \qquad P_{2y} = 20 * 0.8616 = 17.232 \text{ kN} \qquad P_{2z} = 20 * 0.4924 = 9.848 \text{ kN}$$

$$\cos\theta_{3x} = \frac{4}{6} \qquad \cos\theta_{3y} = \frac{-2}{6} \qquad \cos\theta_{3z} = \frac{4}{6}$$

$$P_{3x} = 25 * \frac{4}{6} = 16.667 \text{ kN} \qquad P_{3y} = -\frac{2}{6} * 25 = -8.333 \text{ kN} \qquad P_{3z} = 25 * \frac{4}{6} = 16.667 \text{ kN}$$

$$\cos\theta_{4x} = \frac{-2}{5.385} \qquad \cos\theta_{4y} = \frac{4}{5.385} \qquad \cos\theta_{4z} = \frac{-3}{5.385}$$

$$P_{4x} = 40 * \frac{-2}{5.385} = -14.856 \text{ kN} \qquad P_{4y} = 40 * \frac{4}{5.385} = 29.712 \text{ kN} \qquad P_{4z} = 40 * \frac{-3}{5.385} = -22.284 \text{ kN}$$

$$R_x = \Sigma P_x = 4.867 + 2.462 + 16.667 + (-14.856)$$

$$R_x = 9.14 \text{ kN}$$

$$R_y = \Sigma P_y = 3.245 + 17.232 + (-8.333) + 29.712$$

$$R_y = 41.856 \text{ kN}$$

$$R_z = \Sigma P_z = 8.112 + 9.848 + 16.667 - 22.284$$

$$R_z = 12.343 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(9.14)^2 + (41.856)^2 + (12.343)^2}$$

$$R = 44.585 \text{ kN}$$

The direction of the resultant is given by

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\frac{9.14}{44.585} = 78.17^\circ$$

$$\theta_y = \cos^{-1}\left(\frac{R_y}{R}\right) = \cos^{-1}\frac{41.856}{44.585} = 20.15^\circ$$

$$\theta_z = \cos^{-1}\left(\frac{R_z}{R}\right) = \cos^{-1}\frac{12.343}{44.585} = 73.93^\circ$$

Friction

When two bodies in contact have a tendency to move over each other a resistance to the movement is set up. This resistance to the movement is called the force of friction or simply friction. Friction depends upon the nature of the surfaces of contact. Friction acts parallel to the surface of contact. The direction of this frictional force on any one of the surfaces of contact will be opposite to the direction in which the contact surface tends to move. In other words, friction opposes motion.

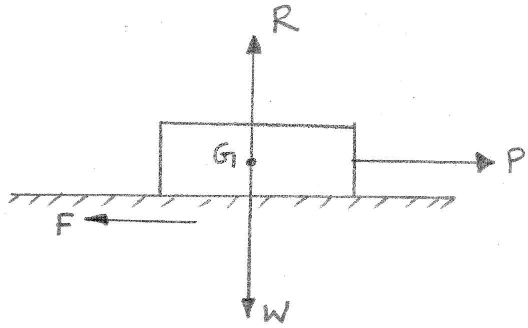


Fig. shows a wooden block resting on a rough horizontal table. Let W be the weight of the block. Let the block be subjected to a horizontal force P. When this applied force is sufficiently small, the block will remain in equilibrium. The rough table surface will exert a normal reaction R and a tangential reaction (Friction) F on the block so as to keep the block in equilibrium.

Resolving the forces on the block vertically & horizontally

$$R - W = 0$$

$$R = W$$

$$P - F = 0$$

$$P = F$$

Suppose the force P is gradually increased. The friction F will also increase, so that at every stage of equilibrium $F = P$. But there is a limit to which friction can increase. We cannot expect the frictional resistance to go on increasing infinitely as the force P is increased. Let F_1 represent the greatest possible friction. Let P_1 be the applied force corresponding to this condition.

At this stage.

$$F_1 = P_1$$

If the applied force exceeds P_1, the block will slip on the table since the frictional resistance cannot increase beyond the value F_1. The greatest possible friction depends upon the normal reaction.

Friction

1. static Friction   2. Dynamic Friction

static friction — It is the friction experienced by a body when it is at rest. Or in other words, it is the friction when the body tends to move.

Dynamic Friction — It is the friction experienced by a body when it is in motion. It is also called kinetic friction. The dynamic friction is of the following two types

sliding friction — It is the friction, experienced by a body when it slides over another body.

Rolling Friction — It is the friction, experienced by a body when it rolls over another body.

Laws of Friction

Prof. Coulomb, after extensive experiments, gave some laws of friction, which may be grouped under the following heads:

1. Laws of static friction
2. Laws of kinetic or dynamic friction

Laws of static friction

1. The force of friction always acts in a direction, opposite to that in which the body tends to move, if the force of friction would have been absent.
2. The magnitude of the force of friction is exactly equal to the force, which tends to move the body.
3. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces.

$$\frac{F}{R} = \text{constant}$$

where F = Limiting friction

R = Normal reaction

4. The force of friction is independent of the area of contact between the two surfaces.
5. The force of friction depends upon the roughness of the surfaces.

Laws of Dynamic friction

1. The force of friction always acts in a direction, opposite to that in which the body is moving.
2. The magnitude of kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.
3. For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

Co-efficient of Friction

It is defined as the ratio of the limiting force of friction (F) to the normal reaction (R) between two bodies. It is denoted by the symbol μ.

$$\mu = \frac{\text{Limiting force of friction}}{\text{Normal reaction}} = \frac{F}{R}$$

$$F = \mu R$$

Angle of Friction

It is defined as the angle made by the resultant of the normal reaction (R) and the limiting force of friction (F) with the normal reaction (R). It is denoted by ϕ.

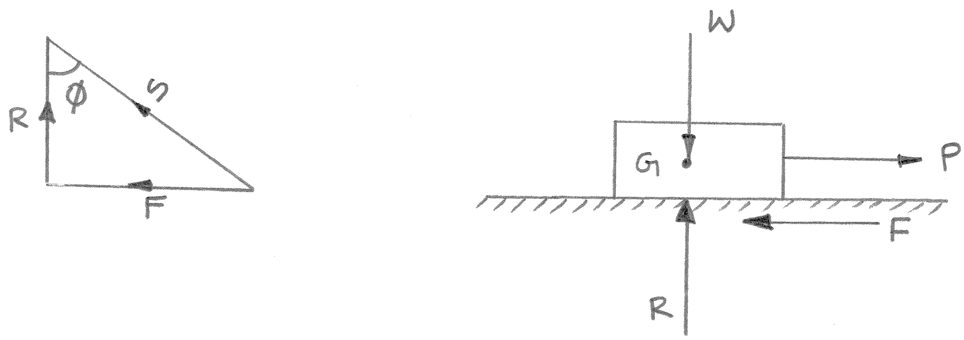


Fig. shows a solid body resting on a rough horizontal plane

Let S = Resultant of the normal reaction (R) and limiting force of friction

Then angle of friction $= \phi$

= Angle between S and R

From the figure, we have

$$\tan\phi = \frac{F}{R} = \frac{\mu R}{R}$$

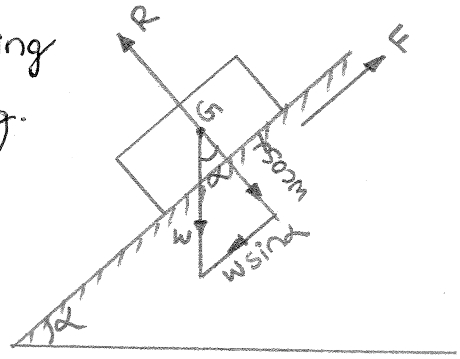
$$\tan\phi = \mu = \text{co-efficient of friction.}$$

Thus the tangent of the angle of friction is equal to the co-efficient of friction.

Angle of Repose

The angle of repose is defined as the maximum inclination of a plane at which a body remains in equilibrium over the inclined plane by the assistance of friction only.

Consider a body of weight W, resting on a rough inclined plane as shown in Fig.



Let R = Normal reaction acting at right angle to the inclined plane

α = Inclination of the plane with the horizontal

F = Frictional force acting upward along the plane.

Let the angle of inclination (α) be gradually increased, till the body just starts sliding down the plane. This angle of inclined plane, at which a body just begins to slide down the plane, is called angle of repose.

Resolving the forces along the plane

$$\Sigma F_x = 0$$

$$F - W\sin\alpha = 0$$

$$F = W\sin\alpha \longrightarrow (1)$$

Resolving the forces normal to the plane

$$\Sigma F_y = 0$$

$$R - W\cos\alpha = 0$$

$$R = W\cos\alpha \longrightarrow (2)$$

Dividing equation (1) by (2)

$$\frac{W\sin\alpha}{W\cos\alpha} = \frac{F}{R} \qquad \because \tan\phi = \frac{F}{R} = \mu$$

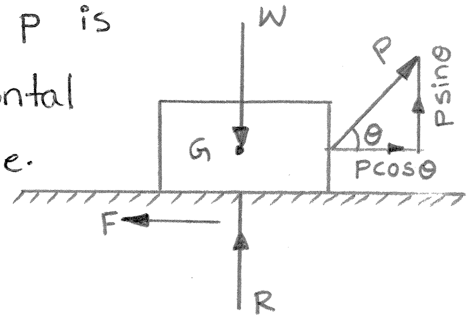
$$\tan\alpha = \frac{F}{R} \qquad \phi - \text{Angle of friction}$$

$$\tan\alpha = \tan\phi$$

$$\alpha = \phi$$

Angle of repose = Angle of friction

A block of weight W is placed on a rough horizontal plane surface as shown in Fig. and a force P is applied at an angle θ with the horizontal such that the block just tends to move.



Let R = Normal reaction

μ = coefficient of friction

F = Force of friction $= \mu R$

In this case the normal reaction R will not be equal to weight of the body. The normal reaction is obtained by resolving the forces on the block horizontally & vertically.

Resolving the forces on the block horizontally

$$\Sigma F_x = 0$$

$$P\cos\theta - F = 0 \qquad F = \mu R$$

$$F = P\cos\theta$$

$$\mu R = P\cos\theta \longrightarrow (1)$$

Resolving the forces on the block vertically

$$R - W + P\sin\theta = 0$$

$$R = W - P\sin\theta \longrightarrow (2)$$

From equation (2), it is clear that normal reaction is not equal to the weight of the block.

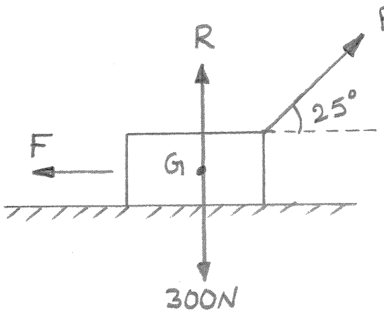
If in equation (2), the values of W, P & θ are known, then value of normal reaction (R) can be obtained. This value of R can be substituted in equation (1) to determine the value of μ.

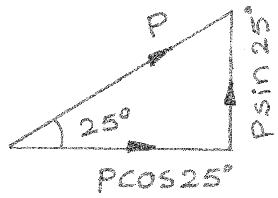
The force of friction is always equal to μR.

$$F = \mu R$$

The normal reaction (R) is not equal to the weight of the body always.

P 25 A body of weight 300N is lying on a rough horizontal plane having a coefficient of friction as 0.3. Find the magnitude of the force, which can move the body, while acting at an angle of 25° with the horizontal.





Weight of the body, $W = 300N$

coefficient of friction, $\mu = 0.3$

Let P = Magnitude of the force, which can move the body

F = Force of friction

Resolving the forces horizontally,

$$\Sigma F_x = 0$$

$$P\cos 25^\circ - F = 0$$

$$F = P\cos 25^\circ$$

Resolving the forces vertically,

$$\Sigma F_y = 0$$

$$R - 300 + P\sin 25^\circ = 0$$

$$R = 300 - P\sin 25^\circ$$

We know that the force of friction, $F = \mu R$

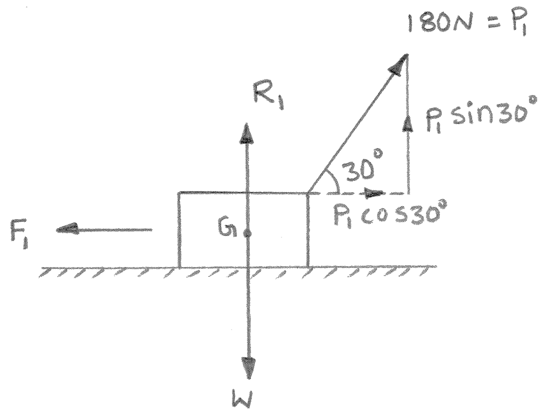
$$P\cos 25^\circ = 0.3\,(300 - P\sin 25^\circ)$$

$$0.9063\,P + 0.1267P = 90$$

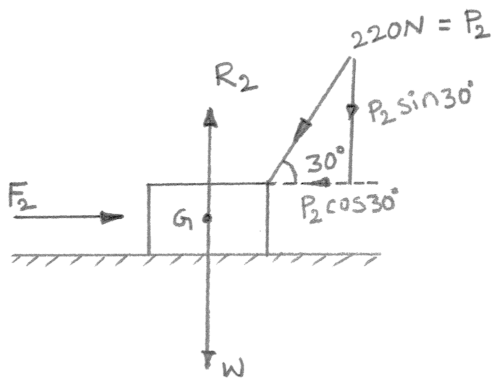
$$1.033\,P = 90$$

$$P = 87.12\ N$$

(P) A body resting on a rough horizontal plane required a pull of 180 N inclined at 30° to the plane just to move it. It was found that a push of 220 N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of friction.



(a) Pull of 180N



(b) Push of 220N

Consider a pull of 180 N acting on the body.

Resolving the forces horizontally,

$$\Sigma F_x = 0$$

$$P_1 \cos 30° - F_1 = 0$$

$$F_1 = 180 \cos 30° = 155.8 \text{ N}$$

Resolving the forces vertically,

$$\Sigma F_y = 0$$

$$P_1 \sin 30° + R_1 - W = 0$$

$$W - 180 \sin 30° = R_1$$

We know that the force of friction, $F = \mu R$

$$F_1 = \mu R_1$$

$$155.8 = \mu (W - 180 \sin 30°)$$

$$155.8 = \mu (W - 90) \longrightarrow ①$$

Consider a push of 220 N acting on the body.

Resolving the forces horizontally,

$$\Sigma F_x = 0$$

$$F_2 - P_2 \cos 30° = 0$$

$$F_2 = 220 \cos 30° = 190.52 \text{ N}$$

Resolving the forces vertically,

$$\Sigma F_y = 0$$

$$R_2 - W - P_2 \sin 30° = 0$$

$$R_2 = W + 220 \sin 30° = W + 110$$

26

R_2 = [illegible]

we know that the force of friction, $F = \mu R$

$$F_2 = \mu R_2$$

$$190.52 = \mu(W + 110) \longrightarrow ②$$

Dividing equation ① by ②

$$\frac{155.8}{190.52} = \frac{\mu(W-90)}{\mu(W+110)}$$

$$155.8(W+110) = 190.52(W-90)$$

$$155.8W + 17138 = 190.52W - 17146.8$$

$$34284.8 = 34.72W$$

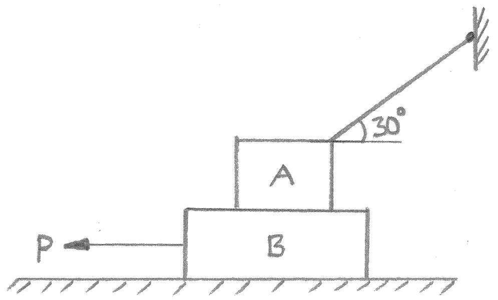
$$W = 987.4 \text{ N}$$

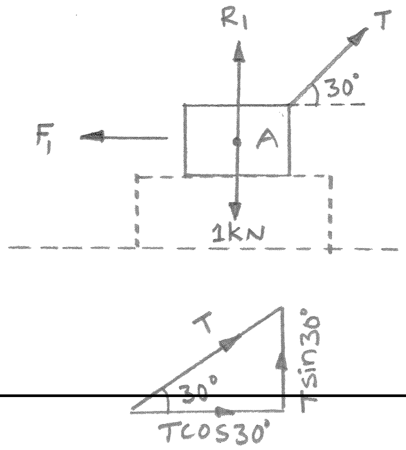
Now substituting the value of W in equation ①

$$155.8 = \mu(987.4 - 90)$$

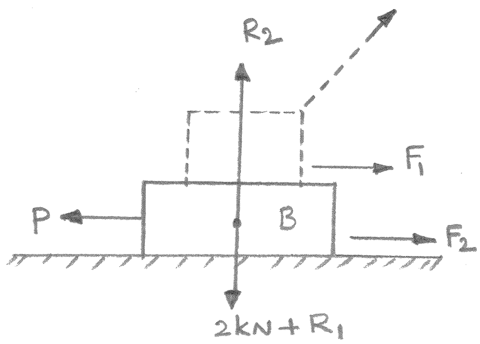
$$\mu = 0.173$$

(P) Two blocks A and B of weights 1kN and 2kN respectively are in equilibrium position as shown in Fig. If the coefficient of friction between the two blocks as well as the block B and the floor is 0.3. find the force (P) required to move the block B.





(a) Block A



b) Block B

consider the block A.

Resolving the forces vertically,

$R_1 + T\sin 30^\circ = 1$

$T\sin 30^\circ = 1 - R_1 \longrightarrow ①$

Resolving the forces horizontally

$T\cos 30^\circ - F_1 = 0$

$F_1 = T\cos 30^\circ \longrightarrow ②$

$F_1 = \mu R_1$

$= 0.3 R_1$

Dividing equation ① by ②

$$\frac{T\sin 30^\circ}{T\cos 30^\circ} = \frac{1-R_1}{\mu R_1}$$

$$\tan 30^\circ = \frac{1-R_1}{0.3R_1}$$

$R_1 = 0.85\ kN$

$F_1 = \mu R_1 = 0.3 * 0.85 = 0.255\ kN$

Now consider the block B. A little consideration will show that the downward force of the block A (equal to R_1) will also act alongwith the weight of the block B.

Resolving the forces vertically, $\Sigma F_x = 0$

$R_2 - 2 - R_1 = 0$

$R_2 = 2 + R_1$

$= 2 + 0.85$

$R_2 = 2.85\ kN$

$F_2 = \mu R_2$

$= 0.3 * 2.85$

$F_2 = 0.855\ kN$

Resolving the forces horizontally,

$\Sigma F_y = 0$

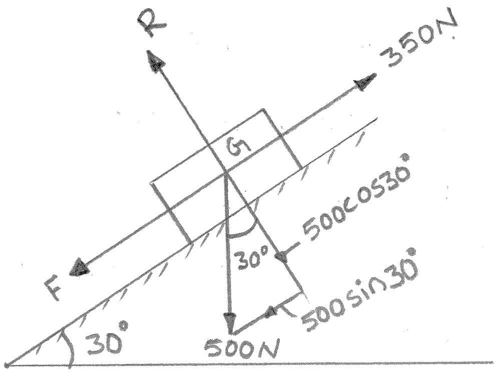
$-P + F_1 + F_2 = 0$

$P = F_1 + F_2$

$= 0.255 + 0.855$

$P = 1.11\ kN$

(P) (27) A body of weight 500 an in a force of 350 N. The inclination of the plane is 30° to the horizontal and the force is applied parallel to the plane. Determine the co-efficient of friction.



The body is in equilibrium under the action of the forces shown in Fig.

Resolving the forces normal to the plane

$$R - 500\cos 30^\circ = 0$$

$$R = 500\cos 30^\circ = 433\,N$$

Resolving the forces along the plane

$$350 - 500\sin 30^\circ - F = 0$$

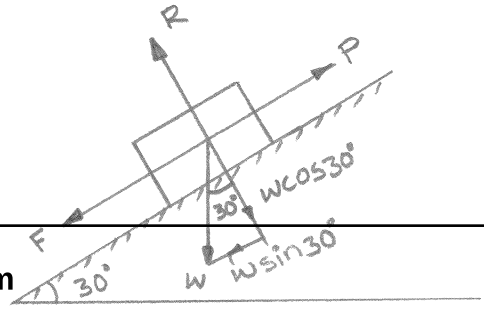
$$F = 350 - 500\sin 30^\circ = 100\ N$$

We know that $F = \mu R$

$$100 = \mu(433)$$

$$\mu = 0.23$$

(P) A body of weight 450 N is pulled up along an inclined plane having inclination 30° to the horizontal at a steady speed. Find the force required if the co-efficient of friction between the body and the plane is 0.25 and force is applied parallel to the inclined plane. If the distance travelled by the body is 10 m along the plane, find the work done on the body.



The body is in equilibrium under the action of forces shown in Fig.

Resolving forces along the plane

$$P - W\sin 30^\circ - F = 0$$

$$P = W\sin 30^\circ + F$$

$$P = 450\sin 30^\circ + \mu R$$

$$P = 450\sin 30^\circ + 0.25R \longrightarrow (1)$$

Resolving forces normal to the plane

$$R - W\cos 30^\circ = 0$$

$$R = W\cos 30^\circ$$

$$R = 450\cos 30^\circ = 389.7 \text{ N}$$

Substituting the value of R in equation (1)

$$P = 450\sin 30^\circ + 0.25(389.7)$$

$$P = 322.425 \text{ N}$$

Work done on the body = Force * Distance travelled in the direction of force

$= 322.425 * 10$ N-m = Joules

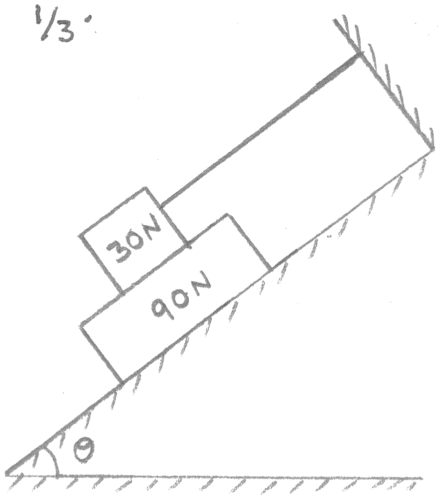
$= 3224.25$ N-m

Work done on the body = 3224.25 Joules

(P) What should be the value of the angle θ in Fig. so that the motion of the 90 N block impends down the plane. The co-efficient of friction μ for all the surfaces is 1/3.

Co-efficient of friction for all surfaces, $\mu = 1/3$

Motion of weight 90 N impends down the plane.



First consider the equilibrium of weight 30 N

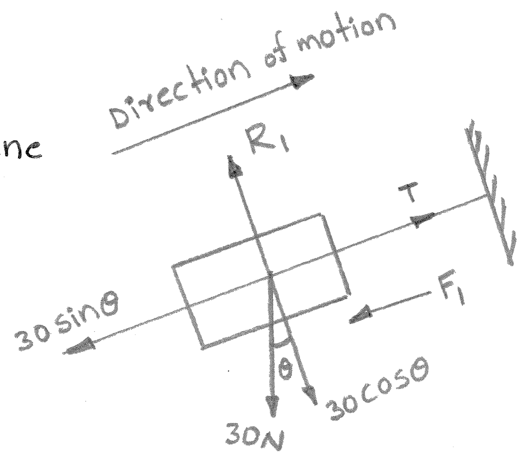
As the weight 90 N tends to move downwards, there will be a rubbing action between the surfaces of weight 90 N & 30 N. Hence a force of friction will be acting between these two surfaces.

The weight 30 N is tied to a string, the other end of the string is fixed to the plane. When the weight 90 N tends to move downwards, the weight 30 N with respect to weight 90 N will move upwards. Hence the force of friction on the lower surface of the weight 30 N will act downward as shown in Fig. The weight 30 N will be in equilibrium under the action of the forces shown in Fig.

T = Tension in the string

R_1 = Normal reaction on the lower surface of weight 30 N

F_1 = Force of friction $= \mu R_1$



Resolving the forces along the plane

$$\Sigma F_x = 0$$

$$T - 30\sin\theta - F_1 = 0$$

$$T = 30\sin\theta + \mu R_1$$

$$T = 30\sin\theta + \frac{1}{3} R_1 \longrightarrow (1)$$

Resolving the forces normal to the plane

$$R_1 - 30\cos\theta = 0$$

$$R_1 = 30\cos\theta \longrightarrow (2)$$

substituting the value of R_1 in equation (1)

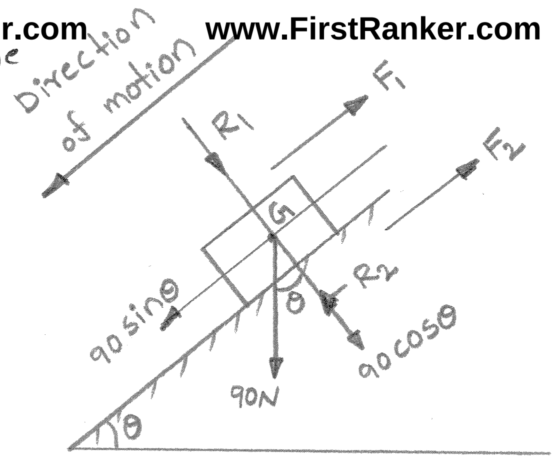
$$T = 30\sin\theta + \frac{1}{3}(30\cos\theta)$$

$$T = 30\sin\theta + 10\cos\theta \longrightarrow (3)$$

Now consider the equilibrium of weight 90 N

The weight 90 N will be in equilibrium under the action of forces shown in Fig.

Resolving the forces along the plane



$$F_1 + F_2 - 90\sin\theta = 0$$

$$90\sin\theta = \mu R_1 + \mu R_2$$

$$= \frac{1}{3}R_1 + \frac{1}{3}R_2$$

$$= \frac{1}{3} \times 30\cos\theta + \frac{1}{3}R_2$$

$$90\sin\theta = 10\cos\theta + \frac{1}{3}R_2 \longrightarrow (4)$$

Resolving the forces normal to the plane

$$R_2 - R_1 - 90\cos\theta = 0 \qquad (\because R_1 = 30\cos\theta)$$

$$90\cos\theta = R_2 - R_1$$

$$R_2 = 90\cos\theta + 30\cos\theta$$

$$R_2 = 120\cos\theta \longrightarrow (5)$$

substituting the value of R_2 in equation (4)

$$90\sin\theta = 10\cos\theta + \frac{1}{3}(120\cos\theta)$$

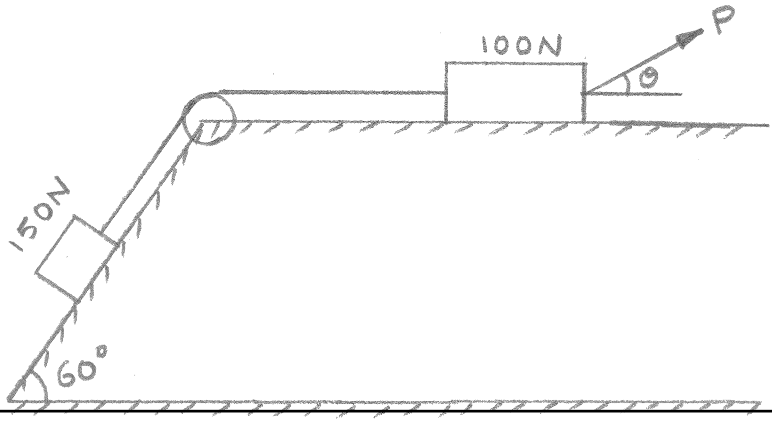
$$= 10\cos\theta + 40\cos\theta$$

$$90\sin\theta = 50\cos\theta$$

$$\tan\theta = 0.5555$$

$$\theta = 29.05^\circ$$

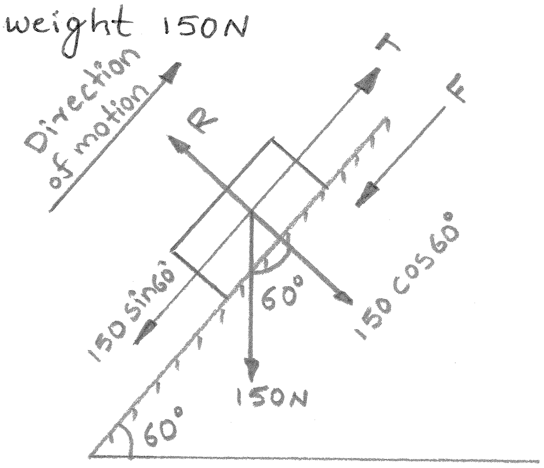
(P) Refering to the fig. given below, determine the least value of the force P to cause motion to impend rightwards. Assume the co-efficient of friction under the blocks to be 0.2 and pulley to be frictionless.



Co-efficient of friction under both blocks, $\mu = 0.2$

Pulley is frictionless. Motion of block of weight 100 N is towards right. Find least value of P.

Consider the equilibrium of block of weight 150N



As the block of weight 100 N tends to move rightwards, the block of weight 150 N will tend to move upwards. Hence force of friction will act downwards as shown in Fig.

Let T = Tension in the string

R = Normal reaction

F = Force of friction = μR

$F = 0.2 R$

The weight 150 N is acting vertically downwards. The body is in equilibrium under the action of forces shown in Fig.

Resolving the forces along the plane

$T - 150 \sin 60° - F = 0$

$T = 150 \sin 60° + 0.2 R \longrightarrow ①$

Resolving the forces normal to inclined plane

$R - 150 \cos 60° = 0$

$R = 150 \cos 60° = 75 \text{ N} \longrightarrow ②$

substituting the value of R in equation ①

$T = 150 \sin 60 + 0.2 \times 75$

$T = 144.9 \text{ N}$

Now consider the equilibrium of block of weight 100 N

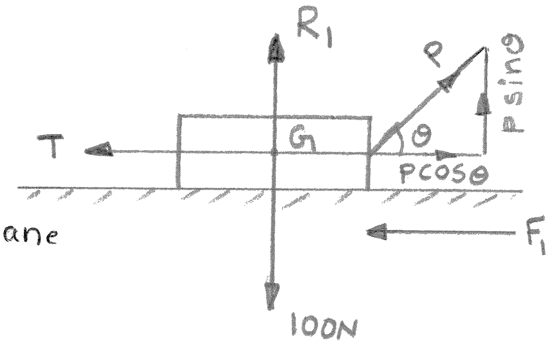
The block of weight 100 N tends to move rightwards, hence force of friction will be acting towards left as shown in Fig. Also the pulley is frictionless. Hence the tension in the string which is attached to the block of weight 100N will be 144.9 N. The body will be in equilibrium under the action of forces shown in Fig.

Direction of motion →

Resolving the forces along to the plane

$$P\cos\theta - T - F_1 = 0$$

$$P\cos\theta = 144.9 + 0.2 R_1 \longrightarrow (3)$$



Resolving the forces normal to the plane

$$R_1 - 100 + P\sin\theta = 0$$

$$R_1 = 100 - P\sin\theta \longrightarrow (4)$$

substituting the value of R_1 in equation (3)

$$P\cos\theta = 144.9 + 0.2(100 - P\sin\theta)$$

$$= 144.9 + 20 - 0.2P\sin\theta$$

$$P\cos\theta + 0.2P\sin\theta = 164.9$$

$$P(\cos\theta + 0.2\sin\theta) = 164.9$$

$$P = \frac{164.9}{\cos\theta + 0.2\sin\theta} \longrightarrow (5)$$

The force P will be minimum, if $(\cos\theta + 0.2\sin\theta)$ is maximum. But $(\cos\theta + 0.2\sin\theta)$ will be maximum if

$$\frac{d}{d\theta}(\cos\theta + 0.2\sin\theta) = 0$$

$$-\sin\theta + 0.2\cos\theta = 0$$

$$0.2\cos\theta = \sin\theta$$

$$0.2 = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = 0.2$$

$$\theta = 11.3^\circ$$

substituting the value of θ in equation (5), the least value of P will be obtained.

$$P_{(least)} = \frac{164.9}{\cos 11.3^\circ + 0.2\sin 11.3^\circ} = 161.88\ N$$

Equilibrium of System of Forces UNIT-2

According to Newton's second law of motion a body starts moving with acceleration if it is acted upon by a resultant force. Hence if another force equal in magnitude and opposite in direction to the force causing motion, is applied to the body, the body comes to rest. The force which is equal and opposite to the resultant force is called Equilibriant. To find the equilibriant of a force system, the resultant of the forces is found and the equilibriant is shown in the opposite direction to that of resultant.

A body is said to be in equilibrium when it is at rest or has uniform motion. According to Newton's law of motion, it means that the resultant of all forces acting on a body in equilibrium is zero. This condition helps in determining the reactions from adjoining bodies acting on the body under consideration.

Analysing equilibrium conditions of various bodies is presented for the following system of forces

1. Coplanar concurrent force system
2. coplanar non-concurrent forces system
3. concurrent system of forces in space

Equilibrium of a body subjected to concurrent forces

The resultant R of a system of concurrent forces is zero only when the following conditions are satisfied

$$\Sigma X = 0$$

$$\Sigma Y = 0$$

It may be observed that only one of the above two conditions is not sufficient. For example $\Sigma X = 0$ means that $R\cos\alpha = 0$. This will ensure that the resultant R cannot exist in any direction except in y direction ($\alpha = 90°$). Hence the condition $\Sigma Y = 0$ also should be satisfied to ensure that the resultant R does not exist, that is, equilibrium condition exists.

Types of Forces on a Body

When applying equilibrium conditions to a body it is essential that all forces acting on the body should be considered. The various forces acting on a body may be grouped as:

- Applied forces
- Non-applied forces

Applied forces

Applied forces are the forces applied externally to a body. Each force has got a point of contact with the body. If a person stands on a ladder, his weight is an applied force. If a temple car is pulled, the force in the rope is an applied for the car.

Non-applied forces

There are two types of non-applied forces:

a) Self-weight b) Reactions

Self-weight

Every body is subjected to gravitational acceleration and hence has got a self-weight

$$W = mg$$

where m is mass of the body and g is gravitational acceleration ($9.81\ m/s^2$ near the earth surface)

Self-weight always acts in vertically downward direction. When analysing equilibrium conditions of a body, self-weight is treated as acting through the centre of gravity of the body. If self-weight is very small, it may be neglected.

Reactions

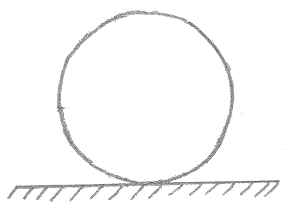
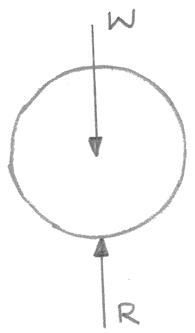
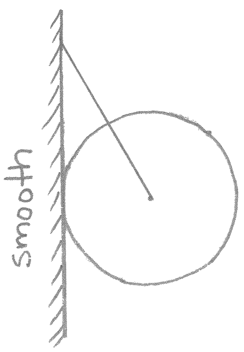
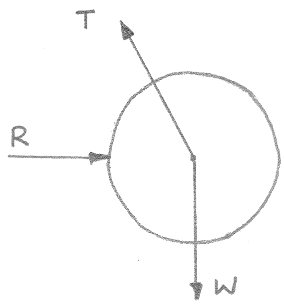
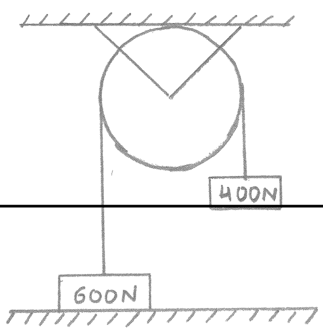
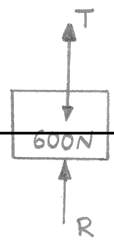
These are self-adjusting forces developed by the other bodies which come in contact with the body under consideration. According to Newton's third law of motion, the reactions are equal and opposite ~~to the actions. The reactions adjust themselves to bring the body to~~ equilibrium.

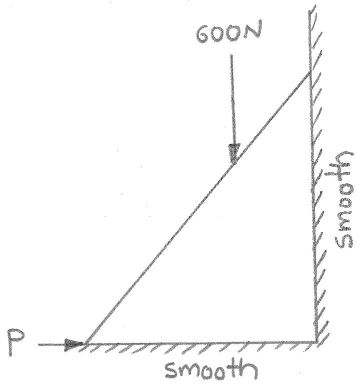
If the surface of contact is smooth, the direction of the reaction is normal to the surface of contact. If the surface of contact is not smooth, apart from normal reaction, there will be frictional reaction also. Hence the resultant reaction will not be normal to the surface of contact.

Free Body Diagram

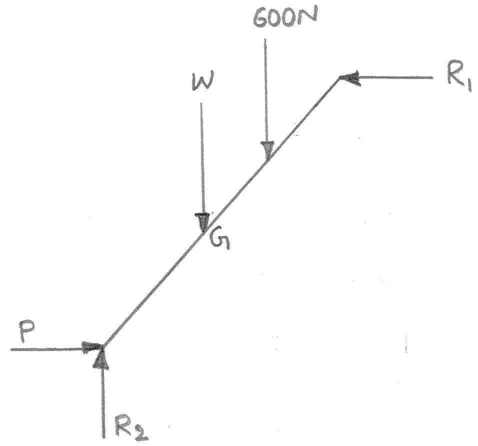
In many problems, it is essential to isolate the body under consideration from the other bodies in contact and draw all the forces acting on the body. For this, first the body is drawn and then applied forces, self-weight and the reactions at the points of contact with other bodies are drawn. Such a diagram of the body in which the body under consideration is freed from all the contact surfaces and all the forces acting on it (including reactions at contact surfaces) are drawn, is called a Free Body Diagram (FBD).

Free Body Diagrams for a few typical cases

Reacting bodies	FBD required for	FBD
	Ball	
	Ball	
	Block weighing 600N	



Ladder



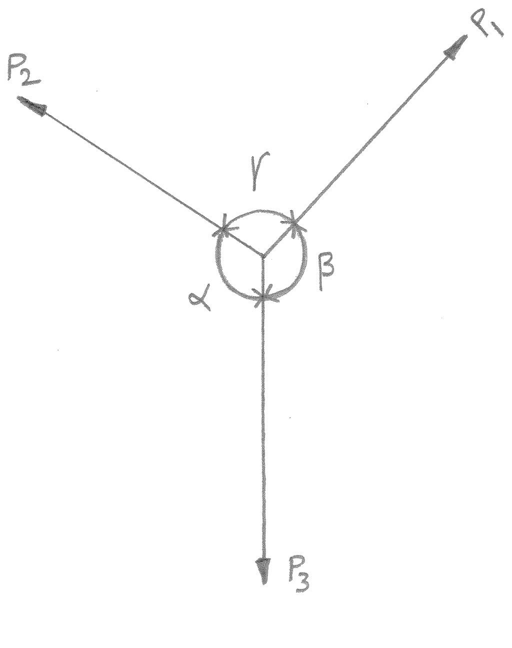
Lami's Theorem

If a body is in equilibrium under the action of a number of forces, it may be analysed using the equations $\Sigma x = 0$ and $\Sigma y = 0$. However, if the body is in equilibrium under the action of only three coplanar concurrent forces, Lami's theorem can be used conveniently.

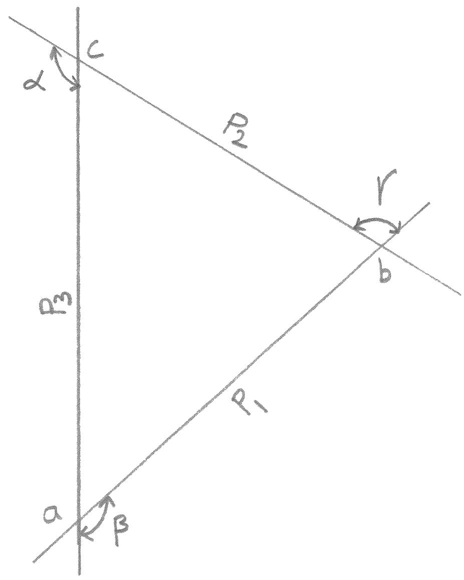
Lami's theorem states : If a body is in equilibrium under the action of three forces, each force is proportional to the sine of the angle between the other two forces.

Thus, for the system of forces shown in Fig.

$$\frac{P_1}{\sin\alpha} = \frac{P_2}{\sin\beta} = \frac{P_3}{\sin\gamma}$$



(a)



(b)

(3) Proof: Draw the three forces P_1, P_2 & P_3 one after the other in direction and magnitude starting from point a. Since the body is in equilibrium (resultant is zero), the last point must coincide with a. Thus, it results in a triangle of forces abc as shown in Fig (b). Now, the external angles at a, b & c are equal to β, γ and α, since ab, bc and ca are parallel to P_1, P_2 & P_3 respectively

In the triangle of forces abc

$$ab = P_1$$

$$bc = P_2$$

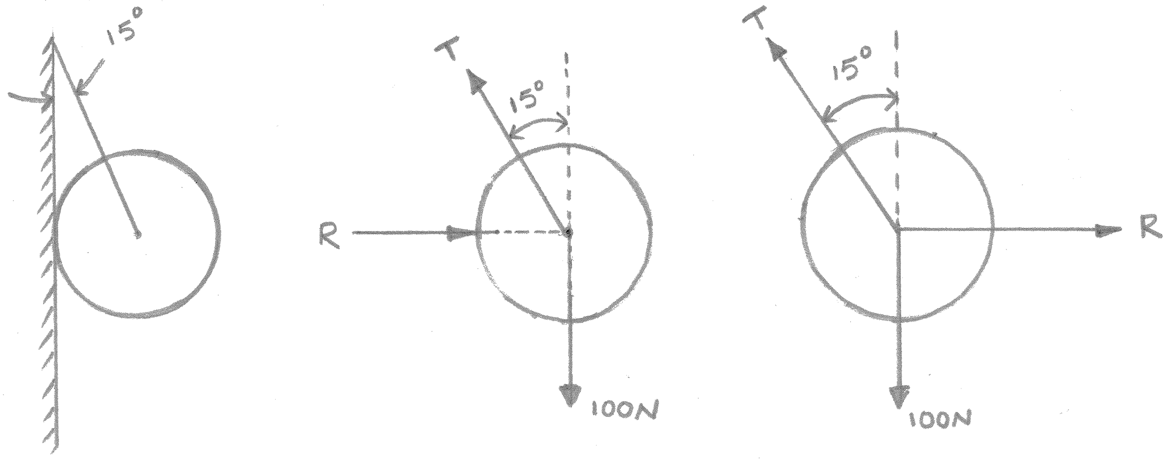
$$ca = P_3$$

Applying sine rule for the triangle abc

$$\frac{ab}{\sin(180-\alpha)} = \frac{bc}{\sin(180-\beta)} = \frac{ca}{\sin(180-\gamma)}$$

$$\frac{P_1}{\sin\alpha} = \frac{P_2}{\sin\beta} = \frac{P_3}{\sin\gamma}$$

P u A sphere of weight 100 N is tied to a smooth wall by a string as shown in Fig. Find the tension T in the string and reaction R of the wall.



(a) (b) (c)

Free Body Diagram of the sphere is as shown in Fig.(b). Fig(c) shows all the forces moving away from the centre of the ball. Applying Lami's theorem to the system of forces,

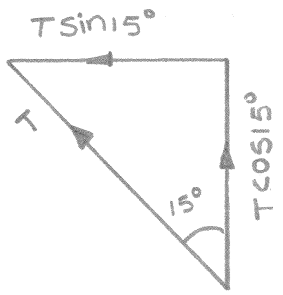
$$\frac{T}{\sin 90^\circ} = \frac{R}{\sin(180-15)} = \frac{100}{\sin(90+15)}$$

$$T = 103.53 \text{ N}$$

$$R = 26.79 \text{ N}$$

The above problem may be solved using equations of equilibrium also,

Taking horizontal direction as x-axis & vertical direction as y-axis,



$$\Sigma Y = 0$$

$$T\cos 15^\circ - 100 = 0$$

$$T = 103.53 \text{ N}$$

$$\Sigma X = 0$$

$$R - T\sin 15^\circ = 0$$

$$R = 26.79 \text{ N}$$

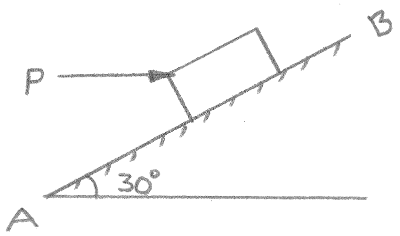
Notes:

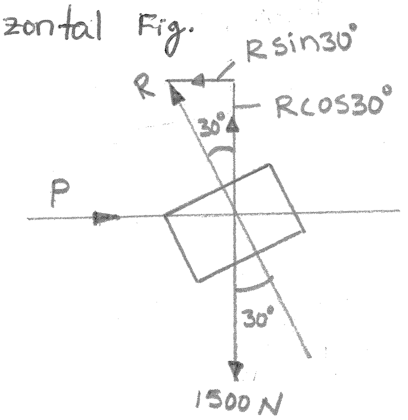
1. The string can have only tension in it (it can pull a body), but cannot have compression in it (cannot push a body).

2. The wall reaction is a push, but cannot be a pull on the body.

3. If the magnitude of reaction comes out to be negative, then assumed direction of reaction is wrong. It is acting exactly in the opposite to the assumed direction. However, the magnitude will be the same. Hence no further analysis is required. This advantage is not there in using Lami's equation. Hence, it is advisable for beginners to use equations of equilibrium, instead of Lami's theorem even if the body is in equilibrium under the action of only three forces.

(P) Determine the horizontal force P to be applied to a block of weight 1500N ~~and~~ to hold it in position on a smooth inclined plane AB which makes an angle of 30° with the horizontal Fig.





The body is in equilibrium under the action of applied force P, self-weight 1500N and normal reaction R from the plane. Since R, which is normal to the plane, makes 30° with the vertical (or 60° with the horizontal),

$$\Sigma Y = 0$$

$$R\cos 30° - 1500 = 0$$

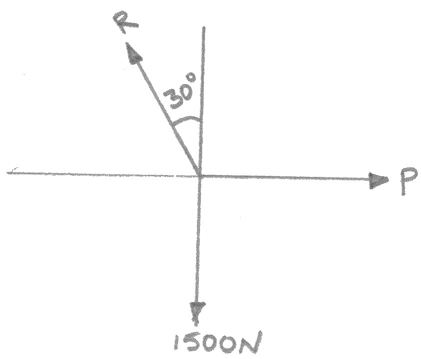
$$R = 1732.06\ N$$

$$\Sigma X = 0$$

$$P - R\sin 30° = 0$$

$$P = R\sin 30°$$

$$= 866.03\ N$$

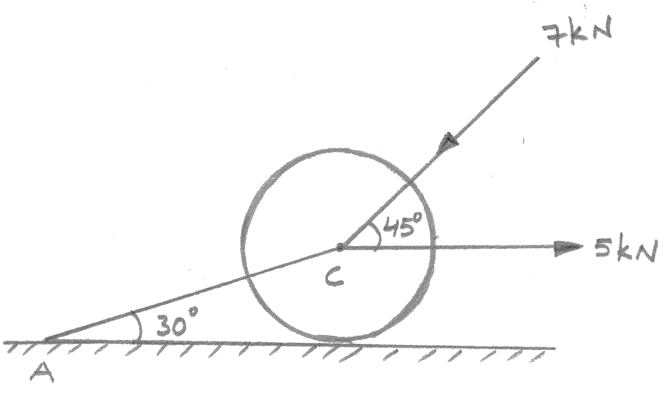


Lami's theorem

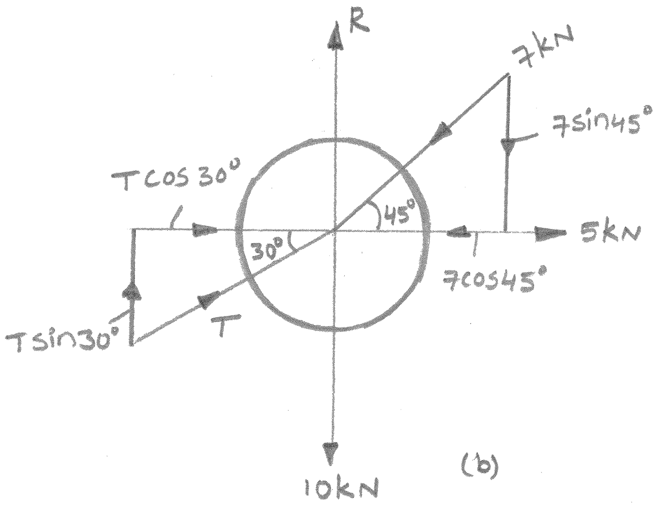
$$\frac{R}{\sin 90°} = \frac{P}{\sin(180-30)} = \frac{1500}{\sin(90+30)}$$

$R = 1732.06\ N$ & $P = 866.03\ N$

P 5 A roller of weight 10 kN rests on a smooth horizontal floor and is connected to the floor by the bar AC as shown in Fig. Determine the force in the bar AC and reaction from floor, if the roller is subjected to a horizontal force of 5 kN and an inclined force of 7 kN as shown in the figure.



(a)



(b)

A bar can develop a tensile force or a compressive force. Let the force developed be a compressive force T (push on the cylinder). Free Body Diagram of the cylinder is as shown in Fig.

Since there are more than three forces in the system, Lami's equations cannot be applied. consider the components in horizontal & vertical directions.

$$\Sigma H = 0$$

$$T\cos 30^\circ + 5 - 7\cos 45^\circ = 0$$

$$T = -0.058 \text{ kN}$$

Since the value of T is negative the force exerted by the bar is not a push, but it is pull (tensile force in bar) of magnitude 0.058kN

$$\Sigma V = 0$$

$$R - 10 - 7\sin 45^\circ + T\sin 30^\circ = 0$$

$$R = 10 + 7\sin 45^\circ - T\sin 30^\circ$$

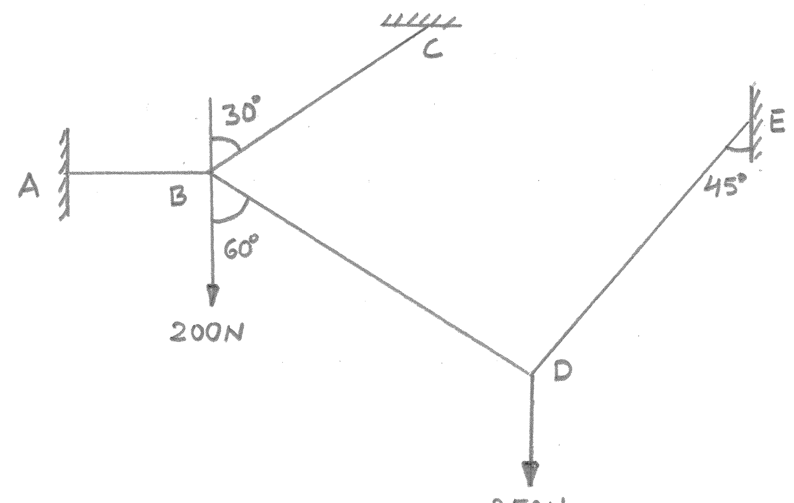
$$= 10 + 7\sin 45^\circ - (-0.058)\sin 30^\circ$$

$$R = 14.979 \text{ kN}$$

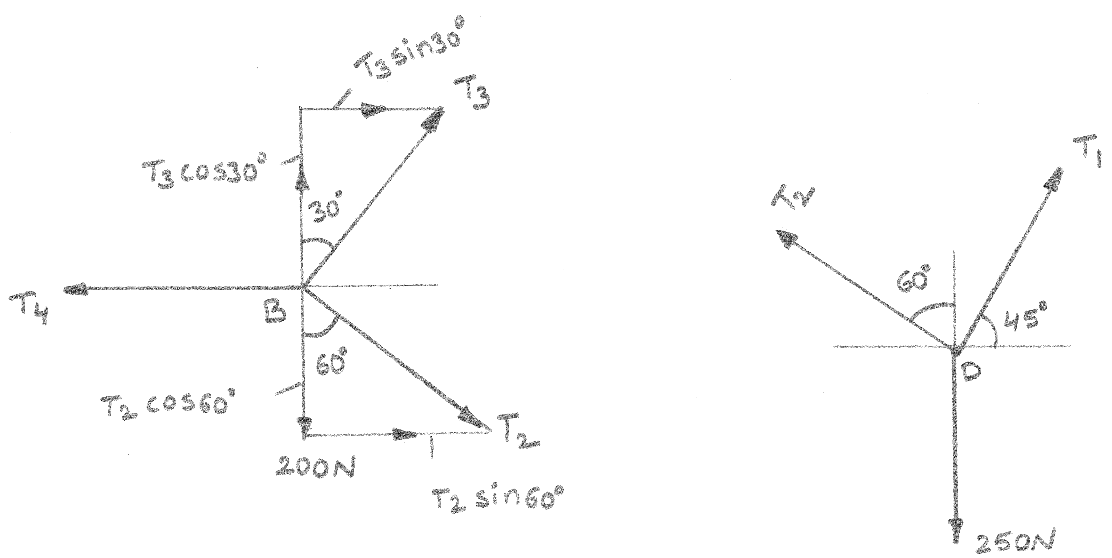
6 Equilibrium of Connected Bodies

When two or more bodies are in contact with one another, the system of forces appears as though it is a nonconcurrent forces system. However, when each body is considered separately, in many situations it turns out to be a set of concurrent force system. In such instances, first, the body subjected to only two unknown forces is to be analysed followed by the analysis of other connected body/bodies.

P A system of connected flexible cables shown in Fig. is supporting two vertical forces 200 N and 250 N at points B and D. Determine the forces in various segments of the cable.



(a)



(b)

Free Body Diagrams of the points B and D are shown in Fig (b). Let the forces in the members be as shown in the figure.

Applying Lami's theorem to the system of forces at point D,

$$\frac{T_1}{\sin 120°} = \frac{T_2}{\sin 135°} = \frac{250}{\sin 105°}$$

$T_1 = 224.$ ... $T_2 = 183.0$ ...

consider the system of forces acting at B

$$\Sigma V = 0$$

$$T_3 \cos 30^\circ - 200 - T_2 \cos 60^\circ = 0$$

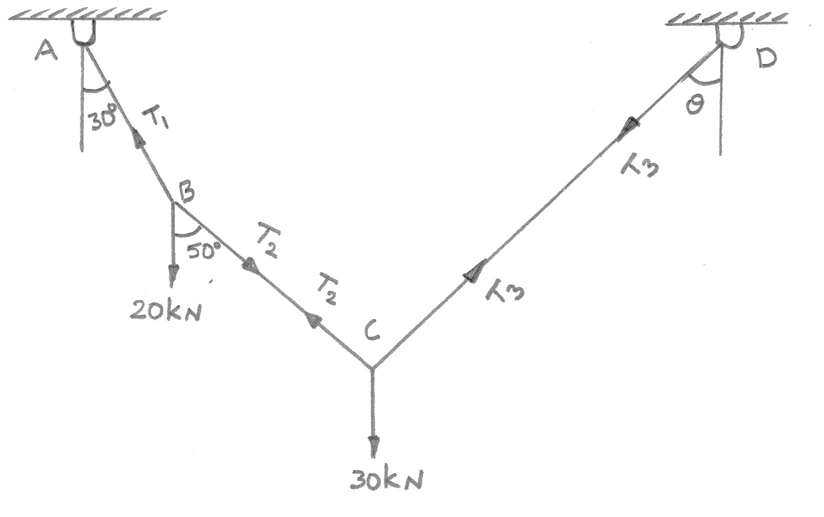
$$T_3 = 336.60 \ N$$

$$\Sigma H = 0$$

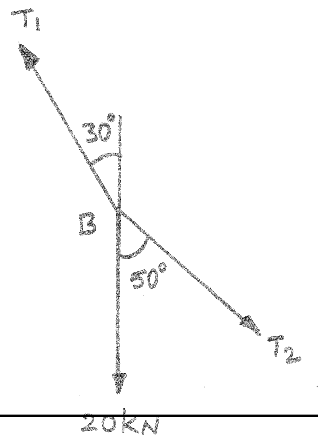
$$T_4 - T_2 \sin 60^\circ - T_3 \sin 30^\circ = 0$$

$$T_4 = 326.79 \ N$$

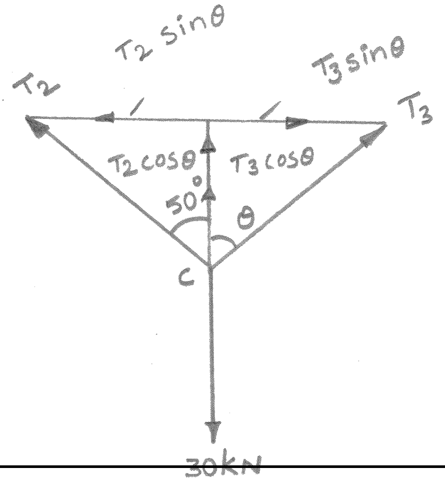
(P) A wire rope is fixed at two points A and D as shown in Fig. Two weights 20kN and 30kN are attached to it at B and C, respectively. The weights rest with portions AB and BC inclined at angles 30° & 50° respectively; to the vertical as shown in figure. Find the tension in the wire in segments AB, BC and CD and also the inclination of the segments CD to vertical.



(a)



(b)



(c)

(7) Applying Lami's theorem for the system of forces at B

$$\frac{T_1}{\sin 50^\circ} = \frac{T_2}{\sin 150^\circ} = \frac{20}{\sin 160^\circ}$$

$$T_1 = 44.79 \text{ kN}$$

$$T_2 = 29.24 \text{ kN}$$

writing equations of equilibrium for the system of forces at C

$$T_3 \sin\theta - T_2 \sin 50^\circ = 0$$

$$T_3 \sin\theta = 22.4 \longrightarrow (1)$$

$$T_3 \cos\theta - 30 + T_2 \cos 50^\circ = 0$$

$$T_3 \cos\theta = 11.20 \longrightarrow (2)$$

From (1) & (2)

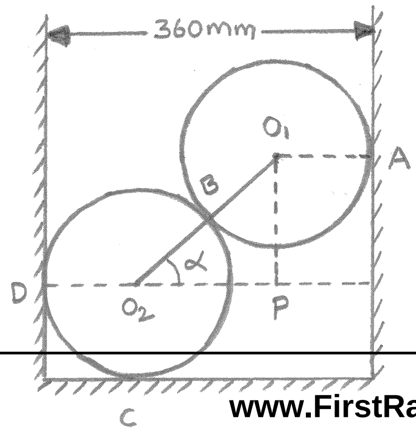
$$\frac{T_3 \sin\theta}{T_3 \cos\theta} = \frac{22.4}{11.20}$$

$$\tan\theta = 1998$$

$$\theta = 63.42^\circ$$

$$T_3 = 25.045 \text{ kN}$$

(P) Two smooth spheres each of radius 100 mm and weighing 100 N, rest in a horizontal channel having walls, the distance between which is 360 mm. Find the reactions at the points of contacts A, B, C and D shown in Fig.



(a)

$$O_1O_2 = O_1B + O_2B$$

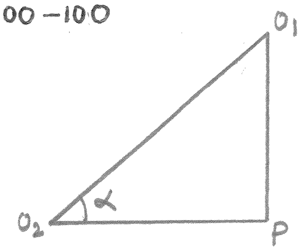
$$= 100 + 100$$

$$O_1O_2 = 200$$

$$O_2P = 360 - O_2D - O_1A$$

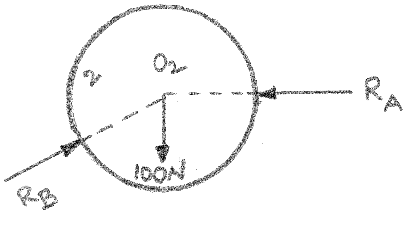
$$= 360 - 100 - 100$$

$$O_2P = 160$$

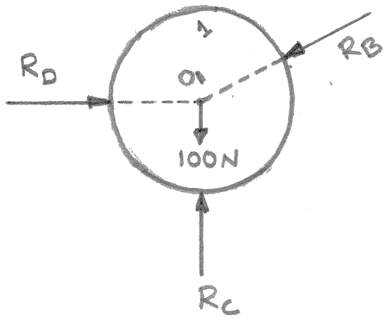


$$(O_1O_2)^2 = (O_1P)^2 + (O_2P)^2$$

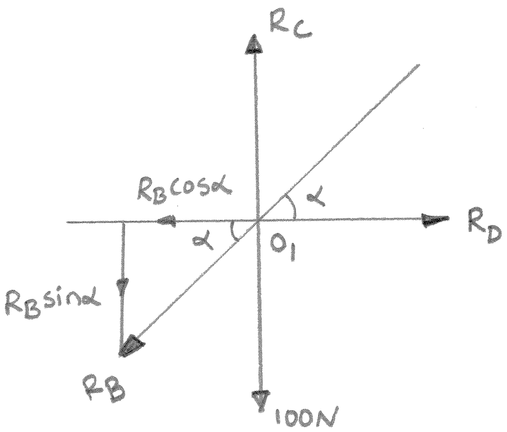
$$O_1P = \sqrt{(O_1O_2)^2 - (O_2P)^2} = 120$$

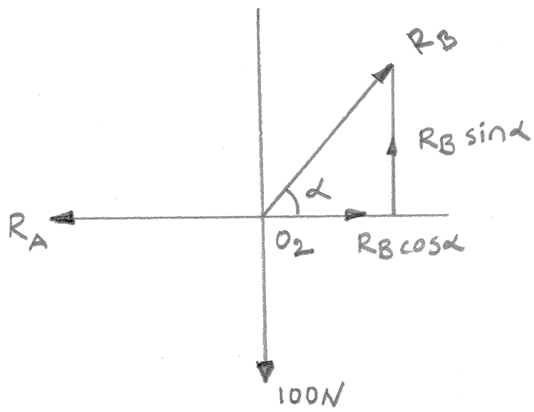


(b)



(c)





Let O_1 and O_2 be the centres of the first and second spheres. Drop perpendicular O_1P to the horizontal line through O_2. Fig (b) & (c) show free body diagram of the sphere 1 and 2, respectively.

Since the surface of contact are smooth, reaction of B is in the radial direction, i.e., in the direction O_1O_2. Let it make angle α with the horizontal. Then,

$$\cos\alpha = \frac{O_2P}{O_1O_2} = \frac{360 - O_1A - O_2D}{O_1B + O_2B} = \frac{360 - 100 - 100}{100 + 100} = 0.8$$

$$\sin\alpha = \frac{O_1P}{O_1O_2} = \frac{120}{200} = 0.6$$

Consider sphere No. 1

$\Sigma V = 0$

$R_C - 100 - R_B \sin\alpha = 0$

$R_C = 200\ N$

$\Sigma H = 0$

$R_D - R_B \cos\alpha = 0$ $R_D = 133.3\ N$

consider sphere No 2

$\Sigma V = 0$

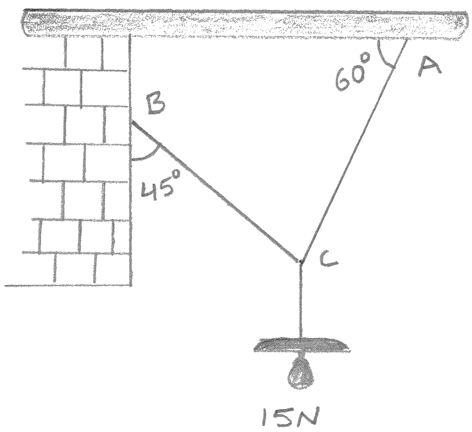
$R_B \sin\alpha - 100 = 0$

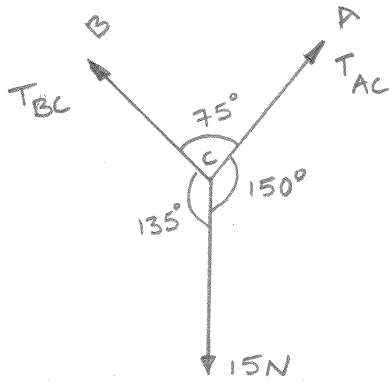
$R_B = 166.67\ N$

$\Sigma H = 0$

$R_B \cos\alpha - R_A = 0$ $R_A = 133.33\ N$

(P) (8) An electric light fixture weighing 15N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Fig. Using Lami's theorem, or otherwise, determine the forces in the strings AC & BC.





Weight at C = 15 N

Let T_{AC} = Force in the string AC

T_{BC} = Force in the string BC

The system of forces is shown in Fig. From the geometry of the figure, we find that angle between T_{AC} and 15N is 150° and angle between T_{BC} and 15N is 135°.

$$\therefore \angle ACB = 180° - (45° + 60°) = 75°$$

Applying Lami's equation at C,

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 135°} = \frac{T_{BC}}{\sin 150°}$$

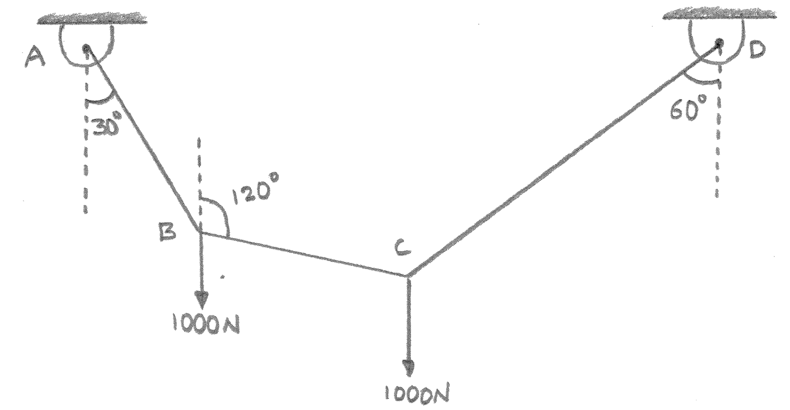
$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 45°} = \frac{T_{BC}}{\sin 30°}$$

$$T_{AC} = \frac{15}{\sin 75°} \times \sin 45° = 10.98 N$$

$$T_{BC} = \frac{15}{\sin 75°} \times \sin 30° = 7.76 N$$

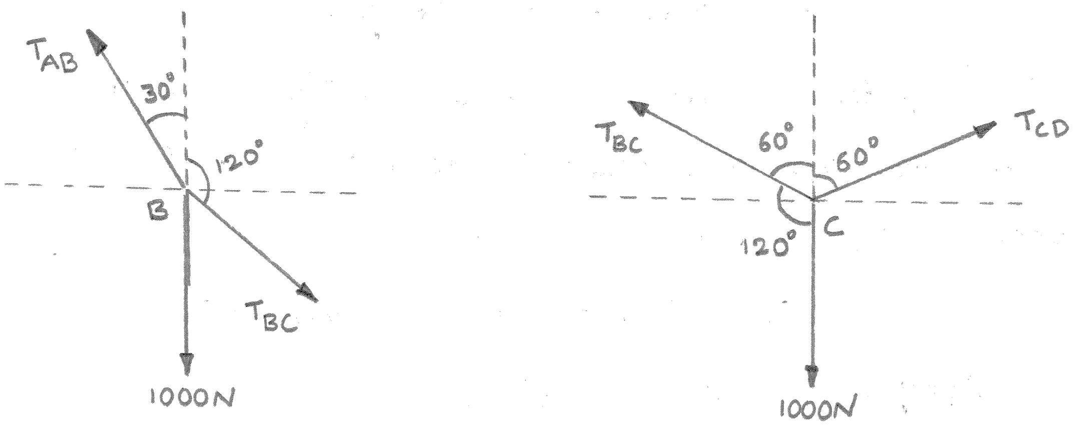
(P) A string ABCD, attached to fixed points A & D has two equal weights of 1000N attached to it at B and C. The weights rest with the portions AB and CD inclined at angles as shown in Figure.

Find the tensions in the portions AB, BC and CD of the string, if the inclination of the portion BC with the vertical is 120°.



Load at B = Load at C = 1000N

For the sake of convenience, let us split up the string ABCD into two parts. The system of forces at joints B and is shown in Fig (a) & (b).



(a) Joint B (b) Joint C

Let T_{AB} = Tension in the portion AB of the string

T_{BC} = Tension in the portion BC of the string

T_{CD} = Tension in the portion CD of the string

Applying Lami's equation at joint B,

$$\frac{T_{AB}}{\sin 60°} = \frac{T_{BC}}{\sin 150°} = \frac{1000}{\sin 150°}$$

$T_{AB} = 1732\text{ N}$ $\qquad T_{BC} = 1000\text{ N}$

Applying Lami's equation at joint C,

$$\frac{T_{BC}}{\sin 120°} = \frac{T_{CD}}{\sin 120°} = \frac{1000}{\sin 120°}$$

$T_{CD} = 1000\text{ N}$

(q) Equilibrium of system of concurrent forces in space

If the system of forces is in equilibrium resultant $R=0$

$\therefore$ In case of concurrent forces in space, it means

$$R_x = 0 \qquad R_y = 0 \qquad R_z = 0$$

i.e., $$\Sigma P_x = 0 \qquad \Sigma P_y = 0 \qquad \Sigma P_z = 0$$

If P is a force in space making angle θ_x, θ_y & θ_z with the coordinate systems when its components in x, y & z directions are given by

$$P_x = P\cos\theta_x$$
$$P_y = P\cos\theta_y$$
$$P_z = P\cos\theta_z$$

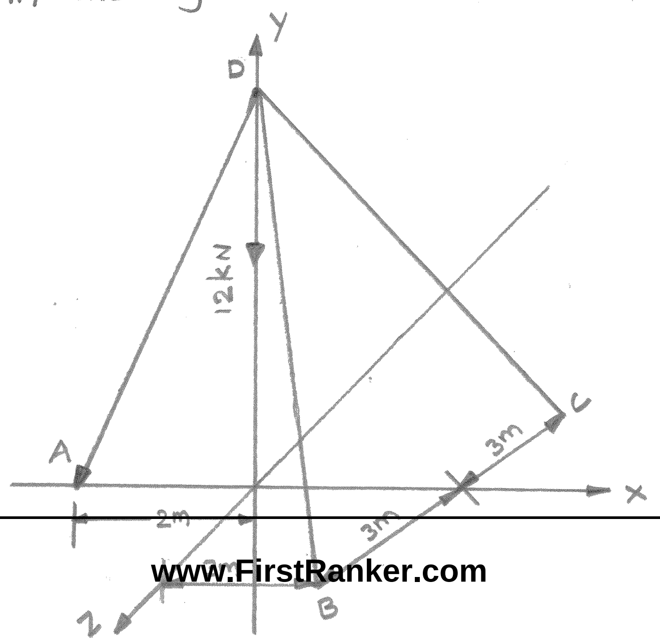
$\cos\theta_x$, $\cos\theta_y$ & $\cos\theta_z$ are termed as direction cosines.

If i and j are the two points on the line of action of a force P, with coordinates (x_i, y_i, z_i) & (x_j, y_j, z_j) then the direction cosines from i to j are given by

$$\cos\theta_x = \frac{x_j - x_i}{L} \qquad \cos\theta_y = \frac{y_j - y_i}{L} \qquad \cos\theta_z = \frac{z_j - z_i}{L}$$

$$L = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}$$

(P) A tripod is resting with its legs on a horizontal plane at points A, B, C as shown in Fig. Its apex point D is 4m above the floor level and carries a vertically downward load of 12kN. Determine the forces developed in the legs.



Let the forces developed in the members DA, DB and DC be P_1, P_2 and P_3 respectively. The point D is in equilibrium under the action of the forces, P_1, P_2, P_3 and 12kN load.

Taking point O, the projection of top point D as the origin and the coordinate system as shown in Fig., we have

$A(-2,0,0)$, $B(3,0,3)$, $C(3,0,-3)$, $D(0,4,0)$

L_1 = length of DA = $\sqrt{(-2-0)^2+(0-4)^2+(0-0)^2} = 4.472$ m

L_2 = length of DB = $\sqrt{(3-0)^2+(0-4)^2+(3-0)^2} = 5.831$ m

$\cos\theta_{1x} = \frac{-2-0}{4.472}$ $\quad\cos\theta_{1y} = \frac{0-4}{4.472}$ $\quad\cos\theta_{1z} = 0$

$P_{1x} = P_1\cos\theta_{1x}$ $\quad P_{1y} = P_1\cos\theta_{1y}$ $\quad P_{1z} = P_1\cos\theta_{1z}$

$= -0.447P_1$ $\quad = -0.894P_1$ $\quad = 0$

$\cos\theta_{2x} = \frac{3-0}{5.831}$ $\quad\cos\theta_{2y} = \frac{0-4}{5.831}$ $\quad\cos\theta_{2z} = \frac{3-0}{5.831}$

$P_{2x} = P_2\cos\theta_{2x}$ $\quad P_{2y} = P_2\cos\theta_{2y}$ $\quad P_{2z} = P_2\cos\theta_{2z}$

$= 0.514P_2$ $\quad = -0.686P_2$ $\quad = 0.514P_2$

$\cos\theta_{3x} = \frac{3-0}{5.831}$ $\quad\cos\theta_{3y} = \frac{0-4}{5.831}$ $\quad\cos\theta_{3z} = \frac{-3-0}{5.831}$

$P_{3x} = P_3\cos\theta_{3x}$ $\quad P_{3y} = P_3\cos\theta_{3y}$ $\quad P_{3z} = P_3\cos\theta_{3z}$

$= 0.514P_3$ $\quad = -0.686P_3$ $\quad = -0.514P_3$

The given load is in downward direction of Y

$\therefore P_{4x} = 0$ $\quad P_{4y} = -12$ $\quad P_{4z} = 0$

The equation of equilibrium are

$R_x = \Sigma P_{ix} = 0$, gives

$$-0.447P_1 + 0.514P_2 + 0.514P_3 = 0 \longrightarrow (1)$$

$R_y = \Sigma P_{iy} = 0$, gives

$$-0.894P_1 - 0.686P_2 - 0.686P_3 = -12 \longrightarrow (2)$$

$R_z = \Sigma P_{iz} = 0$, gives

$$0P_1 + 0.514P_2 - 0.514P_3 = 0 \longrightarrow (3)$$

From equation 3, we get

$$P_2 = P_3 \longrightarrow (4)$$

substituting it in equation 1, we get

$$-0.447 P_1 + 2 * 0.514 P_2 = 0$$

$$P_1 = 2.3 P_2 \longrightarrow (5)$$

substituting for P_1 and P_3 in terms of P_2 in equation 2, we get

$$-0.894 * 2.3 P_2 - 0.686 P_2 - 0.686 P_2 - 12 = 0$$

$$P_2 = -3.649 \text{ kN}$$

From equation 4 we get $P_3 = P_2 = -3.649$ kN

From equation 5 we get $P_1 = 2.3 P_2$

$$= 2.3(-3.649)$$

$$P_1 = -8.393 \text{ kN}$$

UNIT-III

Centroid & Centre of Gravity

Centroid

The plane figures (like triangle, quadrilateral, circle etc.) have only areas, but no mass. The centre of area of such figures is known as centroid.

The method of finding out the centroid of a figure is the same as that of finding out the centre of gravity of a body.

For any object having only one centroid.

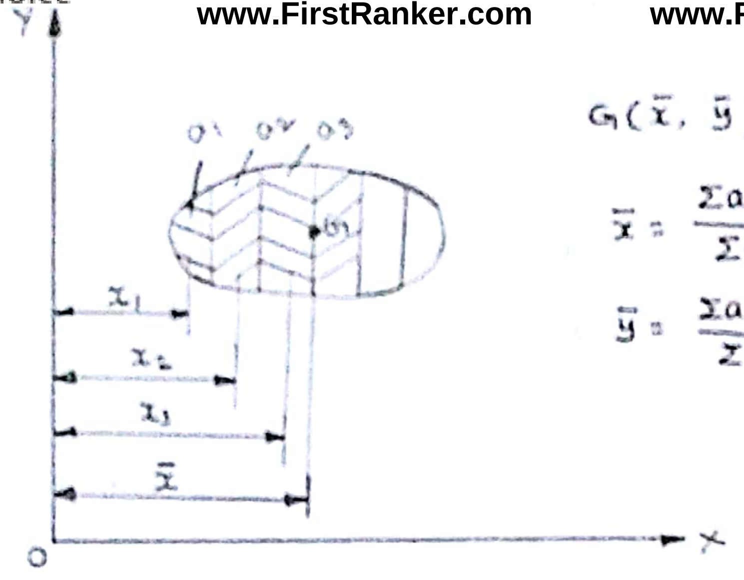
Centre of Gravity

The total weight or mass of the body passing through a common point is called centre of gravity.

(or)

centre of gravity can be defined as the point through which resultant of force of gravity (weight) of the body acts.

plane figures by the method of moments



$G(\bar{x}, \bar{y})$

$$\bar{x} = \frac{\Sigma a_i x_i}{\Sigma a_i}$$

$$\bar{y} = \frac{\Sigma a_i y_i}{\Sigma a_i}$$

Fig. shows a plane figure of total area A whose centre of gravity is to be determined. Let this area A is composed of a number of small areas a_1, a_2, a_3 ---- etc.

$$\therefore \quad A = a_1 + a_2 + a_3 + \text{-----}$$

Let

x_1 = The distance of the C.G of the area a_1 from axis OY

x_2 = The distance of the C.G of the area a_2 from axis OY

and so on.

The moments of all small areas about the axis OY

$$= a_1 x_1 + a_2 x_2 + a_3 x_3 + \text{------} \longrightarrow (1)$$

Let G is the centre of gravity of the total area A whose distance from the axis OY is $\bar{x}$.

Then moment of total area about the axis OY

$$= A\bar{x} \longrightarrow (2)$$

The moments of all small areas about the axis OY must be equal to the moment of total area about the same axis.

Hence equating equations (1) & (2)

$$a_1 x_1 + a_2 x_2 + a_3 x_3 + \text{--------} = A\bar{x}$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + \text{-----}}{A}$$

$A = a_1 + a_2 + a_3 + \text{-----}$

If we take the moments of the small areas about the axis OX and also the moment of total area about the axis OX, we get

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + \cdots}{A}$$

where

$\bar{y}$ = The distance of G from axis OX

y_1 = The distance of C.G of the area a_1 from axis OX

y_2, y_3, y_4 = The distance of C.G of the area a_2, a_3, a_4 from axis OX

Centre of gravity of plane figures by Integration method

Centre of gravity of plane figures by Method of moments

$$\bar{x} = \frac{\Sigma a_i x_i}{\Sigma a_i}, \quad \bar{y} = \frac{\Sigma a_i y_i}{\Sigma a_i} \qquad G(\bar{x}, \bar{y})$$

where

$i = 1, 2, 3, \cdots$

x_i = Distance of C.G of area a_i from axis OY

y_i = Distance of C.G of area a_i from axis OX

The value of i depends upon the number of small areas. If the small areas are large in number, then the summations in the above equations can be replaced by integration. Let small areas are represented by dA instead of a, then the above equations are written as

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\int x\,dA}{A} \qquad \int x\,dA = \Sigma x_i a_i$$

$$\bar{y} = \frac{\int y\,dA}{\int dA} = \frac{\int y\,dA}{A} \qquad \int y\,dA = \Sigma y_i a_i$$

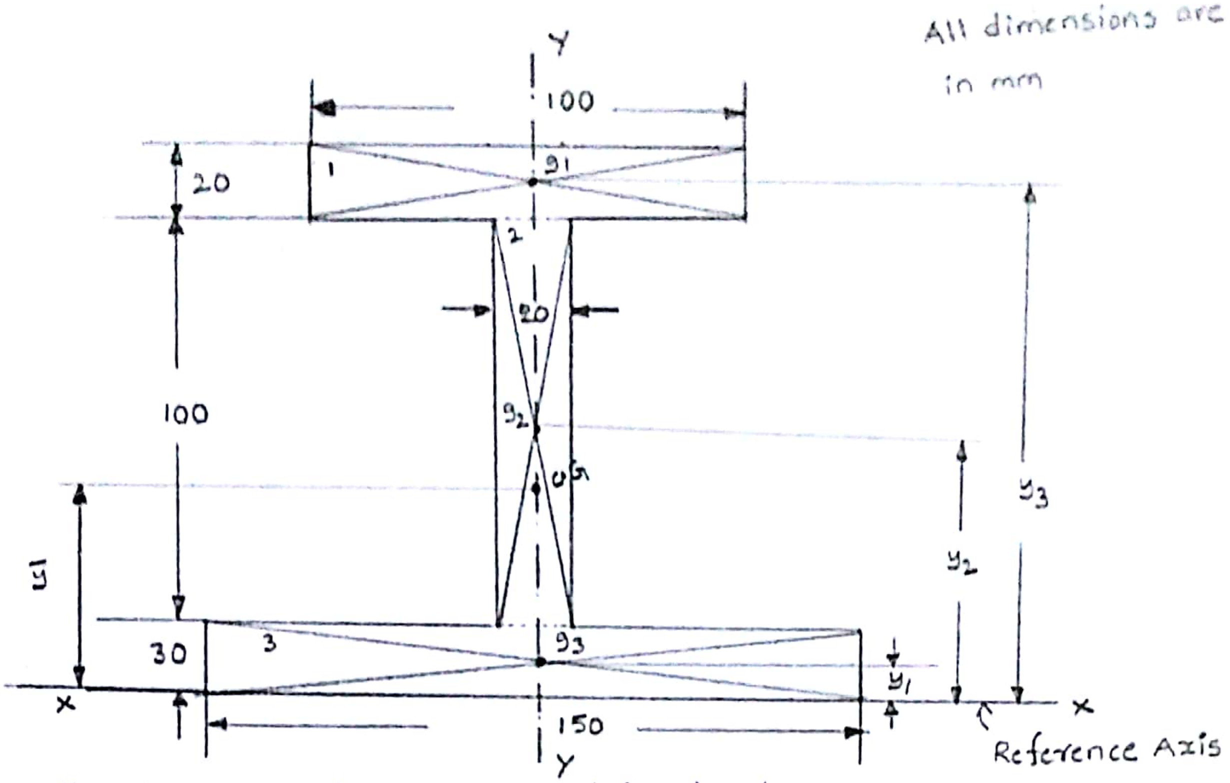
$$\int dA = \Sigma a_i$$

Where

x = Distance of C.G of area dA from axis OY

y = Distance of C.G of area dA from axis OX

Locate the centroid of the I-section shown in Fig.



Here the given I-section is symmetric about Y-axis. So $\bar{x} = 0$. Then centroid lies on y-axis.

Now the given I-section is divided into three parts. Taking bottom of the I-section as reference axis.

1. Rectangle

$b_1 = 100mm$, $d_1 = 20mm$

$a_1 = b_1 d_1 = 100 * 20 = 2000 mm^2$

$y_1 = 30 + 100 + \frac{20}{2} = 140mm$

2. Rectangle

$b_2 = 20mm$, $d_2 = 100mm$

$a_2 = b_2 d_2 = 20 * 100 = 2000 mm^2$

$y_2 = 30 + \frac{100}{2} = 80 mm$

3. Rectangle

$b_3 = 150mm$ $d_3 = 30 mm$

$a_3 = b_3 d_3 = 150 * 30 = 4500 mm^2$

$y_3 = \frac{30}{2} = 15 mm$

$\therefore \bar{y} = \frac{\Sigma a_i y_i}{a_i}$ → The distance of centroid from bottom of the section.

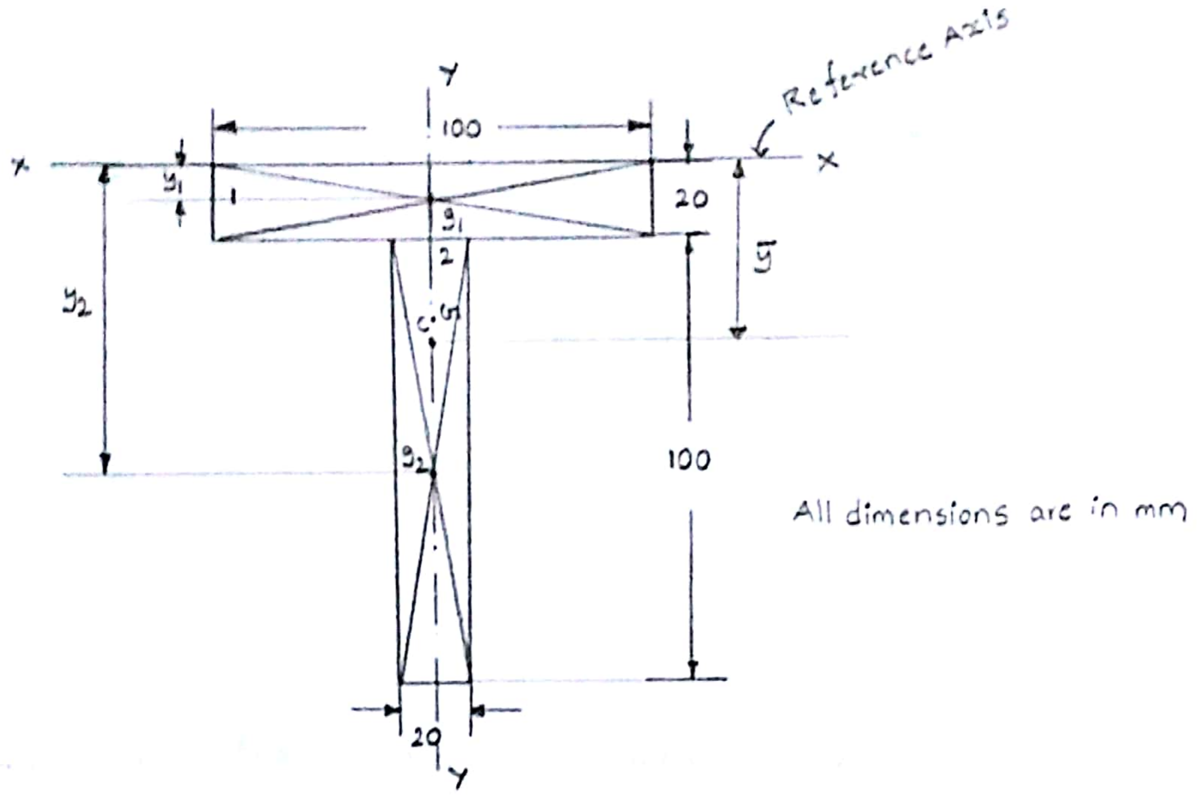
$i = 1, 2, 3$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{2000*140 + 2000*80 + 4500*15}{2000 + 2000 + 4500}$$

$$= 59.71 \text{ mm}$$

Thus, the centroid is on the symmetric axis at a distance 59.71 mm from the bottom.

(P) Locate the centroid of the T-section shown in the figure



Here the given T-section is symmetry about y-axis. So $\bar{x} = 0$.
Then centroid lies on y-axis.

Now the given T-section is divided into two parts. Taking top of the T-section as reference axis.

1. Rectangle

$b_1 = 100\,mm$, $d_1 = 20\,mm$

$a_1 = 100 \times 20 = 2000\,mm^2$

2. Rectangle

$b_2 = 20\,mm$, $d_2 = 100\,mm$

$a_2 = 20 \times 100 = 2000\,mm^2$

$\therefore \quad \bar{y} = \dfrac{\Sigma a_i y_i}{a_i}$ → The distance of centroid from top of the section

$i = 1, 2$

$y_1 = \dfrac{d_1}{2} = \dfrac{20}{2} = 10\,mm$

$y_2 = 20 + \dfrac{100}{2} = 70\,mm$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

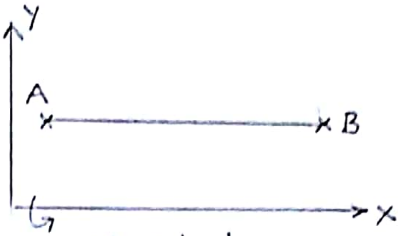
$$= \frac{2000 \times 10 + 2000 \times 70}{2000 + 2000}$$

$$\bar{y} = 40\,mm$$

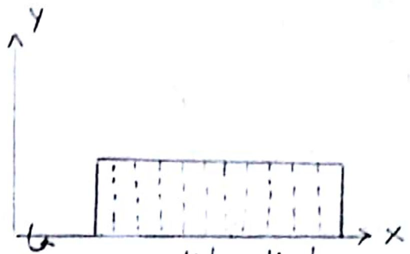
Hence, centroid of T-section is on the symmetric axis at a distance 40mm from the top.

Theorems of Pappus Guldinus

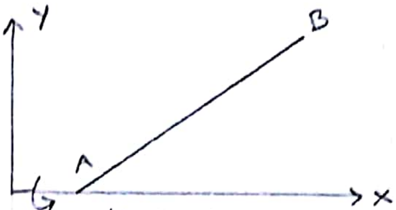
There are two important theorems, first proposed by Greek scientist Pappus and then restated by Swiss mathematician Paul Guldinus for determining the surface area and volumes generated by rotating a curve and a plane area about a non-intersecting axis, some of which are shown in Fig. These theorems are known as Pappus-Guldinus theorems.



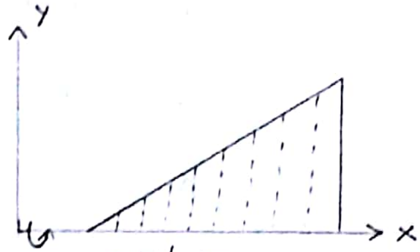
a) Cylinder



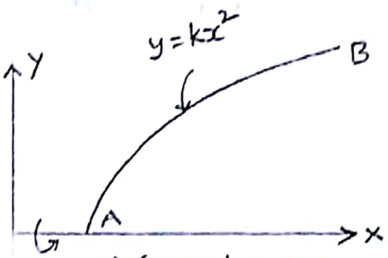
Solid cylinder



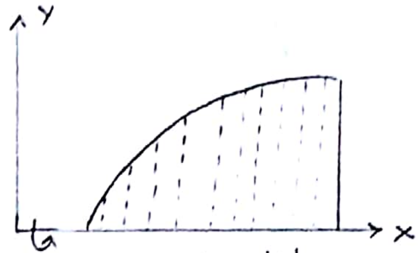
b) cone



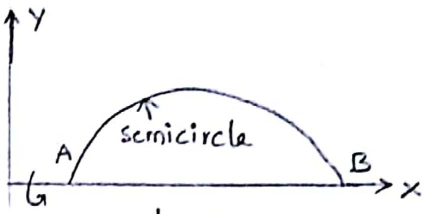
Solid Cone



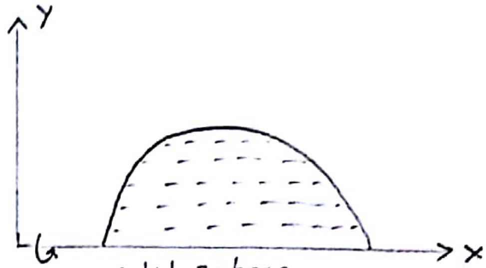
c) General curve



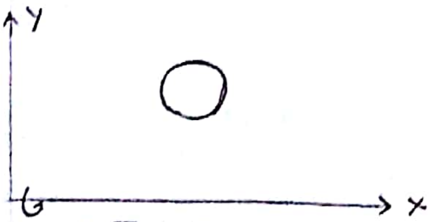
General solid



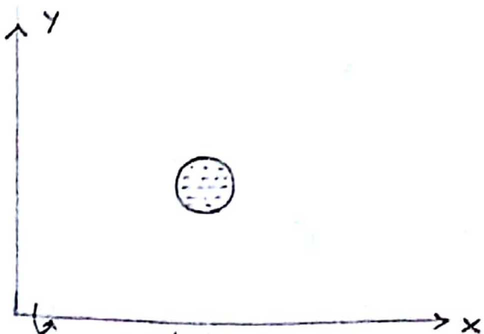
d) sphere



Solid sphere



e) Torus



Solid Torus

Generating surface of revolution

Generating solid of revolution

Theorem 2

The area of surface generated by revolving a plane curve about a non-intersecting axis in the plane of the curve is equal to the length of the generating curve times the distance travelled by the centroid of the curve in the revolution.

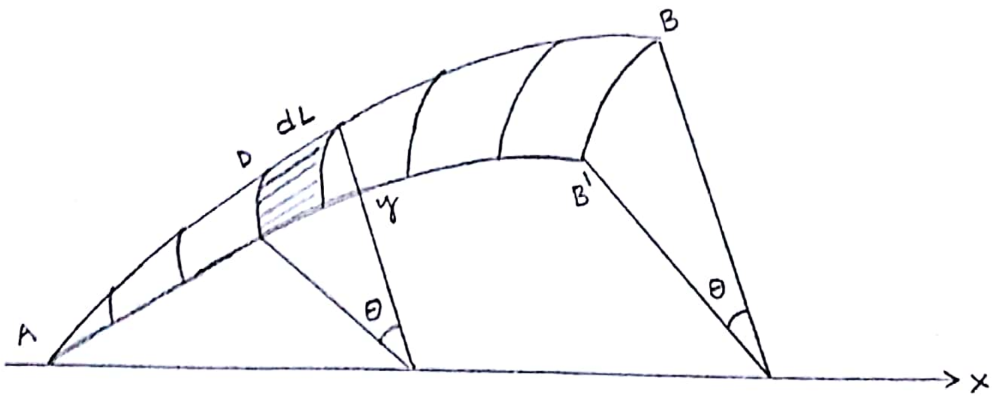
Proof: Fig. shows the isometric view of the plane curve rotated about x-axis by angle θ. We are interested in finding the surface area generated by rotating the curve AB. Let dL be the elemental length on the curve at D. Its coordinate be y. Then the elemental surface area generated by this element at D.

$$dA = dL\,(y\theta)$$

$$A = \int dL\,(y\theta) = \theta \int y\,dL$$

$$A = \theta L y_c = L\,(y_c \theta)$$

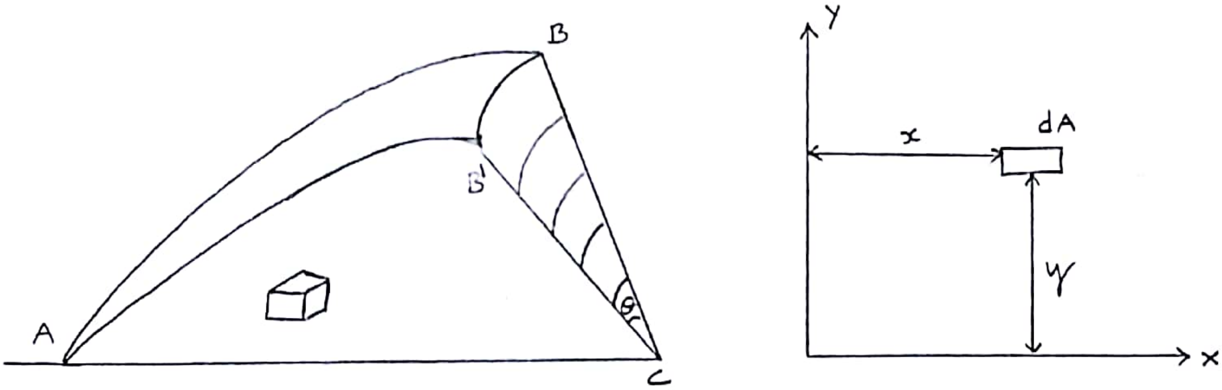
Thus we get area of the surface generated as length of the generating curve times the distance travelled by the centroid.



Theorem II

The volume of the solid generated by revolving a plane area about a non-intersecting axis in the plane is equal to the area of the generating plane times the distance travelled by the centroid of the plane area during the rotation.

Proof: Consider the plane area ABC, which is rotated through an angle θ about x-axis as shown in Fig.



Let dA be the elemental area of distance y from x-axis. Then the volume generated by this area during rotation is given by

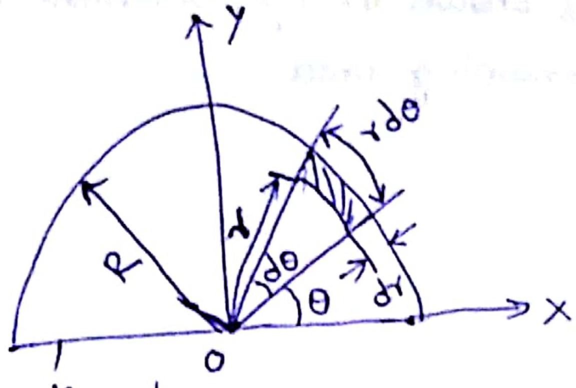
$$dV = dA\, y\, \theta$$

$$V = \int dA\, y\theta = \theta \int y\, dA$$

$$V = \theta A y_c = A(y_c \theta)$$

Thus the volume of the solid generated is area times the distance travelled by its centroid during the rotation.

Centroid of Semi-circle



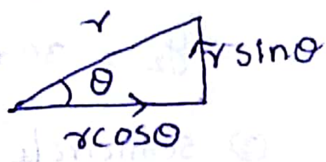
The given section is symmetrical about y-axis, then $\bar{x} = 0$. Centroid lies on Axis of symmetry.

$$A \cdot \bar{y} = \int y \, dA$$

Elemental Area, $dA = r d\theta \times dr$

r - distance of element from 'O'

y - distance of element form diametral axis (i.e; OX)



$$y = r \sin\theta$$

A = Area of semi-circle $= \frac{\pi R^2}{2}$

$$A \cdot \bar{y} = \int y \, dA$$

$$\frac{\pi R^2}{2} \cdot \bar{y} = \int_0^{\pi}\int_0^{R} (r \, d\theta \, dr) \, r \sin\theta$$

$$= \int_0^{\pi} \sin\theta \, d\theta \int_0^{R} r^2 \, dr$$

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$$\frac{\pi R^2}{2} \cdot \bar{y} = \left[-\cos\theta\right]_0^{\pi} \left[\frac{r^3}{3}\right]_0^R$$

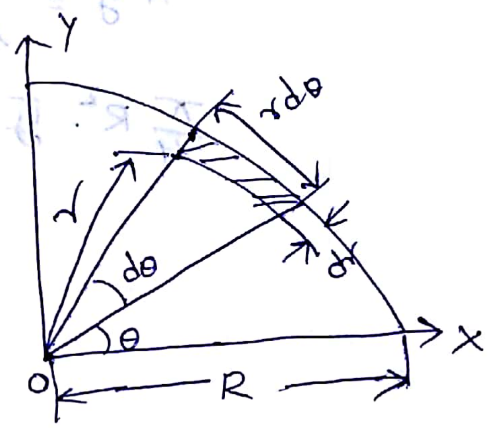
$$= (1+1)\left(\frac{R^3}{3}\right)$$

$$\bar{y} = \frac{2R^3}{3} * \frac{2}{\pi R^2}$$

$$\bar{y} = \frac{4R}{3\pi}$$

$$C.G = \left(0, \frac{4R}{3\pi}\right)$$

Centroid of Quarter circle

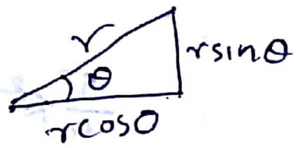


The given section unsymmetrical ~~about~~ either about x-axis or y-axis.

Then Find $\bar{x}$, $\bar{y}$.

$$A \cdot \bar{x} = \int x \, dA$$

$$A \cdot \bar{y} = \int y \, dA$$



x – distance of element from diametral axis, i.e; oy – axis

$$x = r\cos\theta$$

Area of Element, $dA = rd\theta * dr$

Area of Quarter Circle, $A = \frac{\pi}{4} R^2$

$$A \cdot \bar{x} = \int x \, dA$$

$$\frac{\pi}{4} R^2 \cdot \bar{x} = \int_0^{\pi/2}\int_0^R (r d\theta dr)\, r\cos\theta$$

$$\frac{\pi}{4} R^2 \cdot \bar{x} = \int_0^{\pi/2} \cos\theta \, d\theta \int_0^R r^2 \, dr$$

$$= (\sin\theta)_0^{\pi/2} \left(\frac{r^3}{3}\right)_0^R$$

$$\frac{\pi}{4} R^2 \cdot \bar{x} = \frac{R^3}{3} (1)$$

$$\bar{x} = \frac{R^3}{3} \times \frac{4}{\pi R^2}$$

$$\bar{x} = \frac{4R}{3\pi}$$

$$A \cdot \bar{y} = \int y \, dA$$

$$\frac{\pi}{4} R^2 \cdot \bar{y} = \int_0^{\pi/2}\int_0^R (r\,d\theta\,dr)\, r\sin\theta$$

$$= \int_0^{\pi/2} \sin\theta \, d\theta \int_0^R r^2 \, dr$$

$$= (-\cos\theta)_0^{\pi/2} \left(\frac{r^3}{3}\right)_0^R$$

$$\frac{\pi}{4} R^2 \, \bar{y} = \frac{R^3}{3} (1)$$

$$\bar{y} = \frac{R^3}{3} \times \frac{4}{\pi R^2}$$

$$\bar{y} = \frac{4R}{3\pi}$$

$$C.G \left(\frac{4R}{3\pi}, \frac{4R}{3\pi}\right)$$

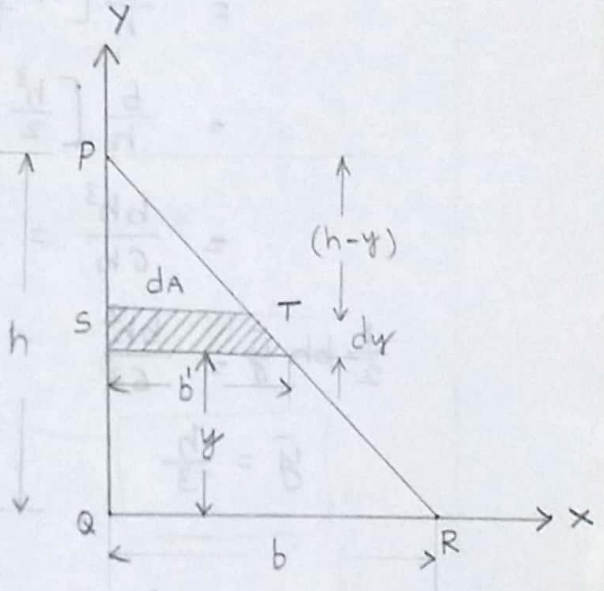
Integration Method

$$A\bar{x} = \int x\,dA \rightarrow \bar{x} = \frac{\int x\,dA}{A} \qquad \bar{y} = \frac{\Sigma A_i y_i}{A}$$

$$A\bar{y} = \int y\,dA \rightarrow \bar{y} = \frac{\int y\,dA}{A} \qquad \bar{x} = \frac{\Sigma A_i x_i}{A}$$

Centroid of a Triangle:-

consider the triangle PQR of base width 'b' & height 'h' as shown in fig.



Let 'b'' be the width of the elemental strip of thickness 'dy' at a distance 'y' from the base.

From similar triangles, ΔPQR, ΔPST

$$\frac{b'}{b} = \frac{h-y}{h}$$

$$b' = \frac{b}{h}(h-y)$$

$\therefore$ Area of the element, $dA = b'dy$

$$dA = \frac{b}{h}(h-y)\,dy$$

Area of the triangle, $A = \frac{1}{2}bh$

From eq, $\bar{y} = \frac{\int y\,dA}{A}$

$$\bar{y} = \frac{\text{Moment of area}}{\text{Total Area}} = \frac{\int y\,dA}{A}$$

$$A\bar{y} = \int y\,dA$$

$$\frac{1}{2}bh\,\bar{y} = \int_0^h y \cdot b'\,dy$$

$$\frac{1}{2} bh\,\bar{y} = \int y \,\frac{b(h-y)}{h}\, dy$$

$$= \int_0^h \frac{b}{h}(hy - y^2)\, dy$$

$$= \frac{b}{h}\left[h\cdot\frac{y^2}{2} - \frac{y^3}{3}\right]_0^h$$

$$= \frac{b}{h}\left[\frac{h^3}{2} - \frac{h^3}{3}\right] = \left(\frac{3h^3 - 2h^3}{6}\right)\frac{b}{h}$$

$$= \frac{bh^3}{6h} = \frac{bh^2}{6}$$

$$\frac{1}{2} bh\,\bar{y} = \frac{bh^2}{6}$$

$$\boxed{\bar{y} = \frac{h}{3}}$$

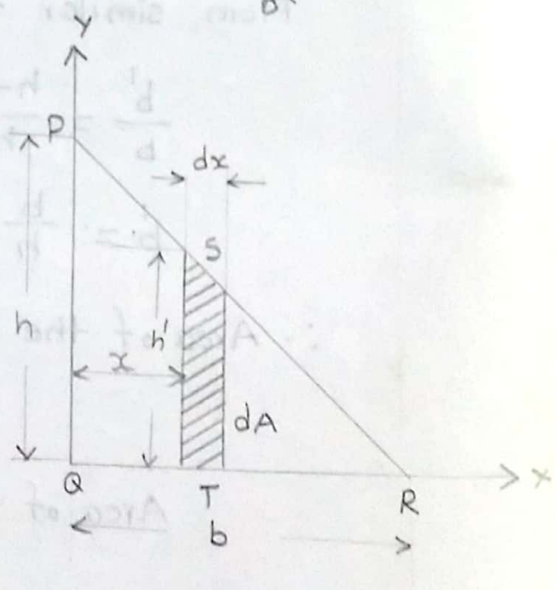
Thus the centroid of a triangle is at a distance $\frac{h}{3}$ from the base (or $\frac{2h}{3}$ from the apex) of the triangle where h is the height of the triangle.

$$A\cdot\bar{x} = \int x\, dA$$

$$\frac{1}{2} bh\,\bar{x} = \int_0^b x\, dA$$

$$= \int_0^b x\, h'\, dx$$

$dA = h'dx$



$$\frac{1}{2} bh\,\bar{x} = \int_0^b x\,\frac{h}{b}(b-x)\, dx$$

$$= \int_0^b \frac{h}{b}(xb - x^2)\, dx$$

$$= \frac{h}{b}\left[b\,\frac{x^2}{2} - \frac{x^3}{3}\right]_0^b$$

$$= \frac{h}{b}\left(\frac{b^3}{2} - \frac{b^3}{3}\right)$$

$$= \left(\frac{3b^3 - 2b^3}{6}\right)\frac{h}{b} = \frac{hb^3}{6b} = \frac{hb^2}{6}$$

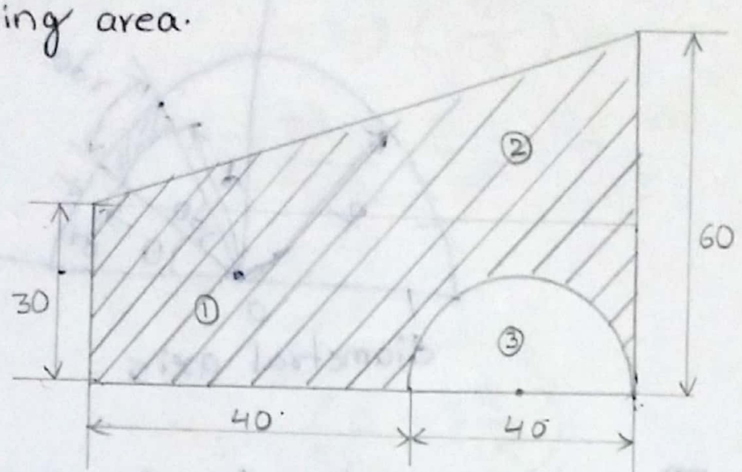
$$\frac{1}{2} bh\,\bar{x} = \frac{hb^2}{6}$$

$$\boxed{\bar{x} = \frac{b}{3}}$$

$$\frac{h}{h'} = \frac{b}{b-x}$$

$$h' = \frac{h}{b}(b-x)$$

3. A semicircular area is removed from a trapezium as shown in fig. Determine the centroid of the remaining area.



Sol:- The section not symmetrical about any axis, we have to find out the values of $\bar{x}$ & $\bar{y}$.

① Rectangle

$a_1 = (40+40) * 30 = 2400\,mm^2$ $\qquad (a+b)h$

$x_1 = \frac{40+40}{2} = 40mm$ $\qquad \frac{a+b}{2}$

$y_1 = \frac{30}{2} = 15mm$ $\qquad \frac{h}{2}$

② Triangle

$a_2 = \frac{1}{2} * (40+40) * 30 = 1200\,mm^2$ $\qquad \frac{1}{2}(a+b)h$

$x_2 = \frac{2b}{3} = \frac{2*80}{3} = 53.3mm$ $\qquad \frac{2b}{3}$

$y_2 = 30 + \frac{h}{3} = 30 + \frac{30}{3} = 40mm$ $\qquad h+\frac{h}{3}$

③ Semicircle

$a_3 = \frac{\pi r^2}{2} = \frac{\pi}{2}(20)^2 = 628.3\,mm^2$

$x_3 = 40 + \frac{40}{2} = 60mm$ $\qquad a+\frac{b}{2}$

$y_3 = \frac{4r}{3\pi} = \frac{4*20}{3\pi} = 8.5mm$

Distance b/w centre of gravity of the area & left face of trapezium

$$\bar{x} = \frac{a_1x_1 + a_2x_2 - a_3x_3}{a_1 + a_2 - a_3} = \underline{\underline{41.1\,mm}}$$

Distance b/w centre of gravity of the area & base of the trapezium

$$\bar{y} = \frac{a_1y_1 + a_2y_2 - a_3y_3}{a_1 + a_2 - a_3} = \underline{\underline{26.5\,mm}}$$