UNIT-IV

Moment Of Inertia

The moment of a force is equal to the product of the force (P) and the perpendicular distance (x) of the point, about which the moment is required and the line of action of the force.

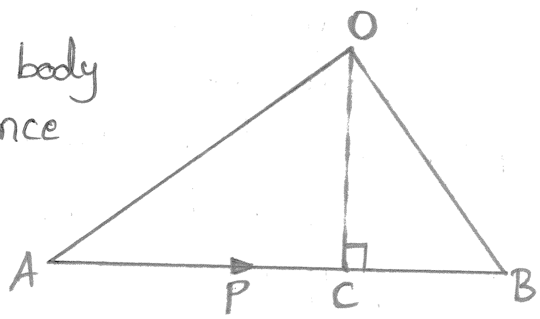
Moment, $M = P*x \rightarrow$ first moment of force

If this moment is again multiplied by the perpendicular distance (x) between the point and the line of action of force. i.e $= P*x^2$, then this quantity is called moment of the moment of force (or) second moment of force (or) moment of inertia.

Sometimes, instead of force, area or mass of a figure or body is taken into consideration. Then the second moment is known as second moment of area or second moment of mass. But all such second moments are broadly termed as moment of inertia.

Let, P – force acting on the body

x – perpendicular distance between the point, about which the moment is required and the line of action of the force.



Moment of the force P about O $= P*OC = AB*OC$

But $AB*OC$ is equal to twice the area of ΔAOB

Moment of Inertia:

Consider the area as shown in fig (a); dA is an elemental area with co-ordinates as x and y

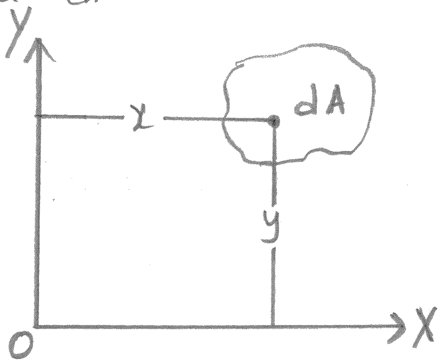
Moment of Inertia about X-axis is

$$I_{xx} = \sum y^2 dA$$

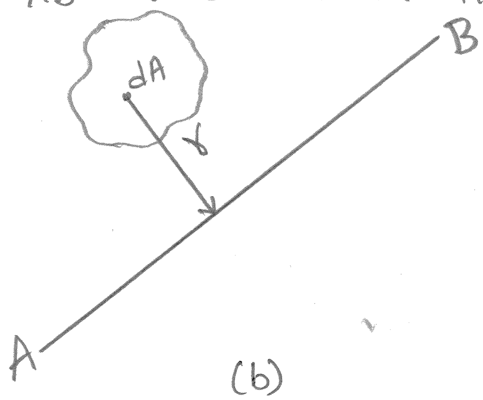
Moment of Inertia of the area about Y-axis is

$$I_{yy} = \sum x^2 dA$$

In general, if "x" is the distance of elemental area dA from the axis AB as shown in fig (b).



(a)



(b)

The sum of the terms $\Sigma r^2 dA$ to cover the entire area is called moment of inertia of the area about the axis AB.

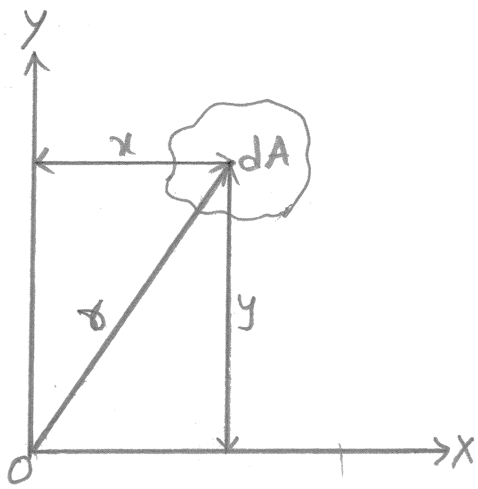
$$I_{AB} = \Sigma r^2 dA = \int r^2 dA.$$

The term rdA may be called as moment of area, similar to the moment of force. The term r^2dA is may be called as moment of moment of area or the second moment of area. Thus, the moment of inertia of a plane figure is nothing but second moment of area.

The moment of inertia is a fourth dimensional term since it is a term obtained by multiplying area by the square of the distance.

Units of Moment of Inertia — $m^4/mm^4/cm^4/ft^4$ (or) in^4

Polar moment of Inertia:



Moment of Inertia about an axis perpendicular to the plane of an area is known as polar moment of inertia. It may be denoted as J or I_{zz}.

Thus, the moment of inertia about an axis perpendicular to the plane of the area at O as shown in figure.

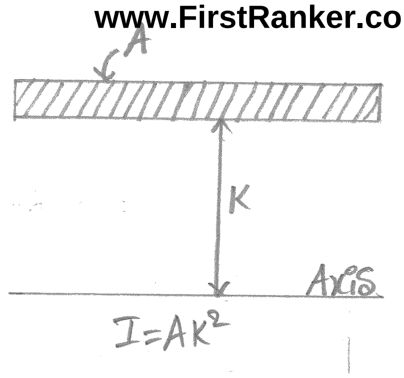
Polar moment of inertia at point "O"

$$I_{zz} = \Sigma r^2 dA.$$

Radius of Gyration:

Radius of Gyration is a mathematical term defined by the relation $K = \sqrt{\frac{I}{A}}$ (or) $I = AK^2$

Where, K is radius of gyration
I is Moment of Inertia
A is cross-sectional area



We can have,

$$K_{xx} = \sqrt{\frac{I_{xx}}{A}} \qquad K_{yy} = \sqrt{\frac{I_{yy}}{A}}$$

$$K_{AB} = \sqrt{\frac{I_{AB}}{A}}$$

The relation between radius of gyration and moment of inertia can be put in the form

$$I = AK^2$$

From the above relation a geometric meaning can be assigned to the term 'radius of gyration'. We can consider K as the distance at which the complete area is squeezed and kept as a strip of negligible width such that there is no change in the moment of inertia.

Methods for Moment of Inertia:

The moment of inertia of a plane area (or a body) may be found out by any of the following methods.

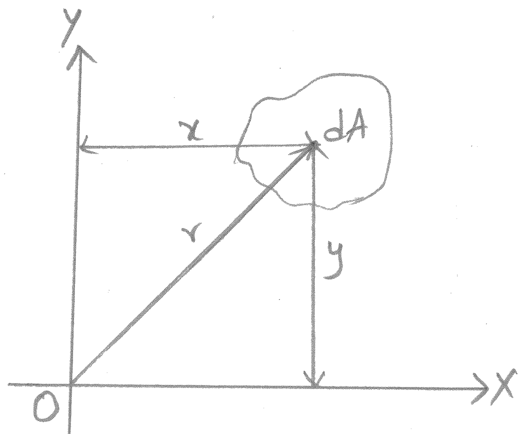
1. By Routh's rule 2. By Integration.

Note: The Routh's rule is used for finding the moment of inertia of a plane area or a body of uniform thickness.

Theorems of Moment of Inertia:

1. Perpendicular axis theorem 2. Parallel axis theorem

Perpendicular axis Theorem:



The moment of inertia of an area about an axis perpendicular to its plane [Polar moment of inertia] at any point 'O' is equal to the sum of moments of inertia about any two mutually perpendicular axis through the same point 'O' and lying in the plane of area.

Let us consider an elemental area dA at a distance r from O. Let the coordinates of dA be x and y.

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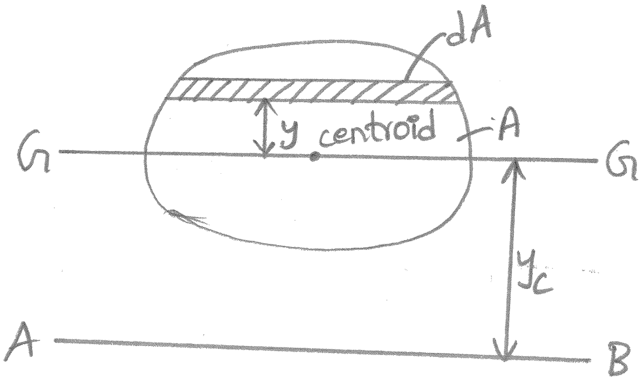
Then from definition

$I_{xx} = \sum y^2 dA$

$I_{yy} = \sum x^2 dA$

$$I_{zz} = \sum r^2 dA$$
$$= \sum (x^2+y^2)\, dA = \sum x^2 dA + \sum y^2 dA$$
$$I_{zz} = I_{xx} + I_{yy}$$

Parallel axis Theorem:



Moment of inertia about any axis in the plane of an area is equal to the sum of moment of inertia about a parallel centroidal axis and product of area and square of the distance between the two parallel axis.

Consider an elemental parallel strip dA at a distance "y" from the centroidal axis. AB be the any axis which is parallel to the centroidal axis located at a fixed distance y_c as shown in figure.

Moment of inertia about centroidal axis

$$I_{GG} = \int y^2 dA$$

Similarly, Moment of inertia about AB axis

$$I_{AB} = \int (y+y_c)^2 dA = \int (y^2 + y_c^2 + 2yy_c)\, dA$$
$$I_{AB} = \int y^2 dA + \int y_c^2 dA + \int 2yy_c dA$$
$$I_{AB} = \int y^2 dA + \int y_c^2 dA = I_{GG} + y_c^2 . A = I_{GG} + A y_c^2$$

$$\sum 2yy_c dA = 2y_c \sum y\, dA = 2y_c A \frac{\sum y dA}{A}$$

In the above term "$2y_cA$" is constant and $\frac{\sum ydA}{A}$ is the distance of centroid from reference axis GG. Since GG is passing through the centroid itself $\frac{ydA}{A}$ is zero. Hence the term $\sum 2yy_c dA$ is zero.

$$I_{AB} = I_{GG} + A . y_c^2$$

Where, I_{AB} = moment of inertia about axis AB

I_{GG} — Moment of inertia about centroidal axis GG parallel to AB

y_c – The distance b/w axis AB & parallel centroidal axis

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A – The area of plane figure given.

Note: The equation $I_{AB} = I_{GG} + A y_c^2$ cannot be applied to any two parallel axis. One of the axis (GG) must be centroidal axis only.

Moment of inertia from First Principles:

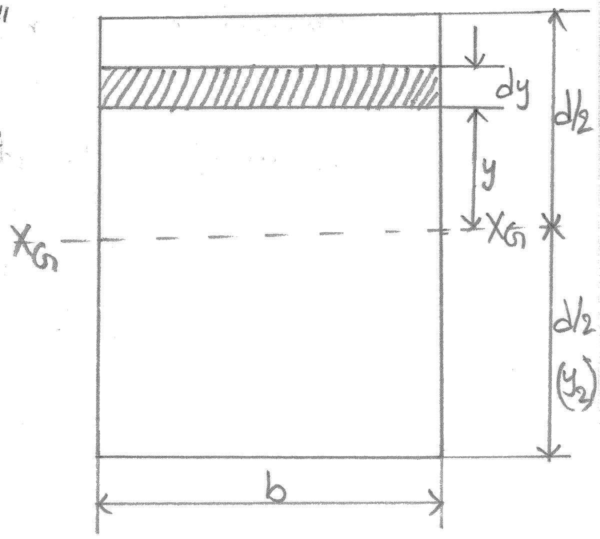
For simple figures, moment of inertia can be obtained by writing the general expression for an element and then carrying out integration so as to cover the entire area. This procedure is illustrated with the following three cases:

1. Moment of inertia of rectangle about the centroidal axis.
2. Moment of inertia of a triangle about the base.
3. Moment of inertia of a circle about a diametral axis.

Moment of inertia of a rectangle about centroidal axis:

Consider a rectangle of width "b" and depth "d". Moment of inertia about the centroidal axis X-G parallel to the short side is required.

Consider an elemental strip of width dy at a distance "y" from the axis.



Moment of Inertia of elemental strip about centroidal axis X-G is

$$= y^2 dA = y^2 b\, dy$$

The elemental strip of the area, $dA = b.dy$.

$$I_{XG} = \int_{-d/2}^{d/2} y^2 b\, dy = b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$$

$$I_{XG} = b\left[\frac{d^3}{24} + \frac{d^3}{24}\right] = \frac{bd^3}{12} \quad \text{(or)} \quad I_{GG} = \frac{bd^3}{12}$$

From parallel axis theorem

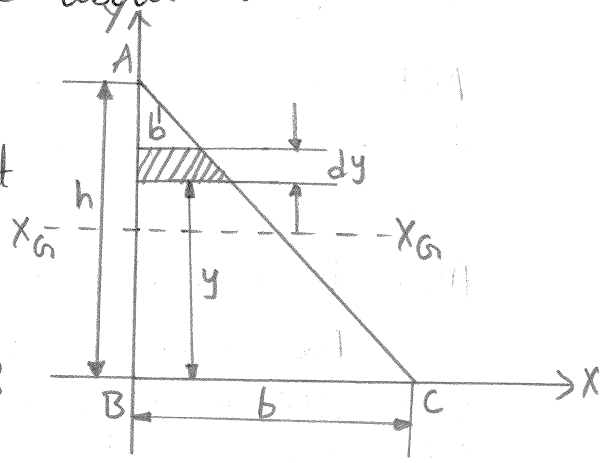
$$I_{xx} = I_{GG} + A y_c^2$$

$$I_{xx} = \frac{bd^3}{12} + (bd)\left(d/2\right)^2$$

$$I_{xx} = \frac{bd^3}{12} + \frac{bd^3}{4} = \frac{3bd^3 + bd^3}{12} = \frac{4bd^3}{12} = \frac{bd^3}{3}$$

$$I_{YG} = \frac{db^3}{12} \quad ; \quad I_{yy} = \frac{db^3}{3}$$

Moment of Inertia of a triangle about the base:



Moment of Inertia of a triangle with base width b and height h is to be determined about the base BC.

Consider an elemental strip at a distance y from base BC. Let dy be the thickness of strip & dA its area.

Width of the strip, $b' = \frac{b}{h}(h-y)$

From similar triangles,

$$\frac{b'}{b} = \frac{h-y}{h}$$

Area of the elemental strip, $dA = b'dy$

Moment of inertia of the strip about BC

$$= y^2 dA = y^2 b' dy = y^2 \cdot \frac{b}{h}(h-y)\,dy$$

Moment of inertia of the triangle about BC

$$I_{BC} = \int_0^h y^2 \frac{b}{h}(h-y)\,dy$$

$$= \int_0^h b y^2 dy - \int_0^h \frac{b}{h} y^3 dy$$

$$= b\left[\frac{y^3}{3}\right]_0^h - \frac{b}{h}\left[\frac{y^4}{4}\right]_0^h$$

$$I_{BC} = \frac{bh^3}{3} - \frac{bh^4}{4h} = \frac{bh^3}{3} - \frac{bh^3}{4} = \frac{bh^3}{12} = I_{xx}$$

From parallel axis theorem

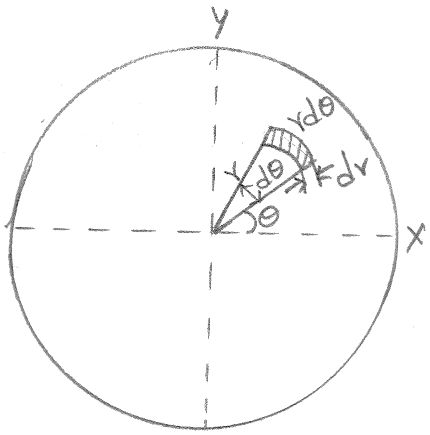
$$I_{xx} = I_{xG} + A y_c^2$$

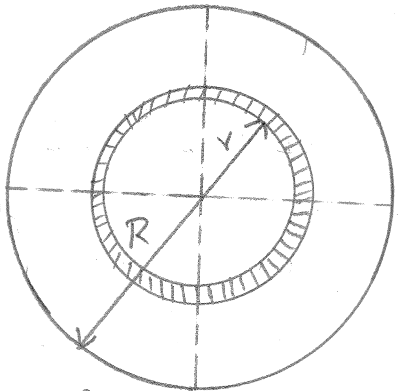
$$\frac{bh^3}{12} = I_{xG} + \left(\frac{1}{2}bh\right)\left(\frac{h}{3}\right)^2 \Rightarrow \frac{bh^3}{12} - \frac{bh^3}{18} = I_{xG}$$

$$\Rightarrow I_{xG} = \frac{18bh^3 - 12bh^3}{216} = \frac{bh^3}{36}$$

$$I_{yy} = \frac{hb^3}{12} \quad ; \quad I_{yG} = \frac{hb^3}{36}$$

(4) Moment of inertia of circle about diametral axis:





Moment of inertia of a circle of radius R is required about it's diametral axis as shown in figure.

Consider an element of sides $r d\theta$ & dr as shown in fig.

Moment of inertia of the circle about x-x

$$I_{xx} = \int_0^R \int_0^{2\pi} r^3 \sin^2\theta \, d\theta \, dr$$

$I_{xx} = \int y^2 dA$

$dA = r\,dr\,d\theta$

$y = r\sin\theta$.

$$= \int_0^R \int_0^{2\pi} r^3 \left(\frac{1-\cos 2\theta}{2}\right) d\theta \, dr$$

$$= \int_0^R \frac{r^3}{2}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{2\pi} dr$$

$$= \left[\frac{r^4}{2*4}\right]_0^R (2\pi - 0 + 0 - 0) = \frac{2\pi R^4}{8}$$

$$I_{xx} = \frac{\pi R^4}{4}$$

d - diameter of circle (or) D

$R = d/2$ or $D/2$.

$$I_{xx} = \frac{\pi}{4}\left(d/2\right)^4 = \frac{\pi d^4}{64}$$

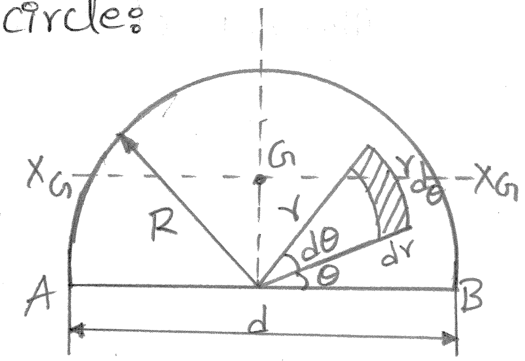
$$dA = 2\pi r\,dr$$

$$I_{zz} = \int_0^R r^2 dA = \int_0^R r^2 \, 2\pi r\,dr = \int_0^R 2\pi r^3 dr$$

$$I_{zz} = 2\pi\left[\frac{r^4}{4}\right]_0^R = \frac{2\pi R^4}{4} = \frac{\pi R^4}{2}$$

$$I_{xx} = I_{yy} = \frac{\pi d^4}{64}$$

Moment of inertia of a semi circle:



① About diametral axis

Moment of inertia of a semi circle about the diametral axis AB.

$$I_{AB} = \frac{1}{2} * \frac{\pi d^4}{64} = \frac{\pi d^4}{128}$$

② About centroidal axis $X_G - X_G$.

The distance of centroidal axis y_c from diametral axis

$$y_c = \frac{4R}{3\pi} = \frac{2d}{3\pi}$$

$$\text{Area, } A = \frac{1}{2} * \frac{\pi d^4}{4} = \frac{\pi d^4}{8}$$

From parallel axis theorem

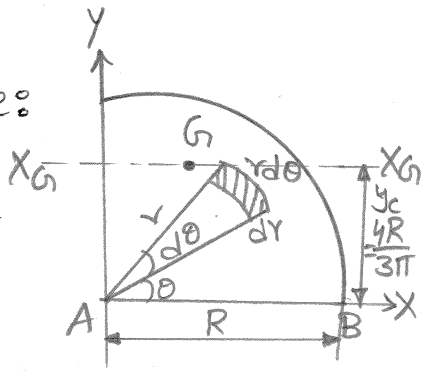
$$I_{AB} = I_{XG} + A y_c^2$$

$$\frac{\pi d^4}{128} = I_{XG} + \frac{\pi d^2}{8}\left(\frac{2d}{3\pi}\right)^2$$

$$I_{XG} = \frac{\pi d^4}{128} - \frac{\pi d^2}{8} * \frac{4d^2}{9\pi^2} = \frac{\pi d^4}{128} - \frac{d^4}{18\pi}$$

$$I_{XG} = 0.0068598\, d^4 = 0.11 R^4$$

Moment of inertia of a Quarter circle:



① About the base

Moment of inertia of a quarter circle about base AB

$$I_{AB} = \frac{1}{4} * \frac{\pi d^4}{64} = \frac{\pi d^4}{256} = \frac{\pi R^4}{16}$$

② About centroidal axis $X_G - X_G$

The distance of centroidal axis y_c from the base

$$y_c = \frac{4R}{3\pi} = \frac{2d}{3\pi} \qquad \text{Area, } A = \frac{1}{4} * \frac{\pi d^2}{4} = \frac{\pi d^2}{16}.$$

From parallel axis theorem,

$$I_{AB} = I_{XG} + A.\, y_c^2$$

$$\frac{\pi d^4}{256} = I_{XG} + \frac{\pi d^2}{16} * \left(\frac{2d}{3\pi}\right)^2$$

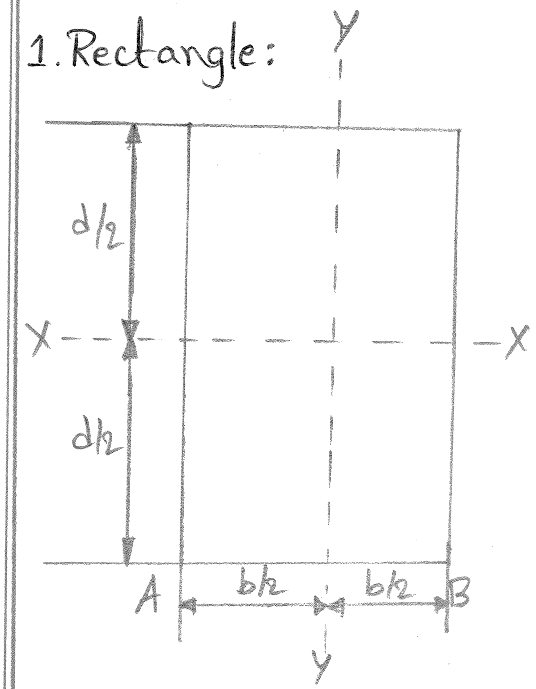
$$I_{XG} = \frac{\pi d^4}{256} - \frac{d^4}{36\pi} = 0.00343\, d^4$$

$$I_{XG} = 0.055 R^4$$

(5)

Moment of inertia of standard sections:

1. Rectangle:



a) Centroidal axis X–X

$$I_{xx} = \frac{bd^3}{12}$$

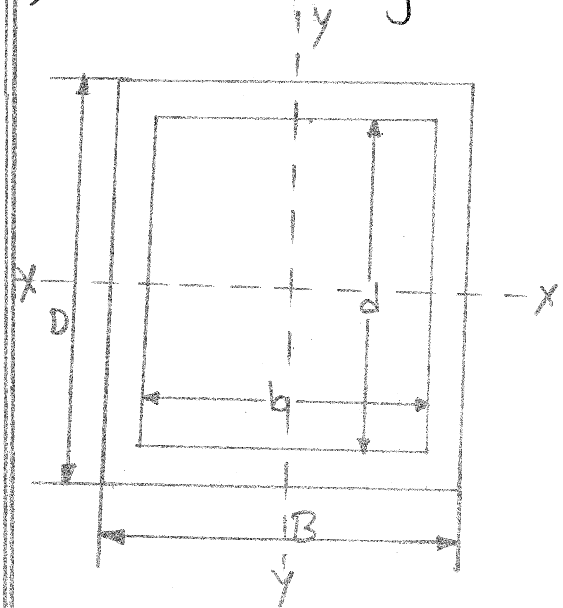
b) Centroidal axis Y–Y

$$I_{yy} = \frac{db^3}{12}$$

c) A–B axis

$$I_{AB} = \frac{bd^3}{3}$$

2) Hollow Rectangle:



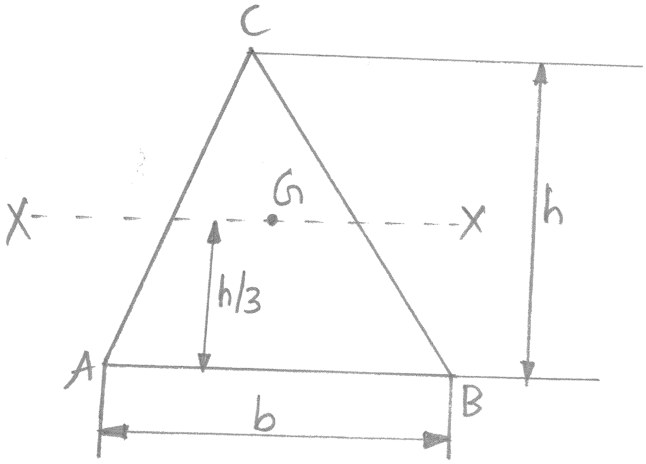
a) centroidal axis X–X

$$I_{xx} = \frac{BD^3 - bd^3}{12}$$

b) centroidal axis Y–Y

$$I_{yy} = \frac{DB^3 - db^3}{12}$$

3) Triangle:



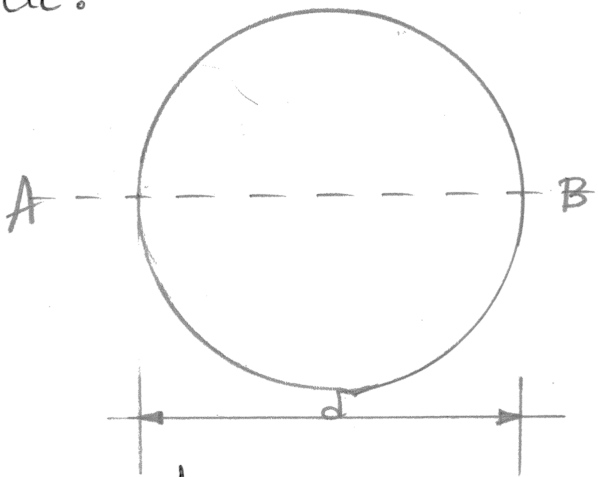
a) Centroidal axis X–X

$$I_{xx} = \frac{bh^3}{36}$$

b) Base AB

$$I_{AB} = \frac{bh^3}{12}$$

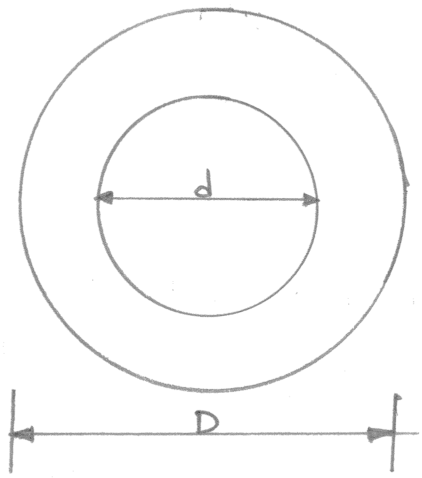
4) Circle:



Diametral axis

$$I = \frac{\pi d^4}{64} = \frac{\pi R^4}{4}$$

5) Hollow circle:



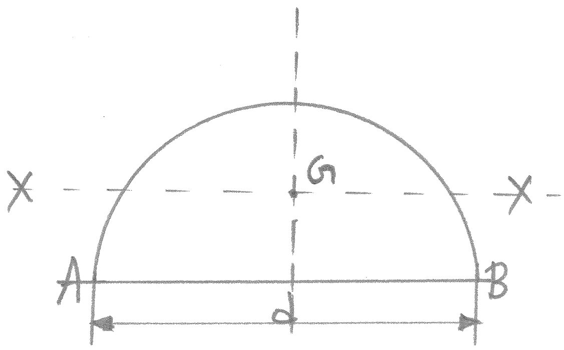
Diametral axis

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{4}(R^4 - r^4)$$

$R = D/2$, $r = d/2$.

6) Semi Circle:



a) A – B

$$I_{AB} = \frac{\pi d^4}{128}$$

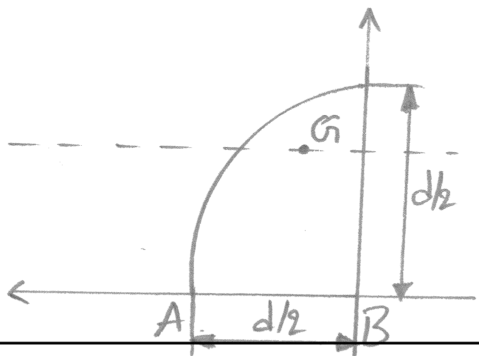
b) Centroidal axis

$$I_{xx} = 0.0068598 d^4$$

$$= 0.11 R^4$$

$R = d/2$.

7) Quarter of a circle:



a) A – B

$$I_{AB} = \frac{\pi d^4}{256}$$

b) Centroidal axis

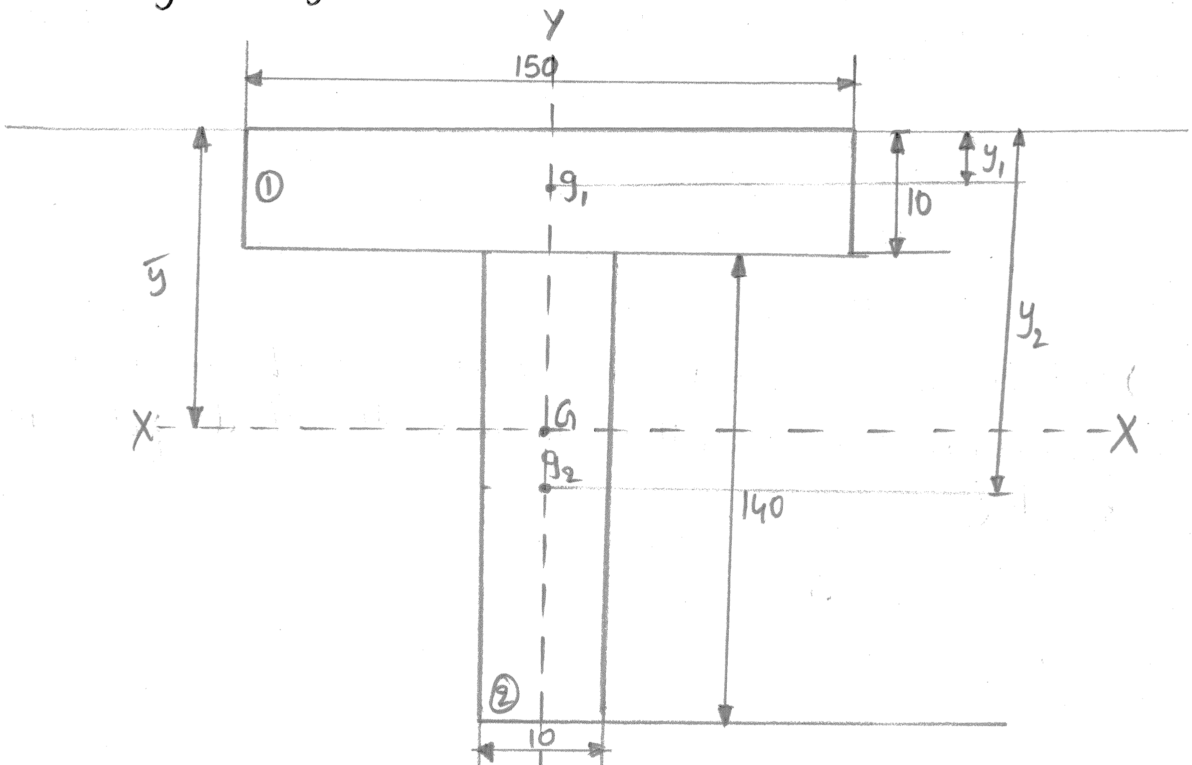
$$I_{xx} = 0.00343 d^4$$

$$= 0.055 R^4$$

$R = d/2$

6

1) Determine moment of inertia of section about an axis passing through centroid and parallel to top most fibre of section. Also determine moment of inertia about the axis of symmetry. Hence find radii of gyration.



Sol: $a_1 = 150 * 10 = 1500\,mm^2$ $\qquad y_1 = \frac{10}{2} = 5\,mm$

$a_2 = 140 * 10 = 1400\,mm^2$ $\qquad y_2 = 10 + \frac{140}{2} = 80\,mm$

Total area, $a = 1500 + 1400 = 2900\,mm^2$

Due to symmetry, centroid lies on symmetric axis $Y-Y$.
The distance of centroid from top most fibre is given by

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{1500 * 5 + 1400 * (70+10)}{2900} = 41.21\,mm$$

From parallel axis theorem,

$$I_{xx} = I_{xG} + A y_c^2 = \frac{bd^3}{12} + (bd)(d/2)^2$$

$$= \left[\frac{bd^3}{12} + (bd)(\bar{y} - y_1)^2\right]_1 + \left[\frac{bd^3}{12} + (bd)(y_2 - \bar{y})^2\right]_2$$

$$= \left[\frac{150 * 10^3}{12} + 1500\,(36.21)^2\right] + \left[\frac{10 * 140^3}{12} + 1400\,(38.79)^2\right]$$

$I_{xx} = 63,72,442.5\,mm^4$

$$I_{yy} = A_{yG} + A x_c^2 = \frac{db^3}{12} + \frac{db^3}{12} = \frac{10 * 150^3}{12} + \frac{140 * 10^3}{12}$$

$I_{yy} = 28,24,166.7\,mm^4$

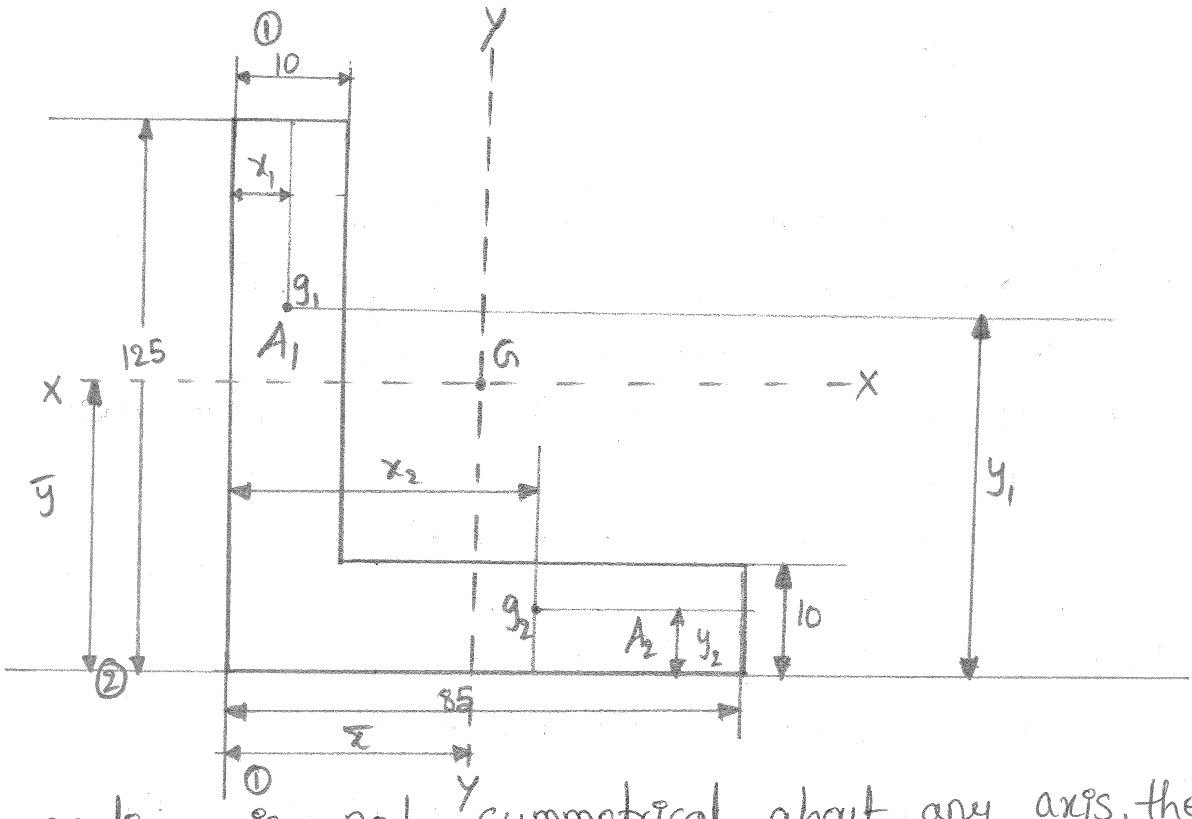
Hence, moment of inertia of section about an axis passing through centroid and parallel to top most fibre is 63,72,442.5 mm^4 & moment of inertia of section about axis of symmetry is 28,24,166.7 mm^4.

Radius of gyration, $K = \sqrt{\frac{I}{A}}$

$$K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{63,72,442.50}{2900}} = 46.88\,mm$$

$$K_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{28,24,166.7}{2900}} = 31.21\,mm.$$

2) Determine moment of inertia of L shown in fig about its centroidal axis parallel to legs. Also find polar moment of inertia.



Any The section is not symmetrical about any axis, therefore we have to find out the values of $\bar{x}$ & $\bar{y}$.

$a_1 = 125 * 10 = 1250\,mm^2$ $\quad a_2 = (85-10)*10 = 750\,mm^2$

Total area, $a = 1250 + 750 = 2000\,mm^2$

$x_1 = \frac{10}{2} = 5\,mm$ $\quad y_1 = \frac{125}{2} = 62.5\,mm$

$x_2 = 10 + \frac{85-10}{2} = 47.5\,mm$ $\quad y_2 = \frac{10}{2} = 5\,mm$

Choosen two reference axes ①-① & ②-②.

7 The distance of centroid from axis ① - ① is

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1+a_2} = 20.94\,mm$$

The distance of centroid from axis ② - ② is

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1+a_2} = 40.94\,mm$$

Centroid of A_1 along X-X is $g_1(15.94, 21.56)$
Centroid of A_2 along Y-Y is $g_2(26.56, 35.94)$

From parallel axis theorem

$$I_{xx} = I_{xG} + A y_c^2 \qquad I_{yy} = I_{yG} + A x_c^2$$

$$I_{xx} = \left[\frac{bd^3}{12} + (bd)(y_1-\bar{y})^2\right] + \left[\frac{bd^3}{12} + (bd)(\bar{y}-y_2)^2\right]$$

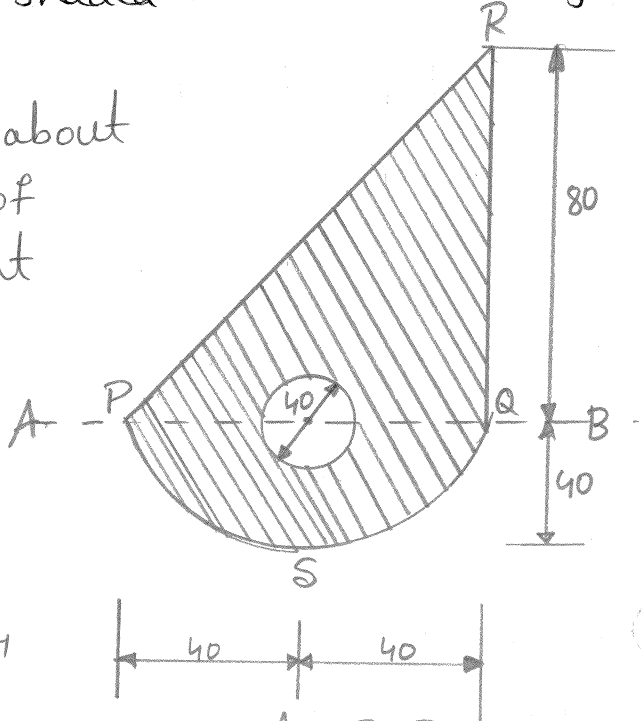
$$I_{xx} = \left[\frac{10*125^3}{12} + 1250(21.56)^2\right] + \left[\frac{75*10^3}{12} + 750(35.94)^2\right] = 3411298.9\,mm^4$$

$$I_{yy} = \left[\frac{db^3}{12} + (bd)(\bar{x}-x_1)^2\right] + \left[\frac{db^3}{12} + (bd)(x_2-\bar{x})^2\right]$$

$$I_{yy} = \left[\frac{125*10^3}{12} + 1250(15.94)^2\right] + \left[\frac{10*75^3}{12} + 750(26.56)^2\right] = 1208658.9\,mm^4$$

polar moment of inertia $I_{zz} = I_{xx} + I_{yy} = 4619957.8\,mm^4$

3) Find moment of inertia of shaded area shown in figure about the axis AB.



Moment of inertia of section about axis AB = [Moment of inertia of triangle PQR about AB + Moment of inertia of semicircle PSQ about AB − Moment of inertia of circle about AB]

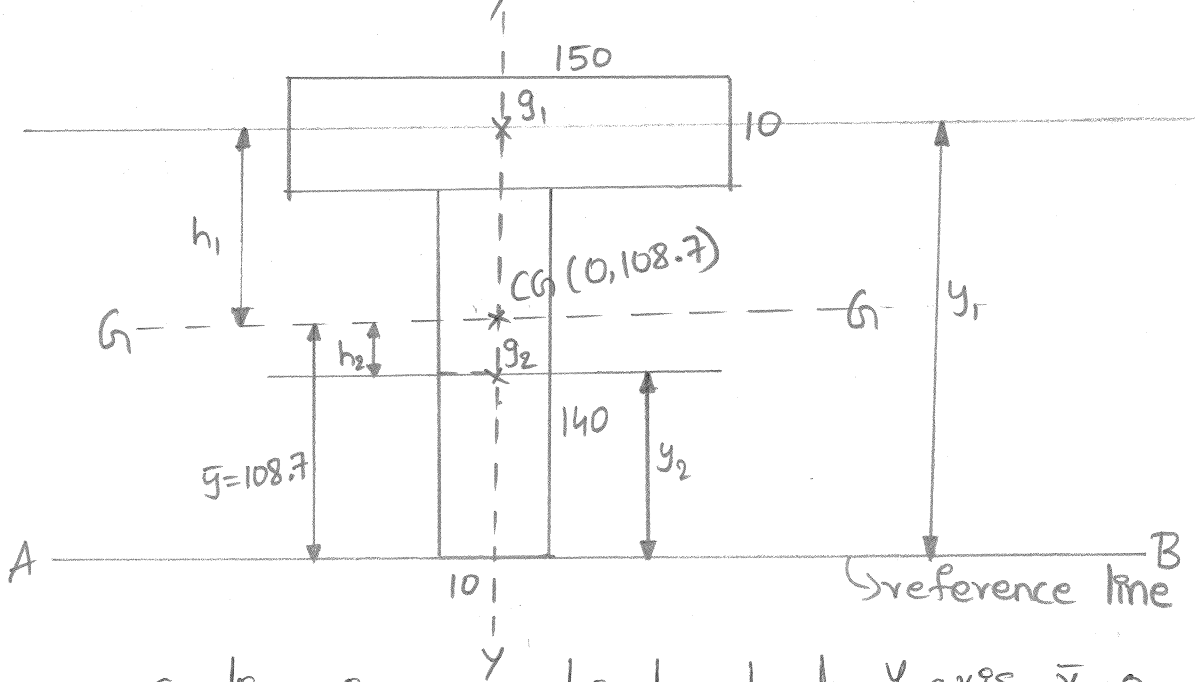
$$I_{AB} = \frac{bd^3}{12} + \frac{\pi d^4}{128} - \frac{\pi d^4}{64}$$

$$= \frac{80*80^3}{12} + \frac{\pi(80)^4}{128} - \frac{\pi(40)^4}{64}$$

$$= 4292979\,mm^4$$

Triangle PQR
Semicircle PSQ
Circle

4) Find moment of inertia of T-Section.



Sol The given section is symmetrical about Y-axis, $\bar{x}=0$.

Taking reference axis from bottom of given section

$$\bar{y}=\frac{a_1y_1+a_2y_2}{a_1+a_2}$$

$a_1=150*10=1500mm^2$ $\quad$ $y_1=140+\frac{10}{2}=145mm$

$a_2=140*10=1400mm^2$ $\quad$ $y_2=\frac{140}{2}=70mm$

$$\bar{y}=\frac{1500*145+1400*70}{1500+1400}=\frac{315500}{2900}=108.7mm$$

$C.G=(0,108.7)$

$I_{AB}=I_G+Ah^2$

$I_{AB}=(I_{G_1}+A_1h_1^2)+(I_{G_2}+A_2h_2^2)$

$$=\frac{b_1d_1^3}{12}+A_1(y_1-\bar{y})^2+\frac{b_2d_2^3}{12}+A_2(\bar{y}-y_2)^2$$

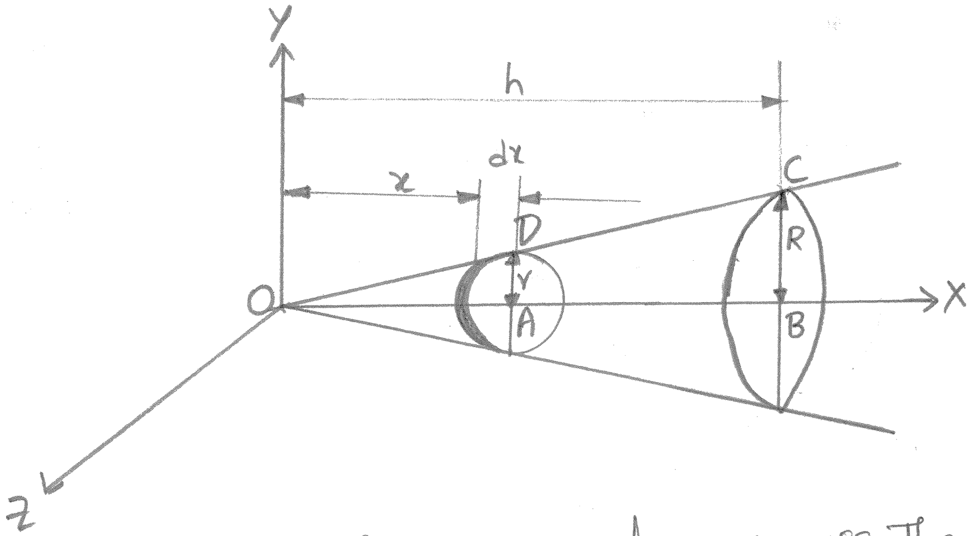
$$=\frac{150*10^3}{12}+1500(145-108.7)^2+\frac{10*140^3}{12}+1400(108.7-70)^2$$

$$=12500+1976535+2286666.6+2096766$$

$I_{AB}=6372467.6\ mm^4.$

5) A right circular cone of radius R at base and of height "h" is placed as shown in figure. Find the location of centroid of the volume of the cone.

(8)



In fig. the axis of cone is along x-axis. The centroid will be at the x-axis. Hence $\bar{y}=0$ & $\bar{z}=0$.

To find $\bar{x}$, consider a small volume dv. For this, take a thin circular plate at distance x from O. Let thickness of plate is dx as shown in fig. and radius of plate is r. The centroid of the plate is at a distance $\bar{x}$ from O.

Volume of thin plate, $dv = \pi r^2 dx$

From similar Δ^{les} OBC, OAD

$$\frac{R}{r} = \frac{h}{x} \Rightarrow r = \frac{R * x}{h}$$

$$dv = \pi \left(\frac{Rx}{h}\right)^2 dx = \frac{\pi R^2}{h^2} x^2 dx$$

$$\int_0^h dv = \frac{\pi R^2}{h^2} \int_0^h x^2 dx = \frac{\pi R^2}{h^2}\left[\frac{x^3}{3}\right]_0^h = \frac{\pi R^2}{h^2}\left[\frac{h^3}{3}\right] = \frac{1}{3}\pi R^2 h$$

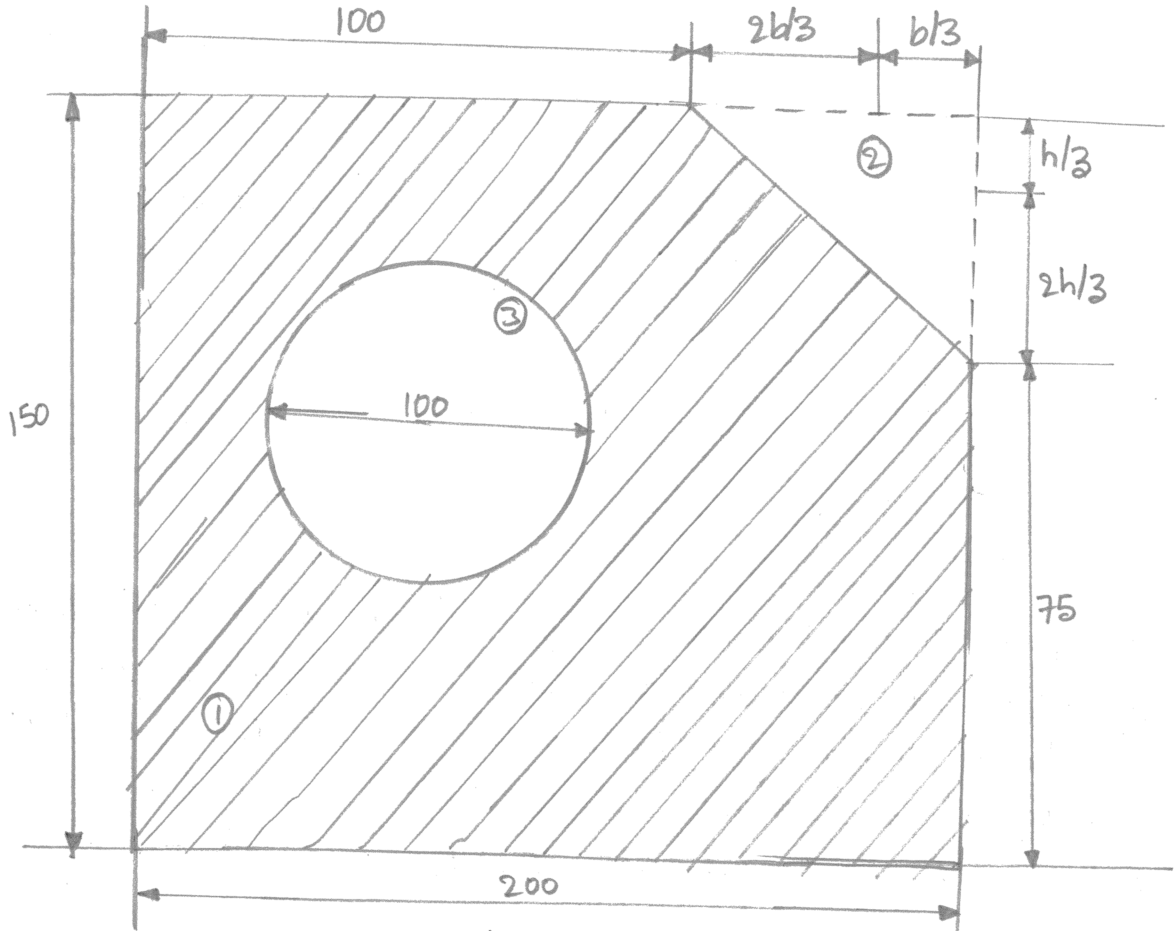
$$V * \bar{x} = \int x \, dv$$

$$V * \bar{x} = \int_0^h x \frac{\pi R^2}{h^2} x^2 dx = \frac{\pi R^2}{h^2}\left[\frac{x^4}{4}\right]_0^h = \frac{\pi R^2 h^2}{4}$$

$$\bar{x} = \frac{\pi R^2 h^2}{4} * \frac{3h^2}{\pi R^2 h^3} = \frac{3h}{4}$$

$$\therefore \bar{x} = \frac{3h}{4}.$$

6) With respect to the coordinate axis x & y. Locate the centroid of the shaded area shown in figure.



Sol:

If x_c & y_c are coordinates of the center of circle, centroid also must have coordinates x_c & y_c as per the condition laid down in the problem.

The shaded area may be considered as a rectangle of size 200mm x 150mm minus a triangle of sides 100mm x 75mm and a circle of diameter 100 mm.

$a_1 = b*h = 200*150 = 30000 mm^2$ $\quad a_2 = \frac{1}{2}bh = \frac{1}{2}*100*75 = 3750 mm^2$

$a_3 = \frac{\pi}{4}(100)^2 = \frac{\pi}{4}d^2 = 7853.98 mm^2$

Total area, $a = a_1 - a_2 - a_3 = 30000 - 3750 - 7853.98 = 18396 mm^2$

$\therefore \quad a = 18396 mm^2$

$x_1 = \frac{b}{2} = \frac{200}{2} = 100 mm$ $\quad x_2 = 100 + \frac{2b}{3} = 100 + \frac{2*100}{3} = 166.66 mm$

$x_3 = x_c$

$y_1 = \frac{150}{2} = 75 mm \left[\frac{h}{2}\right]$ $\quad y_2 = 75 + \frac{2h}{3} = 75 + \frac{2*75}{3} = 125 mm$ $\quad y_3 = y_c$

$\bar{x} = x_c = \frac{a_1 x_1 - a_2 x_2 - a_3 x_3}{a_1 - a_2 - a_3}$

9

$$x_c = \frac{30000*100 - 3750*166.66 - 7853.98*x_c}{30000 - 3750 - 7853.98}$$

$$(18396)x_c + (7853.98)x_c = 2375025$$

$$x_c = 90.47\text{ mm}$$

$$\bar{y} = y_c = \frac{a_1y_1 - a_2y_2 - a_3y_3}{a_1 - a_2 - a_3}$$

$$= \frac{30000*75 - 3750*\overset{125}{\cancel{166.66}} - 7853.98*y_c}{30000 - 3750 - 7853.98}$$

$$y_c = \frac{1781250 - 7853.98y_c}{18396}$$

$$(18396)y_c + (7853.98)y_c = 1781250$$

$$y_c = 67.85\text{ mm}$$

Center of circle should be located at (90.47, 67.85) so that this point will be centroid of the remaining shaded area shown in figure.

Kinematics is the branch of dynamics dealing with motion of particles / bodies, without referring to the forces acting.

Rectilinear Motion

A particle is said to be in rectilinear motion, if the path traced by it is a straight line. Many kinematic problems can be solved just by using the definition of displacement, velocity and acceleration.

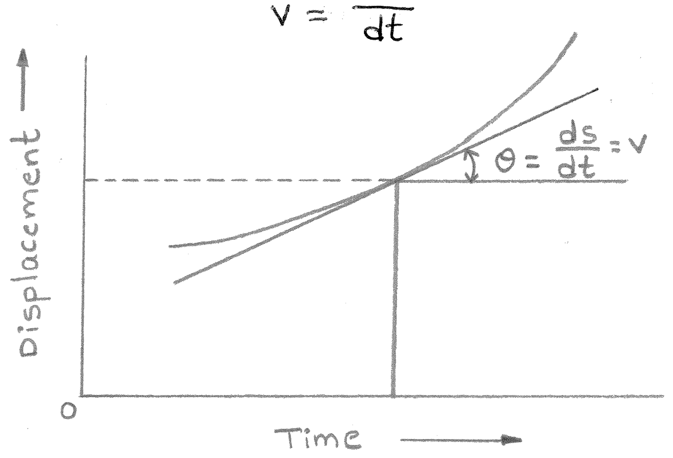
Motion Curves

Motion curves are the graphical representation of the displacement, velocity and acceleration with time.

Displacement – Time Curve (s–t curve)

Displacement – Time curve is a curve with time as abscissa and displacement as ordinate. At any instant of time t, velocity v is given by

$$v = \frac{ds}{dt}$$



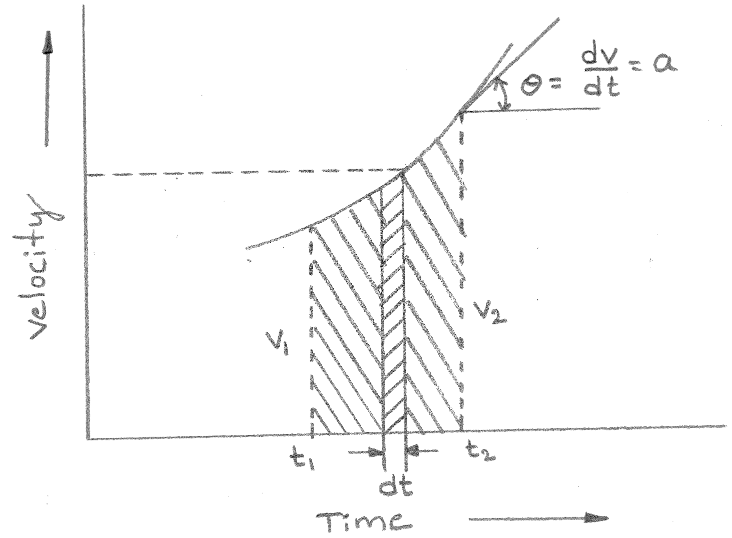
If a body is having non-uniform motion, its displacement at various time interval may be observed and s–t curve plotted. Velocity at any time may be found from the slope of s–t curve.

Velocity – Time curve (v–t curve)

In velocity – Time curve diagram, the abscissa represents time and ordinate represents the velocity of the motion. Such a curve is shown in Fig.

Acceleration is given by the slope of the v-t curve

$$a = \frac{dv}{dt} = \theta$$



Thus, acceleration at any time is the slope of v-t curve at the time, as shown in Fig.

$$\text{Now,} \quad \frac{ds}{dt} = v$$

$$ds = v\,dt$$

$$s = \int v\,dt$$

Referring to Fig. vdt is the elemental area under the curve at time t in the interval dt. Hence the shaded area under the curve between t_1 and t_2 shown in Fig. represents displacement s of the moving body in the time interval between t_1 and t_2. Thus in v-t curve;

1. slope of the curve represents acceleration
2. Area under the curve represents displacement

Acceleration - Time Curve (a-t curve)

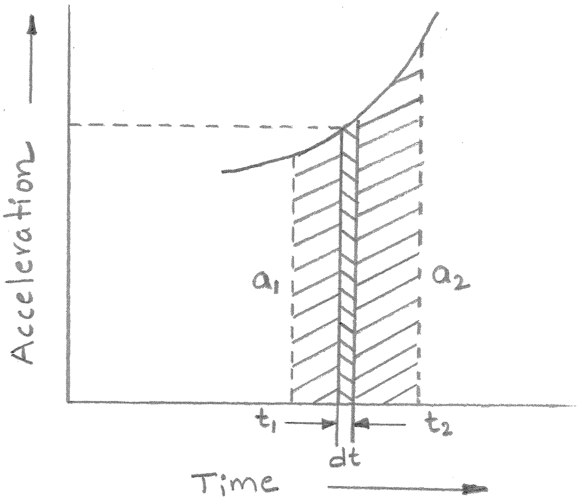
If a body is moving with varying acceleration, its motion can be studied more conveniently by drawing a curve with time as abscissa and acceleration as ordinate. Such a curve is called acceleration - time curve.

Now, $\frac{dv}{dt} = a$

$$dv = a\,dt$$

$$v = \int a\,dt$$

Hence the area under the curve represents velocity.



Motion with constant velocity

Consider the motion of a body moving with uniform velocity v. Now,

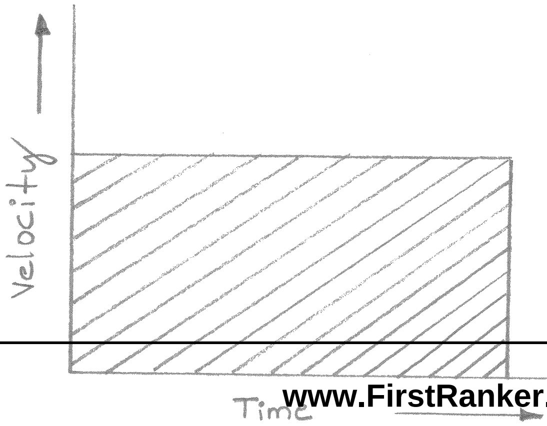
$$\frac{ds}{dt} = v$$

$$s = \int v\,dt \qquad \text{since } v \text{ is constant}$$

$$s = vt$$

v-t curve for such a motion is shown in Fig. It can be easily seen that the distance travelled s, from starting point in time t is given by the shaded area, which is a rectangle

$$s = vt$$



Motion with constant Acceleration

Consider the motion of a body with uniform acceleration

Let u – initial velocity

v – final velocity

t – time taken for change of velocity from u to v

Acceleration is defined as the rate of change of velocity. Since it is uniform, we can write

$$a = \frac{v-u}{t} \quad (or) \quad v = u + at \longrightarrow (1)$$

Displacement s is given by

$$s = \text{average velocity} * \text{time}$$

$$s = \frac{u+v}{2} * t \longrightarrow (2)$$

Substituting the value of v from Eqn (1) into Eqn (2),

$$s = \frac{u+u+at}{2} * t$$

$$s = ut + \frac{1}{2}at^2 \longrightarrow (3)$$

From Eqn (1) $\quad t = \frac{v-u}{a}$

Substituting it into Eqn (2)

$$s = \frac{u+v}{2} * \frac{v-u}{a} = \frac{v^2-u^2}{2a}$$

$$v^2 - u^2 = 2as \longrightarrow (4)$$

Thus equations of motion of a body moving with constant acceleration are

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 - u^2 = 2as$$

③ Acceleration due to Gravity

The acceleration due to gravity is constant for all practical purposes when we treat the motion of the bodies near earth's surface. Its value is found to be $9.81\ m/s^2$ and is always directed towards centre of earth, i.e. vertically downwards. Hence, if vertically downward motion of a body is considered, the value of acceleration is $9.81\ m/s^2$ and if vertically upward motion is considered, then

$$a = -g = -9.81\ m/s^2$$

Rate of change of velocity with respect to time is called acceleration.

$$a = \frac{dv}{dt}$$

The acceleration may be +ve or −ve. The positive acceleration is simply referred as acceleration and the negative acceleration is called as retardation or deceleration.

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right)$$

$$a = \frac{d^2s}{dt^2}$$

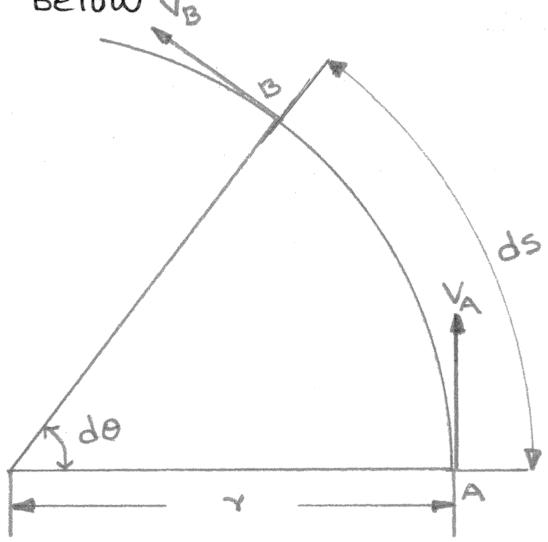
Motion with Varying Acceleration

A vehicle is normally not accelerated uniformly. Initially it starts with zero acceleration, then the rate of acceleration is increased and when the desired speed is nearing, the rate of acceleration is reduced. By the time desired speed is picked up acceleration is brought to zero. Thus there are situations with varying acceleration. If the variation of acceleration or velocity or displacement with respect to time is known, such problems can be solved using the differential equations

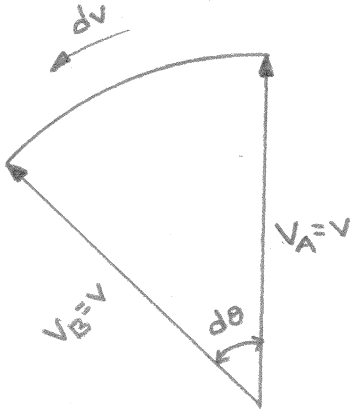
$$v = \frac{ds}{dt}, \quad a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v\frac{dv}{ds}$$

Curvilinear Motion

Whenever there is change in the directions of Roads/ railways, curves are provided to get smooth change over of the direction. Though vehicle moves with uniform velocity, there will be acceleration towards centre of the circle. The expression for this acceleration during curvilinear motion can be derived as explained below



(a)



(b)

Let a body move with uniform velocity along a curved path of radius r as shown in Fig(a). In time interval dt, let it move from A to B. Since the body has uniform velocity, tangential velocity V_A at A is equal to tangential velocity V_B at B in magnitude, say $V_A = V_B = V$. However there is change in direction equal to $d\theta$. Drawing the vector diagram of velocities as shown in Fig(b). we find that there is change in velocity dv at right angles to V_A (i.e., in radial inward direction) of magnitude dv.

$$\text{Now} \quad dv = v\,d\theta$$

$$dv = v\frac{ds}{r} = \frac{v}{r}ds$$

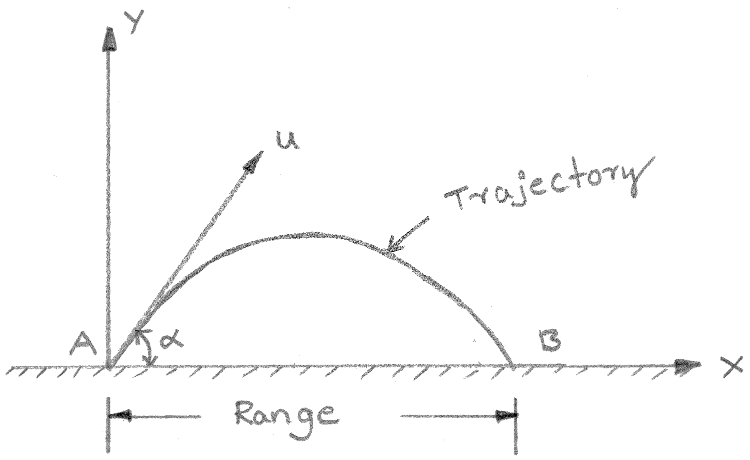
$$\therefore \text{Acceleration} = \frac{dv}{dt} = \frac{v}{r}\frac{ds}{dt} = \frac{v}{r}\cdot v$$

$$a = \frac{v^2}{r}$$

Thus when a body moves with uniform velocity v along a curved path of radius r, there is a radial inward acceleration of magnitude $\frac{v^2}{r}$

Rectangular Components of Curvilinear Motion in a plane

We observe that a particle moves along a curved path if it is freely projected in the air in the direction other than vertical. These freely projected particles which are having the combined effect of a vertical and a horizontal component. The vertical component of the motion is subjected to gravitational acceleration/retardation while horizontal component remains constant, if air resistance is neglected. The motion of a projectile can be analysed independently in vertical and horizontal directions and then combined suitably to get the total effect.



Velocity of projection

The velocity with which the particle is projected is called as velocity of projection (u m/s)

Angle of Projection

The angle between the direction of projection and horizontal direction is called an angle of projection (α).

Trajectory

The path traced by the projectile is called as its trajectory.

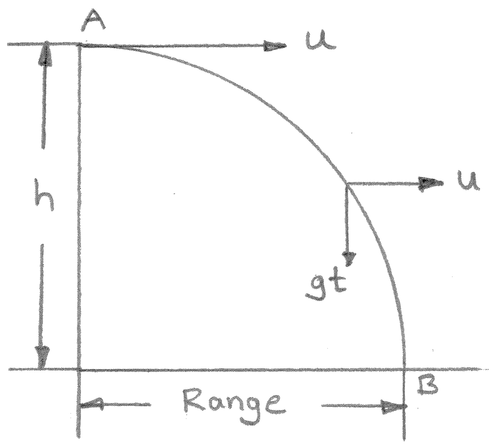
Horizontal Range or Range of the projectile

The horizontal distance through which the projectile travels in its flight is called the horizontal range.

5 Time of Flight

The time interval during which the projectile is in motion is called the time of flight.

Motion of body projected horizontally



consider a particle thrown horizontally from point A with a velocity u m/sec as shown in Fig. At any instant the particle is subjected to :

1. Horizontal motion with constant velocity u m/s
2. Vertical motion with initial velocity zero and moving with acceleration due to gravity g.

Let h be the height of A from the ground

Considering vertical motion,

$$s = ut + \frac{1}{2}at^2$$

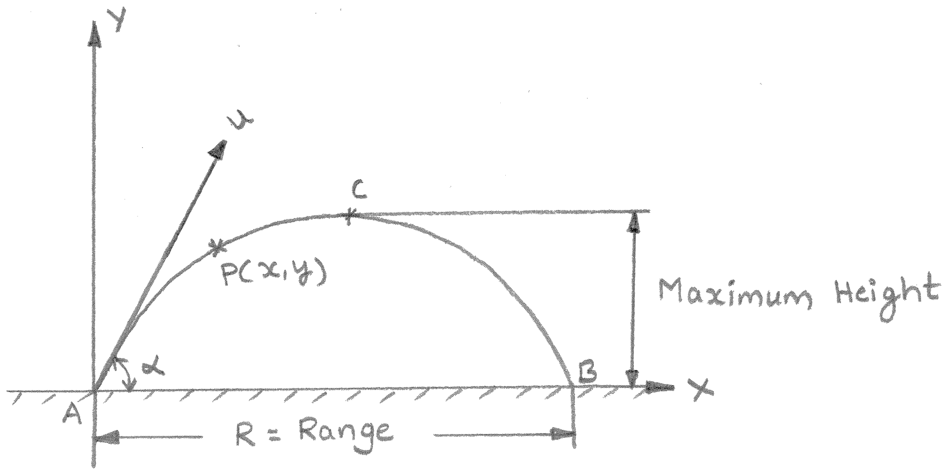
$$h = 0 * t + \frac{1}{2}gt^2$$

$$h = \frac{1}{2}gt^2$$

This expression gives the time of flight. During this period, the particle moves horizontally with uniform velocity, u m/s.

Range = ut

Inclined Projection on level Ground



Consider the motion of a projectile, projected from point A with velocity of projection u and angle of projection α, as shown in Fig. Let the ground be a horizontal surface.

The particle has motion in vertical as well as horizontal directions.

Vertical Motion

Initial velocity $= u \sin\alpha$, upward

Gravitational acceleration $= g = 9.81\ m/s^2$, downward

$$a = -g = -9.81\ m/s^2$$

Hence initially the particle moves upward with velocity $u\sin\alpha$ and retardation $9.81\ m/s^2$.

Velocity becomes zero after some time (at C) and then the particle starts moving downward with gravitational acceleration.

Horizontal Motion

Horizontal component of velocity $= u\cos\alpha$

Neglecting air friction, we can say that the projectile is having uniform velocity $u\cos\alpha$ during its entire flight.

⑥ Equation of the Trajectory

Let $P(x, y)$ represent the position of projectile after t seconds. Considering the vertical motion,

$$s = ut + \frac{1}{2}at^2$$

$$y = (u\sin\alpha)t - \frac{1}{2}gt^2 \longrightarrow ①$$

considering horizontal motion,

$$s = vt$$

$$x = (u\cos\alpha)t \longrightarrow ②$$

$$t = \frac{x}{u\cos\alpha}$$

substituting this value in Eqn ①, we get

$$y = (u\sin\alpha)\frac{x}{u\cos\alpha} - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$$

$$y = x\tan\alpha - \frac{1}{2}\frac{gx^2}{u^2\cos^2\alpha} \longrightarrow ③$$

But $\frac{1}{\cos^2\alpha} = \sec^2\alpha = (1+\tan^2\alpha)$

Hence Eqn ③ reduces to the form

$$y = x\tan\alpha - \frac{1}{2}\frac{x^2}{u^2}(1+\tan^2\alpha) \longrightarrow ④$$

This is an equation of a parabola. Hence the equation of the trajectory is a parabola.

Maximum Height

When the particle reaches maximum height, the vertical component of the velocity will be zero. Considering vertical motion,

initial velocity $= u\sin\alpha$

final velocity $= 0$

acceleration $= -g$

Using the equation of linear motion $v^2 - u^2 = 2as$

$$0 - (u\sin\alpha)^2 = -2gh$$

$$h = \frac{u^2\sin^2\alpha}{2g} \longrightarrow (5)$$

Time required to reach Maximum Height

Using first equation of motion ($v = u + at$), when projectile reaches maximum height,

$$0 = u\sin\alpha - gt$$

$$u\sin\alpha = gt$$

$$g = \frac{u\sin\alpha}{t}$$

$$t = \frac{u\sin\alpha}{g} \longrightarrow (6)$$

Time of flight of projectile

Motion of the projectile in vertical ~~motion~~ direction is given by Eqn (1) as

$$y = (u\sin\alpha)t - \frac{1}{2}gt^2$$

At the end of flight, $y = 0$

$$0 = (u\sin\alpha)t - \frac{1}{2}gt^2 = t\left[u\sin\alpha - \frac{1}{2}gt\right]$$

$$t = 0$$

or $$u\sin\alpha - \frac{1}{2}gt = 0$$

$$t = \frac{2u\sin\alpha}{g}$$

$t = 0$, gives initial position '0' of the projectile. Hence time of flight is given by

$$t = \frac{2u\sin\alpha}{g} \longrightarrow (7)$$

Horizontal Range

During the time of the flight, projectile moves in horizontal direction with uniform velocity $u\cos\alpha$. Hence the horizontal distance traced by the projectile in this time is given by

$$R = (u\cos\alpha)t = (u\cos\alpha)\frac{2u\sin\alpha}{g}$$

$$R = \frac{u^2\sin 2\alpha}{g} \longrightarrow (8)$$

Maximum Range

In the Eqn (8) $\sin 2\alpha$ can have maximum value of 1.

Hence the maximum range $= \frac{u^2}{g} \longrightarrow (9)$

and the angle of projection for this is when

$$\sin 2\alpha = 1$$

$$2\alpha = 90^\circ$$

$$\alpha = 45^\circ \longrightarrow (10)$$

Angle of projection for the required range

In Eqn (8), we have the expression for range as

$$R = \frac{u^2\sin 2\alpha}{g}$$

$$\sin 2\alpha = \frac{gR}{u^2}$$

since $\sin 2\alpha = \sin(180 - 2\alpha)$

There are two values of α which give the same result.

$2\alpha_1 = 2\alpha$ i.e., $\alpha_1 = \alpha$

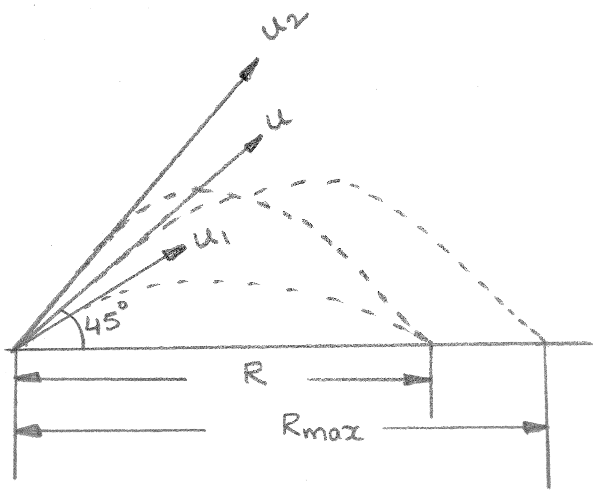
and another is $2\alpha_2 = 180 - 2\alpha$ i.e., $\alpha_2 = 90^\circ - \alpha$

$$\therefore \alpha_1 + \alpha_2 = 90^\circ$$

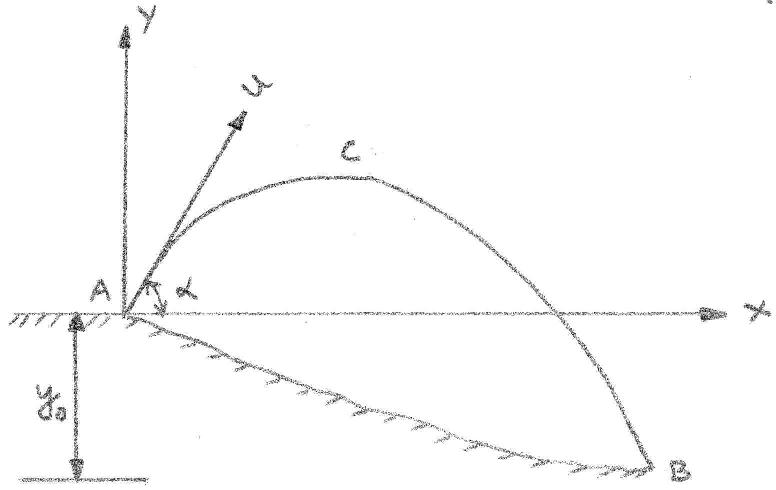
Hence if $\alpha_1 = 45^\circ + \theta$

$\alpha_2 = 45^\circ - \theta$

Thus there are two angles of projection for the required range as shown in Fig.



Inclined projection with point of projection and point of strike at different levels



Equations $t = \frac{u \sin\alpha}{g}$ and $t = \frac{2u \sin\alpha}{g}$ are to be used only when the point of projection and the point of striking the ground are at the same level. Now let us consider the case, when the point of projection is at a height y_0 above the point of strike as shown in Fig.

Any one of the following three methods can be used to analyse such cases :

Method 1

From equation of motion in vertical ~~motion~~ direction

$$y = (u\sin\alpha)\,t - \frac{1}{2}gt^2$$

By putting $y = -y_0$ in the above equation, the time required to reach B (time of flight) is obtained.

$$-y_0 = (u\sin\alpha)\,t - \frac{1}{2}gt^2$$

Once the time of flight is known, horizontal range can be found from the relation

$$R = (u\cos\alpha)\,t$$

Maximum height above the point of projection and time required to reach it can be found as usual from Equations

$$h = \frac{u^2\sin^2\alpha}{2g} \quad \text{and} \quad t = \frac{u\sin\alpha}{g}$$

Method 2

Total time of flight can be split into two parts t_1, time required to reach maximum height (point C) and t_2, time required to descend from point C to B.

Considering the vertical motion and noting that vertical component of velocity is zero at C, we get,

$$0 = u\sin\alpha - gt_1$$

$$t_1 = \frac{u\sin\alpha}{g}$$

Height reached in this time interval

$$h = (u\sin\alpha)\,t_1 - \frac{1}{2}gt_1^2$$

$$= \frac{u^2\sin^2\alpha}{g} - \frac{1}{2}\,\frac{gu^2\sin^2\alpha}{g^2}$$

$$h = \frac{u^2\sin^2\alpha}{2g}$$

While descending, the vertical distance to be covered $d = h + y_0$. Considering downward motion

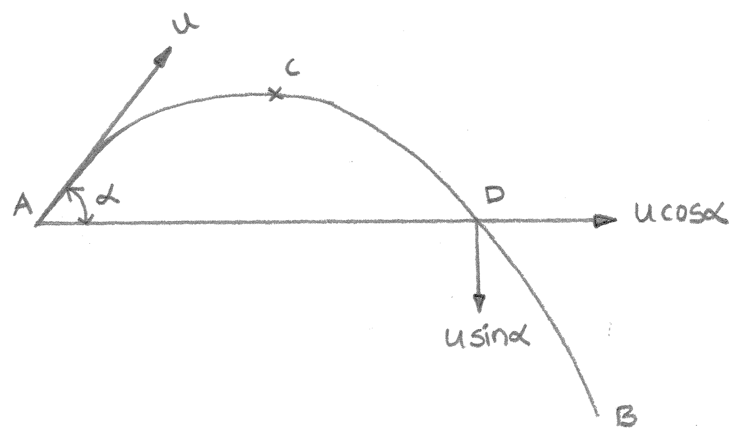
$$h + y_0 = 0 \times t_2 + \frac{1}{2} g t_2^2$$

Hence t_2 can be found. Therefore total time of flight $t = t_1 + t_2$

Horizontal range is given by

$$R = (u\cos\alpha)\, t$$

Method 3



The motion can be split into two parts; AD & DB where D is the point on the trajectory at the same level as A, the point of projection.

For the portion AD, the equations can be used.

$$h = \frac{u^2 \sin^2\alpha}{2g}, \quad t = \frac{u\sin\alpha}{g}, \quad t = \frac{2u\sin\alpha}{g}, \quad R = \frac{u^2 \sin 2\alpha}{g}$$

For the portion DB:

The vertical component of the velocity of the projectile at point D will be equal to the vertical component of the velocity at A, but downwards, since A and D are at the same elevation. The horizontal component of the velocity remains equal to $u\cos\alpha$ throughout.

Hence for vertical motion in the portion DB

initial velocity = $u\sin\alpha$, downward

gravitational acceleration = $g = 9.81\ m/s^2$, downward

$$\therefore \quad y_0 = (u\sin\alpha)\,t_2 + \frac{1}{2} g t_2^2$$

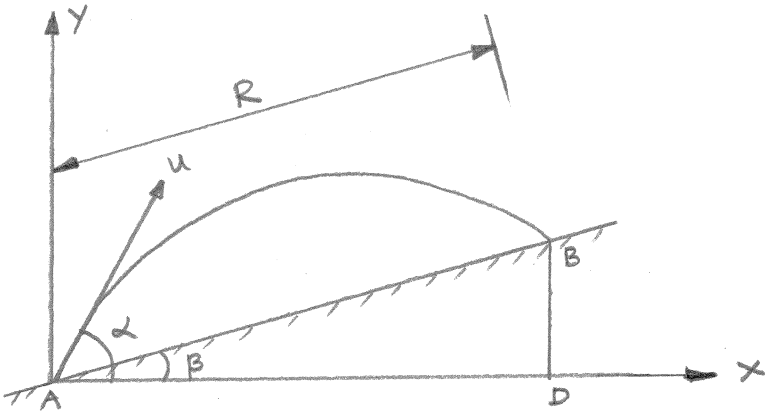
where, t_2 is the time taken to travel portion DB.

Horizontal distance moved during this time

$$= (u\cos\alpha)\,t_2$$

Total result may be obtained by combining the motion in the two portions.

Projection on Inclined Plane



Let AB be a plane inclined at angle β to the horizontal as shown in the Fig. A projectile is fired up the plane from the point A with initial velocity u m/s and an angle α. Now, the range on the inclined plane AB and the time of flight are to be determined.

Let the inclined range AB be denoted by R. AD be the corresponding horizontal range

$$\therefore \quad AD = R\cos\beta \quad \text{and} \quad DB = R\sin\beta$$

The equation of trajectory of the projectile is given by

$$y = x\tan\alpha - \frac{1}{2}\frac{gx^2}{u^2\cos^2\alpha}$$

Applying this equation to point B, we get

$$R\sin\beta = R\cos\beta\tan\alpha - \frac{1}{2}\frac{gR^2\cos^2\beta}{u^2\cos^2\alpha}$$

$$R\left(\frac{1}{2}\frac{g\cos^2\beta}{u^2\cos^2\alpha}\right) = \cos\beta\tan\alpha - \sin\beta$$

$$R = \frac{2u^2\cos^2\alpha}{g\cos^2\beta}(\cos\beta\tan\alpha - \sin\beta)$$

$$R = \frac{2u^2\cos^2\alpha}{g\cos^2\beta}\;\frac{\cos\beta\sin\alpha - \sin\beta\cos\alpha}{\cos\alpha}$$

$$= \frac{2u^2\cos\alpha}{g\cos^2\beta}\left[\sin(\alpha-\beta)\right] \longrightarrow (1a)$$

$$R = \frac{u^2}{g\cos^2\beta}\left[\sin(2\alpha-\beta) - \sin\beta\right] \longrightarrow (1b)$$

since $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

Time of flight

Let t be the time of flight. The horizontal distance covered during the flight

$$\Rightarrow AD = (u\cos\alpha)\,t$$

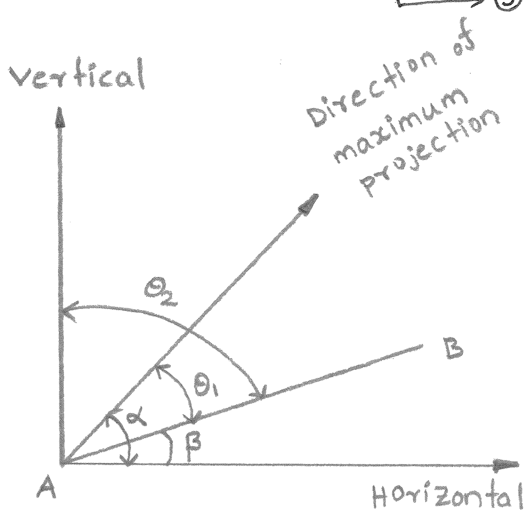
$$t = \frac{AD}{u\cos\alpha} = \frac{R\cos\beta}{u\cos\alpha}$$

$$= \frac{2u^2\cos\alpha}{g\cos^2\beta} \times \frac{\sin(\alpha-\beta)}{u\cos\alpha}\cos\beta = \frac{2u\sin(\alpha-\beta)}{g\cos\beta} \longrightarrow (2)$$

For the given values of u and β, the range is maximum when:

$$\sin(2\alpha-\beta) = 1 \quad \text{i.e.} \quad 2\alpha - \beta = \frac{\pi}{2} \quad \text{or} \quad \alpha = \frac{\pi}{4} + \frac{\beta}{2} \longrightarrow (3)$$

Refering to Fig.



$$\theta_1 = \frac{\pi}{4} + \frac{\beta}{2} - \beta = \frac{\pi}{4} - \frac{\beta}{2}$$

$$\theta_2 = \frac{\pi}{2} - \beta$$

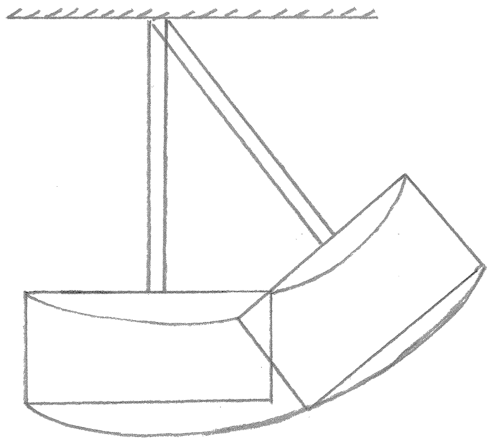
Thus $\theta_2 = 2\theta_1$

i.e., the range on the given plane is maximum, when the angle of projection bisects the angle between the vertical and inclined plane.

If the projection is down the plane, the Eqn (1) to (3) can be used, but the value of β should be taken negative.

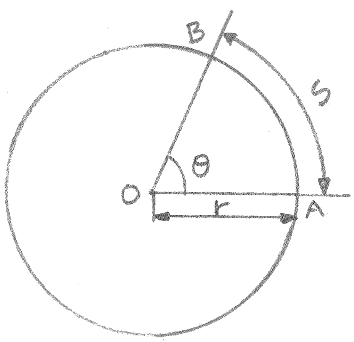
Rotation about Fixed Axis

Rigid bodies motion having rectilinear or curvilinear translation. During translation a straight line drawn on the rigid body remains parallel to its original position at any time. The motion of body rotating about a fixed axis. In such motion every particles of rigid body move along concentric circles as shown in Fig.



Angular Motion

Translation has been treated as linear motion and now we treat rotation as angular motion. The displacement of the body in rotation is measured in terms of angular displacement θ, where θ is in radians.



When a particle in a body moves from position A to B, the displacement is θ as shown in Fig. This displacement is a vector quantity since it has magnitude as well as direction. The direction is a rotation either clockwise or counter clockwise.

The rate of change of angular displacement with time is called angular velocity and is denoted by ω. Thus

$$\omega = \frac{d\theta}{dt}$$

The rate of change of angular velocity with time is called angular ~~velocity~~ acceleration and is denoted by α. Thus

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

The angular acceleration may be expressed in another useful form. Now

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt}$$

Thus $$\alpha = \omega \frac{d\omega}{d\theta}$$

Relationship between Angular motion and Linear motion

When the particle moves from A to B, the distance travelled by it is 's'. If 'r' is the distance of the particle from the centre of rotation then

$$s = r\theta$$

The tangential velocity of the particle is called as linear velocity and is denoted by v. Then

$$v = \frac{ds}{dt} = r\frac{d\theta}{dt}$$

The linear acceleration of the particle in tangential direction a_t is given by

$$a_t = \frac{dv}{dt} = r\frac{d^2\theta}{dt^2}$$

While treating the curvilinear motion, it has been shown that, if v is the tangential velocity, then there is radial acceleration $\frac{v^2}{r}$.

Denoting radial acceleration by a_n, we get

$$a_n = \frac{v^2}{r} = r\omega^2$$

Uniform Angular Velocity

If the angular velocity is uniform, the angular distance moved in t seconds by a body having angular velocity ω rad/s is given by

$$\theta = \omega t \text{ radians}$$

Note that the uniform angular velocity is characterized by zero angular acceleration.

Many a times the angular velocity is given in terms of number of revolution per minute (rpm). Since there are 2π radians in one revolution and 60 seconds in one minute, the angular acceleration ω is given by

N is in rpm

$$\omega = \frac{2\pi N}{60} \text{ rad/sec}$$

Since angular velocity is uniform, the time taken for one revolution T is given by

$$2\pi = \omega T \quad (\text{or}) \quad T = \frac{2\pi}{\omega}$$

Uniformly Accelerated Rotation

Let us consider the uniformly accelerated motion with angular acceleration α. From the definition

$$\frac{d\omega}{dt} = \alpha$$

C_1 is constant of integration

$$\omega = \alpha t + C_1$$

If the initial velocity is ω_0 then

$$\omega_0 = \alpha * 0 + C_1 \quad (\text{or}) \quad C_1 = \omega_0$$

$$\omega = \omega_0 + \alpha t \longrightarrow (1)$$

Again from the definition of angular velocity

$$\frac{d\theta}{dt} = \omega = \omega_0 + \alpha t$$

C_2 is constant of integration

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 + C_2$$

Measuring the angular displacement from the instant of reckoning the interval of time, we get

$$0 = 0 + 0 + C_2$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \longrightarrow (2)$$

From Eqn $\alpha = \omega \dfrac{d\omega}{d\theta}$ (or) $\alpha d\theta = \omega d\omega$

Integrating, we get

$$\alpha\theta = \frac{\omega^2}{2} + C_3$$

C_3 is constant of integration

Initially $\theta = 0$ and $\omega = \omega_0$

Hence we get,

$$\alpha \times 0 = \frac{\omega_0^2}{2} + C_3 \quad (\text{or}) \quad C_3 = -\frac{\omega_0^2}{2}$$

$$\alpha\theta = \frac{\omega^2}{2} - \frac{\omega_0^2}{2} \quad (\text{or}) \quad \omega^2 - \omega_0^2 = 2\alpha\theta \longrightarrow (3)$$

Thus for uniformly accelerated angular motion

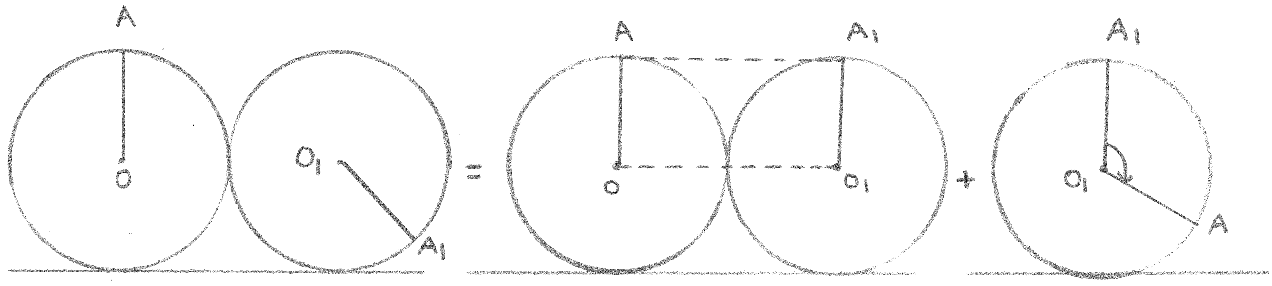
$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 - \omega_0^2 = 2\alpha\theta$$

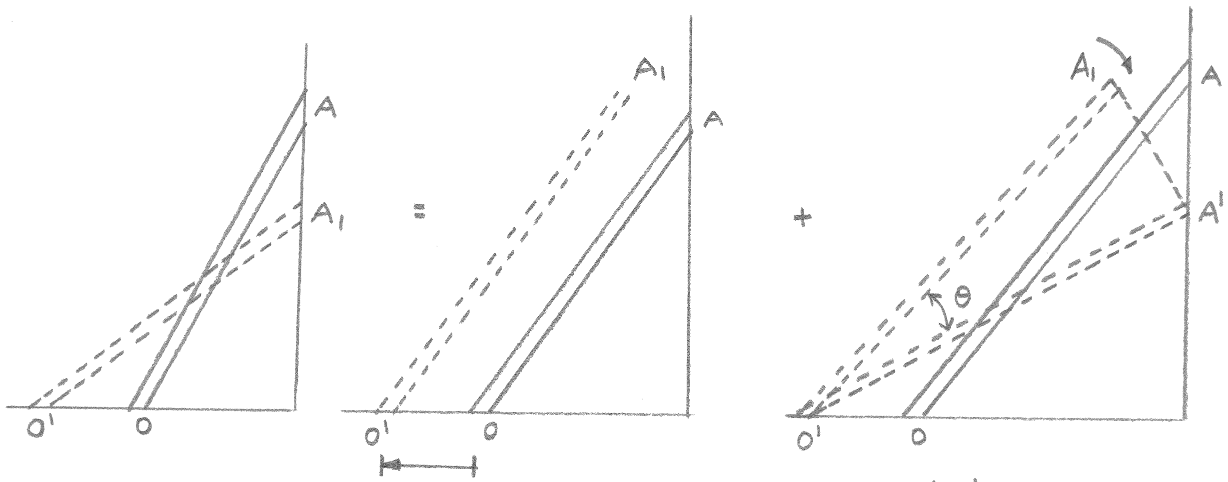
Kinematics of Plane Motion

A body is said to have general plane motion if it possesses translation and rotation simultaneously. Common examples of such motion are a wheel rolling on straight line and a rod sliding at one end against wall and the floor at other end.



Plane Motion = Translation + Rotation

For the analysis of general plane motion, it is convenient to split the motion into translation and pure rotation cases. The analysis for these two cases is carried out separately and then combined to get the final motion.



Plane Motion = Translation + Rotation

Kinetics

Kinetics is the branch of dynamics which deals with motion of bodies and considering the forces acting on them. We will consider

1. Bodies in rectilinear translation
2. Bodies in curvilinear translation
3. Bodies rotating about fixed axis
4. Bodies in plane motion

Bodies in Rectilinear Translation

From Newton's second law, we know

Force = mass * acceleration

Instead of a single force, if a system of forces acts on a particle Newton's second law reduces to resultant force is equal to the product of mass and acceleration in the direction of the resultant force. Mathematically

$$R = \Sigma F = ma \longrightarrow ①$$

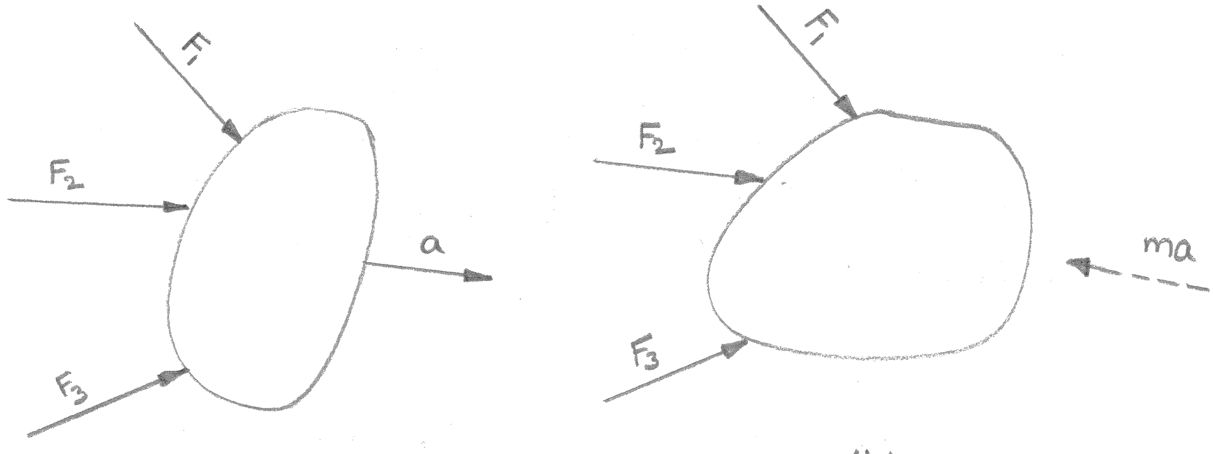
Hence many times Newton's second law is stated as a particle acted upon by an unbalanced system of forces has an acceleration directly proportional and in line with the resultant force.

French mathematician Jean le Rond d'Alembert (1743) proved that the Newton's second law of motion is applicable not only to the motion of a particle but also to the motion of a body and looked at equation ①, from different angle. The equation R=ma may be written as

$$R - ma = 0 \longrightarrow ②$$

The term '$-ma$' may be looked as a force of magnitude $m*a$ applied in the opposite direction of motion and is termed as the inertia force or reverse effective force.

D'Alembert looks at equation (1) as an equation of equilibrium and states that the system of forces acting on a body in motion is in dynamic equilibrium with the inertia force of the body. This is known as D'Alembert's principle



(a) (b)

Let the body shown in Fig (a). be subjected to a system of forces causing the body to move with an acceleration 'a' in the direction of the resultant. Then apply a force equal to ma in the reversed direction of acceleration as shown in Fig (a). Now according to D'Alembert's principle, the equations of equilibrium $\Sigma F_x = 0$ and $\Sigma F_y = 0$ may be used for the system of forces shown in Fig (b).

The inertia force $-ma$ has a physical meaning. According to Newton's first law of motion, a body continues to be in the state of rest or of uniform motion in a straight line unless acted by an external force. That means every body has a tendency to continue in its state of rest or of uniform motion. This tendency is called inertia. Hence inertia force is the resistance offered by a body to the change in its state of rest or of uniform motion.

Many scientists are critical of D'Alembert's principle. Usually equilibrium equations are applied to a system of forces acting on a body. Inertia force is not acting on the moving body. Actually this is the force exerted by the moving body to resist the change in its state. Hence D'Alembert is criticized for messing-up

(14) the concept of equations of equilibrium.

However, many engineers prefer to use D'Alembert's principle, since just by applying a reverse effective force, the moving body can be treated as a body in equilibrium and can be analysed using equations of static equilibrium.

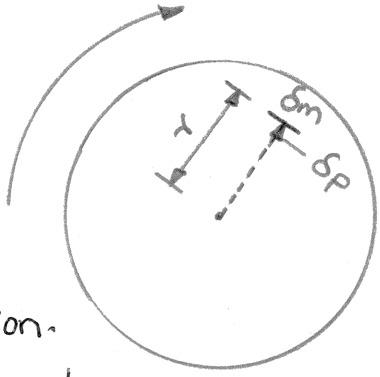
Bodies in Curvilinear Translation

When a body moves with uniform velocity along a curved path, there is a radial inward acceleration of magnitude $a = \frac{v^2}{r}$. If we apply D'Alembert principle to get equilibrium condition, an inertia force of magnitude $\frac{W}{g} a = \frac{W}{g} \frac{v^2}{r}$ must to be applied in radial outward direction. This force is called as centrifugal force.

(15)

Kinetics of bodies rotating about fixed axis

Consider the wheel shown in Fig. rotating about its axis in clockwise direction with an angular acceleration α. Let δm be mass of an element at a distance r from the axis of rotation. If δp is the resulting force on this element.



$$\delta p = \delta m * a \quad , \text{ where } a \text{ is tangential acceleration}$$

But $a = r\alpha$

$$\therefore \delta p = \delta m * r\alpha$$

Rotational moment δM_t due to this force δp is given by

$$\delta M_t = \delta p\, r = \delta m\, r^2 \alpha$$

$$\therefore M_t = \Sigma \delta M_t = \Sigma \delta m\, r^2 \alpha$$

$$= \alpha \Sigma \delta m\, r^2$$

$$M_t = I\alpha \longrightarrow (1)$$

Where I is mass moment of inertia of the body about its centroidal axis.

Note the similarity between the expressions $M_t = I\alpha$ and $F = ma$. Forces causes rectilinear motion while rotational moment causes angular motion.

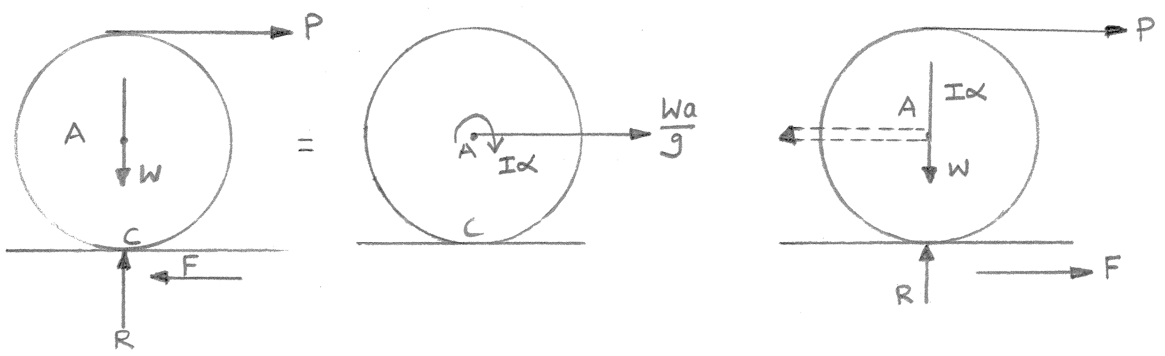
The force is equal to the product of mass and the linear acceleration where as rotational moment is the product of mass moment of inertia and the angular acceleration.

To solve the kinetic problems of bodies rotating about fixed axis according to D'Alemberts principle we apply reverse effective moment M_t and treat as if the body is in equilibrium.

Kinetics of Rolling Bodies

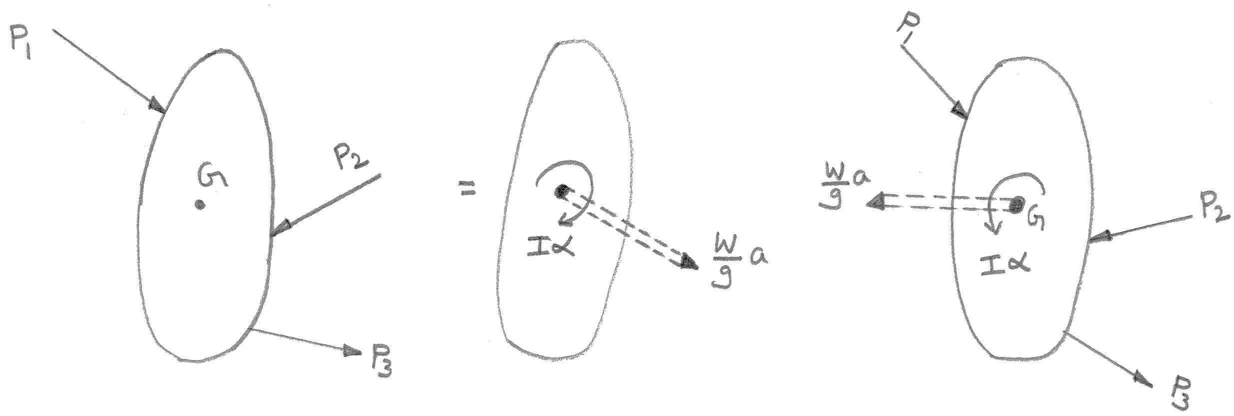
The plane motion of a rolling body can be split into linear motion and rotation about the geometric axis. Hence the system of forces acting on the body are equal to an effective force $\frac{W}{g}a$ through the geometric centre in the direction of the resultant forces (the direction of motion) and a rotational moment $I\alpha$ as shown in Fig (a). The reverse effective forces may be applied on the body along with the system of forces acting on it Fig (b). and then the system may be treated as in equilibrium. This is the D'Alembert's principle for rolling bodies.

Note: The relation $a_A = r\alpha$, holds good only for the bodies rolling without slipping. If slipping takes place, this relation is not applicable.



(16)

Kinetics of General Plane Motion



(a) (b) (c)

Consider the body shown in Fig (a). which has got plane motion. The plane motion of the body may be split into linear motion and rotation about its mass centre. Then the effective force on the body is a force $\frac{W}{g}a$ at mass centre and a moment $I\alpha$ as shown in Fig (b).

Hence by equating the component of forces acting on the body to the respective components of effective force, $\frac{W}{g}a$, and equating moment of the force about any point to that of effective moment about the same point, kinetic equations may be formed and solved. Another method is one can use D'Alembert's principle i.e., apply reverse effective force to a given force system and solve the equation of equilibrium. The system of force to be considered in this approach are shown in Fig (c).

Work-Energy Method

Work-Energy approach, is used to solve kinetic problems. This method is advantageous over D'Alembert's method when the problem involves velocities, rather than Acceleration.

By using work-Energy equation a number of kinetic problems are solved.

Work:

The work done by a force on a moving body is defined as the product of the force and the distance moved in the direction of the force.

workdone = Force * Distance

Units → N-m (or) Joule

kilo joules — kJ (kN-m)

milli joules — mJ (N-mm)

Energy:

Energy is defined as the capacity to do work.

There are many forms of energy like heat energy, mechanical energy, electrical energy & chemical energy.

Mechanical Energy < Potential Energy / Kinetic Energy

Potential energy is the capacity to do work due to the position of the body.

$$P.E = Wh$$

Kinetic energy is the capacity to do work due to motion of the body.

$$K.E = \frac{1}{2}mv^2$$

Unit of energy is same as that of work, since it is nothing but capacity to do work.

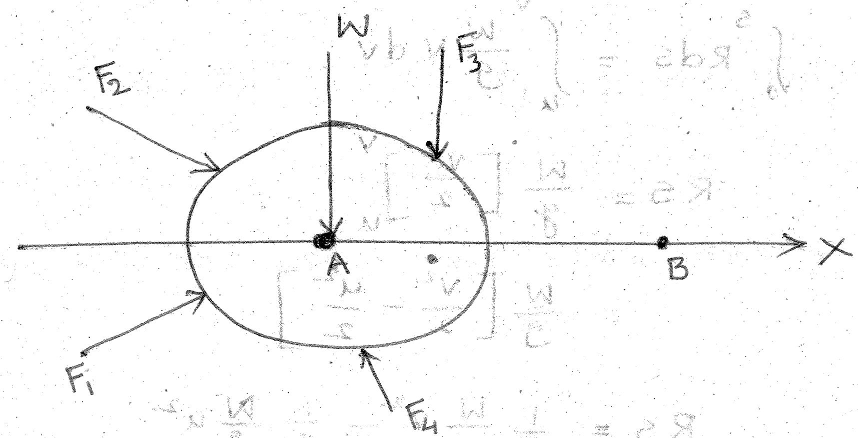
Power

Power is defined as time rate of doing work.

Unit of power is watt.

Horse power is the unit used in MKS & FPS

Work-Energy Equation For Translation



consider the body shown in Fig. subject to a system of forces F_1, F_2 --- and moving with an acceleration 'a' in x- direction.

Let its initial velocity at A be u and Final velocity when it moves distance $AB = s$ be V.

The resultant of system of the forces must be in x- direction.

$$R = \Sigma F_x$$

From Newton's second Law of motion

$$R = ma = \frac{W}{g} a$$

$R = ma$
(or)
$F = ma$

Multiplying both sides by elementary distance ds,

$$a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v\frac{dv}{ds}$$

$$v = \frac{ds}{dt}$$

$$R\,ds = \frac{W}{g} a\, ds$$

$$= \frac{W}{g} v \frac{dv}{ds} \cdot ds$$

$$R\,ds = \frac{W}{g} v\, dv$$

Integrating for the

from A to B:

$$\int_0^S R\,ds = \int_u^v \frac{W}{g} v\,dv$$

$$RS = \frac{W}{g}\left[\frac{v^2}{2}\right]_u^v$$

$$= \frac{W}{g}\left[\frac{v^2}{2} - \frac{u^2}{2}\right]$$

$$RS = \frac{1}{2}\frac{W}{g}v^2 - \frac{1}{2}\frac{W}{g}u^2$$

R·S – workdone by the forces acting on the body

$\frac{W}{2g}v^2$ – Final kinetic energy

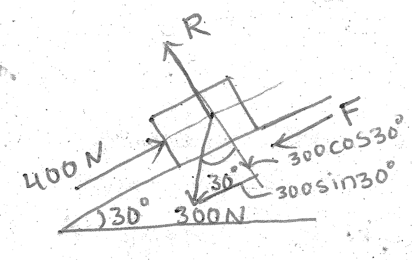
$\frac{W}{2g}u^2$ – Initial kinetic energy

$$\boxed{\text{Workdone} = \text{Final kinetic energy} - \text{Initial kinetic energy}}$$

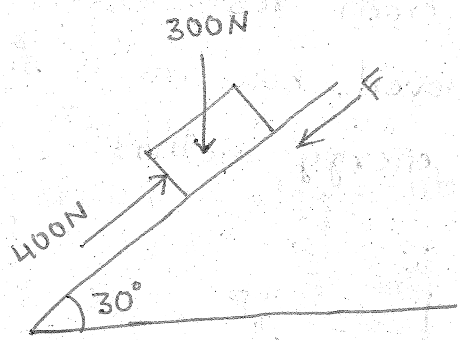
→ work – Energy Equation

"Work energy principle may be stated as the work done by a system of forces acting on a body during a displacement is equal to the change in kinetic energy of the body during the same displacement.

(P) A body weighing 300N is pushed up a 30° plane by a 400N force acting parallel to the plane. If the initial velocity of the body is 1.5 m/s and coefficient of kinetic friction is $\mu = 0.2$, what velocity will the body have after moving 6m ?



FBD



Σ Forces normal to plane = 0

$$R = 300\cos 30° = 259.8 \text{ N}$$

Frictional force, $F = \mu R$

$$= 0.2 * 259.8$$

$$F = 51.96 \text{ N}$$

Initial velocity, $u = 1.5$ m/s

Final velocity, $v = ?$

Displacement, $s = 6$ m

Equating the workdone by forces along the plane to change in kinetic energy

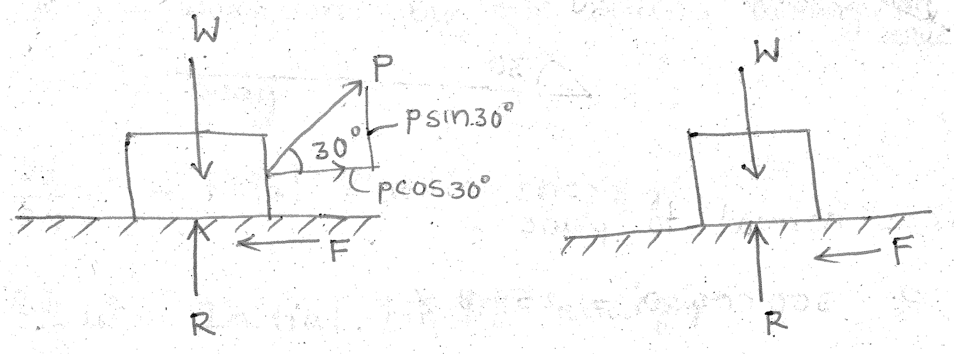
$$RS = \frac{1}{2}\frac{W}{g}v^2 - \frac{1}{2}\frac{W}{g}u^2 = \frac{1}{2}\frac{W}{g}(v^2 - u^2)$$

$$(400 - F - 300\sin 30)6 = \frac{1}{2} * \frac{300}{9.81}(v^2 - 1.5^2)$$

$$77.71 = v^2 - 2.25$$

$$v = 8.942 \text{ m/s}$$

Ⓟ A block weighing 2500 N rests on a level horizontal plane for which coefficient of friction is 0.20. This block is pulled by a force of 1000 N acting at an angle of 30° to the horizontal. Find the velocity of the block after it moves 30 m starting from rest. If the force of 1000 N is then removed, how much further will it move? Use work energy method.



(a) (b)

Free Body diagrams of the block for the two cases are shown in Fig. (a) & (b)

When pull P is acting

$$R = W - P\sin 30^\circ$$

$$= 2500 - 1000 \sin 30^\circ = 2000\ N$$

$$F = \mu R$$

$$= 0.2 * 2000 = 400\ N$$

Initial velocity, $u = 0$

Final velocity, $v = ?$

Displacement, $s = 30\ m$

Applying work energy equation for the horizontal motion

$$(P\cos 30 - F)\,s = \frac{W}{2g}(v^2 - u^2)$$

$$(1000*0.866 - 400)\,30 = \frac{2500}{2*9.81}(v^2 - 0)$$

$$v = 10.47 \text{ m/s}$$

Now, if the force 1000N is removed Fig (b), Let the distance moved be 's' before the body comes to rest.

Initial velocity = 10.47 m/s

Final velocity = 0

Applying work energy equation for the motion in horizontal direction,

$$-F*s = \frac{W}{2g}(v^2 - u^2)$$

$$-400*s = \frac{2500}{2*9.81}(0 - 10.47^2)$$

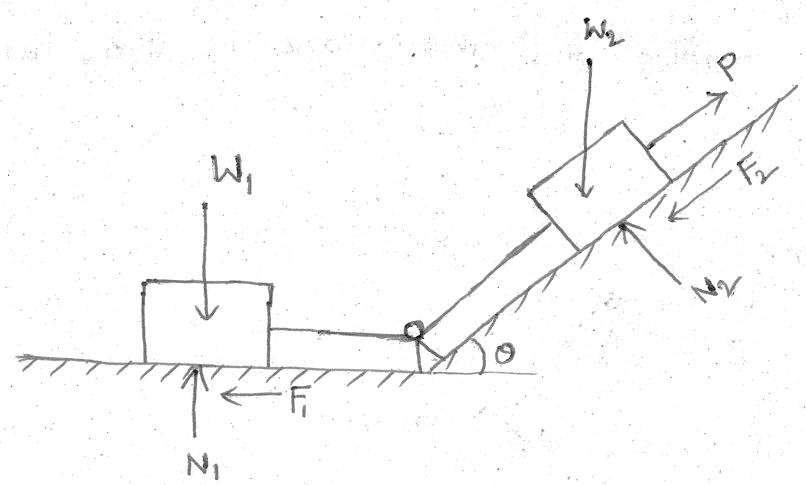
$$s = 27.96 \text{ m}$$

Motion of connected bodies

Work - Energy equation may be applied to the connected bodies also. There is no need to separate connected bodies and work out for forces in the connecting member. Note the various forces acting on connected bodies. Equate summation of work done by forces acting on bodies to the summation of change in kinetic energy of the bodies.

While writing down work done note that force components in the direction of motion are to be multiplied by the distance moved.

Consider the two connected bodies shown in Fig.



Under the pull P, both bodies move the same ~~direction~~ distance and with the same velocity. Hence, initial velocity, final velocity and displacement are the same for the two bodies.

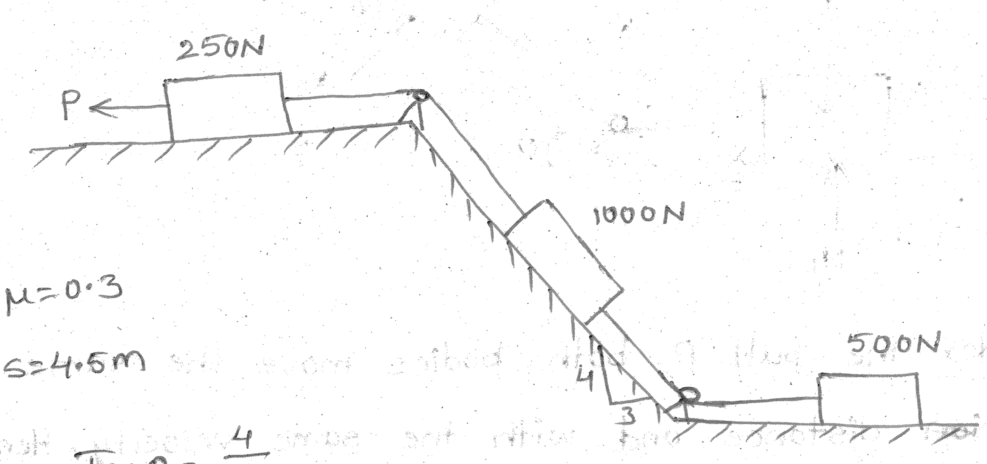
out of the three forces W_1, N_1 and F_1 acting on first body only the frictional force F_1 will do the work. Among the various forces acting on the second body, the applied force P, frictional force F_2 and the down the plane component of weight, $W_2 \sin\theta$ will do the work.

Work - Energy equation for that system

$$-F_1 s + (P - F_2 - W_2 \sin\theta) s = \frac{W_1}{2g}(v^2 - u^2) + \frac{W_2}{2g}(v^2 - u^2)$$

$$(-F_1 + P - F_2 - W_2 \sin\theta) s = \frac{W_1 + W_2}{2g}(v^2 - u^2)$$

(P) Determine the constant force P that will give the system of bodies shown in Fig. a velocity of 3 m/s after moving 4.5m from rest. Coefficient of friction between the blocks and the plane is 0.3. Pulleys are smooth.



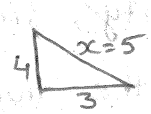
$\mu = 0.3$

$s = 4.5m$

$\tan\theta = \frac{4}{3}$

$\sin\theta = \frac{4}{5}$ $\quad u = 0$

$\cos\theta = \frac{3}{5}$ $\quad v = 3\ m/s$



$x = \sqrt{4^2 + 3^2} = 5$

Sol:-

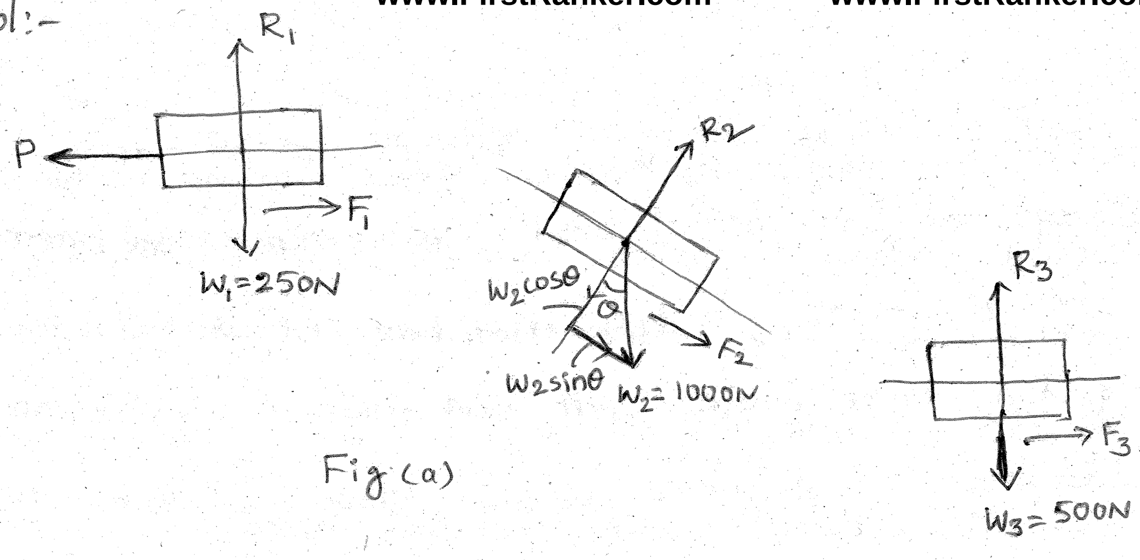


Fig (a)

The system of forces acting on connecting bodies is shown in Fig (a).

Work – Energy Equation

$$\left(-P + F_1 + F_2 + F_3 + W_2\sin\theta\right)s = \frac{W_1 + W_2 + W_3}{2g}\left(v^2 - u^2\right)$$

$F_1 = \mu R_1$
$= 0.3 * 250$
$= 75N$

$F_2 = \mu R_2$
$= 0.3 * W_2 \cos\theta$
$= 0.3 * 1000 * \frac{3}{5}$
$= 180N$

$F_3 = \mu R_3$
$= 0.3 * 500$
$= 150N$

$$\left(-P + 75 + 180 + 150 + 1000 * \sin\theta\right)4.5 = \frac{250 + 1000 + 500}{2 * 9.81}\left(3^2 - 0\right)$$

$$\left(-P + 1205\right)4.5 = 802.75$$

$$-P * 4.5 + 1205 * 4.5 = 802.75$$

$$-P * 4.5 + 4619.75 = 0$$

$$P = 1026.6\ N$$

$$\left(P - 75 - 180 - W_2\sin\theta\right)s = \frac{W_1 + W_2 + W_3}{2g}\left(v^2 - u^2\right)$$

$$\left(P - 75 - 180 - 1000 * 0.8 - 150\right)4.5 = \frac{250 + 1000 + 500}{2 * 9.81}\left(3^2 - 0\right)$$

$$P = 1383.39\ N$$

5

(P) In what distance will body A of Fig. attain a velocity of 3 m/sec. starting from rest?

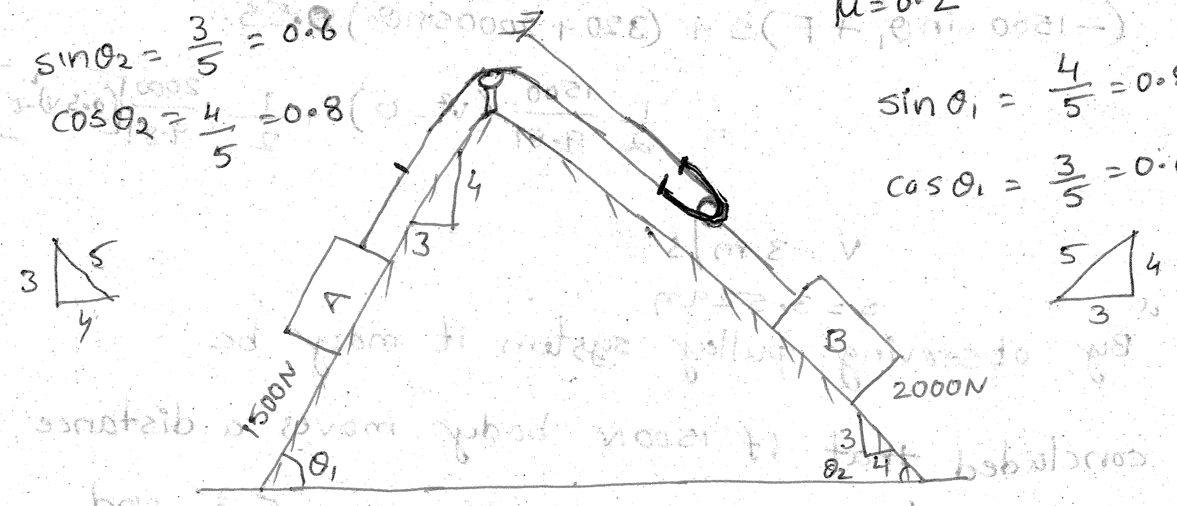
$\mu = 0.2$

$\sin\theta_2 = \frac{3}{5} = 0.6$

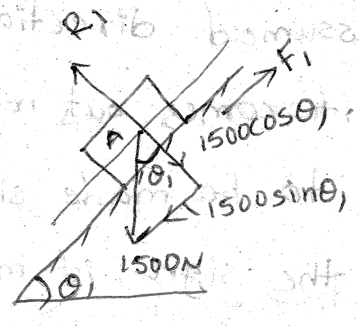
$\cos\theta_2 = \frac{4}{5} = 0.8$

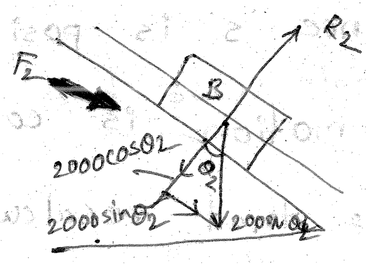
$\sin\theta_1 = \frac{4}{5} = 0.8$

$\cos\theta_1 = \frac{3}{5} = 0.6$



Sol: Assuming 2000N body moves up the plane and 1500N body moves down the plane.





(a)

$F_1 = \mu R_1$

$= 0.2 * 1500 * \cos\theta_1$

$= 0.2 * 1500 * 0.6$

$F_1 = 180\ N$

$F_2 = \mu R_2 = \mu\ 2000\cos\theta_2$

$= 0.2 * 2000 \cos\theta_2$

$= 0.2 * 2000 * 0.8$

$F_2 = 320\ N$

$\underbrace{F_1,\ 1500\sin\theta_1}_{W.D}$

$\underbrace{F_2,\ 2000\sin\theta_2}_{W.D}$

work – energy Equation

$$R \cdot s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}m(v^2 - u^2)$$

$$(-1500\sin\theta_1 + F)s + (320 + 2000\sin\theta_2)\,0.5s$$

$$= \frac{1}{2}\frac{1500}{9.81}(v^2 - 0) + \frac{1}{2}\frac{2000}{9.81}\left[(0.5v)^2 - 0\right]$$

$v = 3\ m/s$

$s = 3.529m$

"By observing pulley system it may be concluded that if 1500N body moves a distance 's', 2000N body moves a distance 0.5 s, and if velocity of 1500N block is v that of 2000N block will be 0.5v"

since 's' is positive the assumed direction of motion is correct. If it comes out negative note that recalculations are to be made since the frictional force changes the sign, if motion is reversed.

$$(1200 - 180)s - (1200 + 320)\,0.5\,s = \frac{1500}{2 \times 9.81}(v^2 - 0) + \frac{2000}{2 \times 9.81}\left[(0.5v)^2 - 0\right]$$

$v = 3\ m/s$

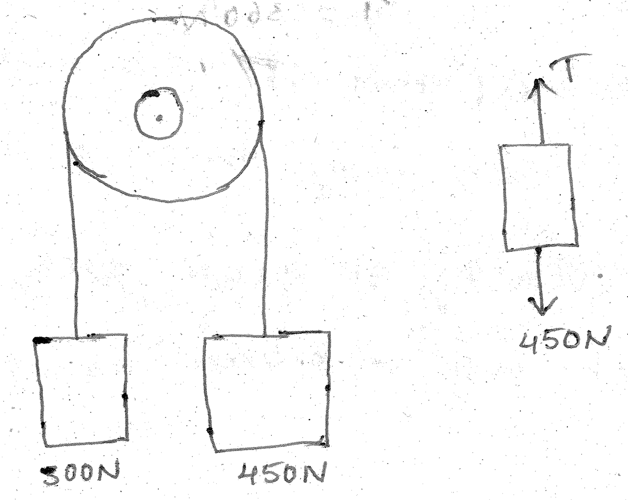
$$260\,s = \frac{1500}{2 \times 9.81} \times 3^2 + \frac{2000}{2 \times 9.81} \times 1.5^2$$

$s = 3.529\ m$

(P) Two bodies weighing 300N and 450N are hang to the ends of a rope passing over an ideal pulley as shown in Fig. How much distance the blocks will move in increasing the velocity of system from 2 m/sec to 4 m/sec? How much is the tension in the string? Use work-energy method.

$u = 2\,m/s$

$v = 4\,m/s$



Sol:- 450 N block moves down and 300N block moves up.

The arrangement is such that both bodies will be having same velocity & both will move by the same distance.

$$450S - 300S = \frac{450}{2\times9.81}(v^2-u^2) + \frac{300}{2\times9.81}(v^2-u^2)$$

$$150S = \frac{450}{2\times9.81}(4^2-2^2) + \frac{300}{2\times9.81}(4^2-2^2)$$

$$S = 3.058\ m$$

Let T be the tension in the spring.

consider work energy equation for any one body, say 450N body

$$450S - TS = \frac{450}{2 \times 9.81}(4^2 - 2^2)$$

$$(450 - T)\,3.058 = \frac{450}{2 \times 9.81} \times 12$$

$$T = 360\,N$$

Impulse Momentum

Impulse momentum method is dealt which is useful for solving the problems involving force, time and velocity.

Impulse - Momentum Equation

If R is the resultant force acting on a body of mass m, then from Newton's second law,

$$R = ma \qquad a = \frac{dv}{dt}$$

$$R = m \cdot \frac{dv}{dt}$$

$$R\,dt = m\,dv$$

$$\int R\,dt = \int m\,dv$$

If initial velocity u and after time interval 't' it becomes v, then

$$\int_0^t R\,dt = \int_u^v m\,dv$$

$$= m[v]_u^v$$

$$= m[v-u]$$

$$\int_0^t R\,dt = mv - mu$$

If R is constant during time interval t, then Impulse = R×t

$\int_0^t R\,dt \rightarrow$ Impulse

unit of impulse will be N-sec.

Impulse = Final momentum − Initial momentum

mass × velocity = momentum

$$mv = \frac{W}{g} \cdot V$$

$$= \frac{N}{m/s^2} \cdot m/s = N\text{-}sec$$

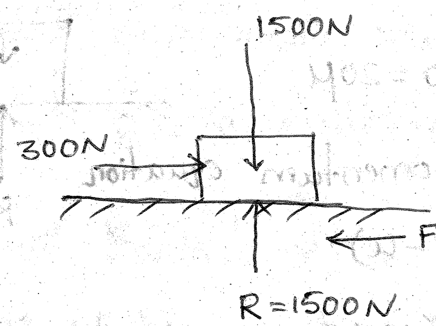
unit of momentum is N-sec.

Since the velocity is a vector, impulse is also a vector. The impulse momentum equation holds good when the directions of R, u & v are the same.

"The component of resultant linear impulse along any direction is equal to change in the component of momentum in that direction."

The impulse-momentum equation ~~cab~~ can be applied in any convenient direction and the kinetic problems involving force, velocity and time ~~cane~~ can be solved easily.

(P) A 1500 N block is in contact with a level plane, the coefficient of friction between two contact surfaces being 0.1. If the block is acted upon by a horizontal force of 300N, what time will elapse before the block reaches a velocity of 16 m/s starting from rest? If 300N force is then removed, how much longer will the block continue to move? solve the problem using impulse momentum equation.



Sol:

$F = \mu R$ $\qquad$ $v = 16\ m/s$

$= 0.1 \times 1500 = 150N$ $\qquad$ $u = 0$

Applying impulse momentum equation in the horizontal direction $\qquad Rt = m(v-u)$

$$(300-150)t = \frac{1500}{9.81}(v-u)$$

$$t = 16.31\ sec$$

If the force is then removed only horizontal force is $F = 150N$. Applying impulse momentum equation for the motion towards right $\qquad v=0$, $u = 16 m/s$

$$-150t = \frac{150}{9.81}(0-16) \Rightarrow t = 16.31\ sec$$

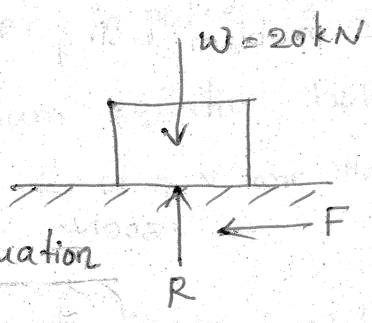
The block takes another 16.31 sec before it comes to rest.

(P) A 20 kN automobile is moving at a speed of 70 kmph when the brakes are fully applied causing all four wheels to skid. Determine the time required to stop the automobile (a) on concrete road for which $\mu = 0.75$ (b) on ice for which $\mu = 0.08$

Sol:- Initial velocity of the vehicle

$$u = 70\,kmph = \frac{70 \times 1000}{60 \times 60} = 19.44\ m/s$$

Final velocity, $v = 0$



$F = \mu R = \cancel{\mu} \times 20 = 20\mu$

Applying impulse momentum equation

$$-Ft = \frac{W}{g}(v-u)$$

$Rt = m(v-u)$

$$-20\mu t = \frac{20}{9.81}(19.44 - 0)$$

$$t = \frac{1.982}{\mu}$$

$t = \frac{39.5}{\mu_1}$

(i) On concrete road, $\mu = 0.75$

$$t = \frac{1.982}{0.75} = 2.64\ sec$$

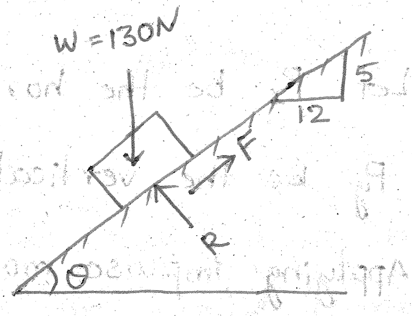
(ii) On ice, $\mu = 0.08$

$$t = \frac{1.982}{0.08} = 24.78\ sec$$

(P) A block weighing 130N is on an incline, whose slope is 5 vertical to 12 horizontal. Its initial velocity down the incline is 2.4 m/sec. what will be its velocity 5 sec. later? Take coefficient of friction at contact surface = 0.3

Sol:-

$\tan\theta = \frac{5}{12}$ $\theta = 22.62°$

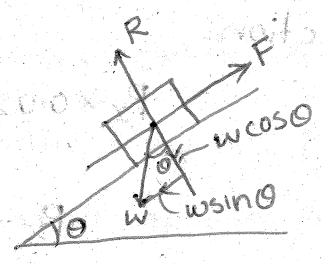


$R = W\cos\theta$

$= 130\cos 22.62$

$R = 120N$

$F = \mu R = 0.3 * 120 = 36N$



Initial velocity, $u = 2.4$ m/s

Final velocity, $v = ?$

$t = 5$ sec

$Rt = m(v-u)$

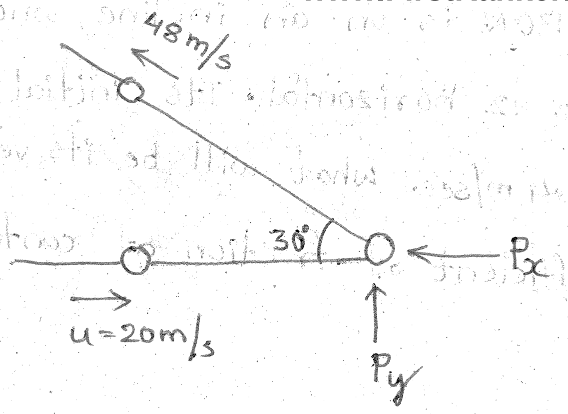
$(Rt) = m(v-u)$

$(w\sin\theta - F)t = \frac{W}{g}(v-u)$

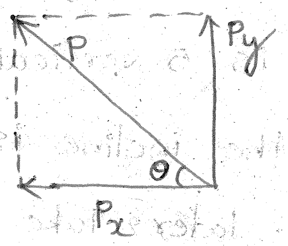
$(130\sin 22.62 - 36)5 = \frac{130}{9.81}(v-2.4)$

$v = 7.68$ m/s

(P) A 1N ball is bowled to a batsman. The velocity of ball was 20m/s horizontally just before batsman hit it. After hitting it went away with a velocity of 48 m/s at an inclination of 30° to horizontal as shown in Fig. Find the average force exerted on the ball by the bat if the impact lasts for 0.02 sec



(a)



(b)

sol:- Let P_x be the horizontal component of the force and P_y be the vertical component.

Applying impulse momentum equation in horizontal direction

$$P_x * 0.02 = \frac{1}{9.81}\left[48\cos 30° - (-20)\right]$$

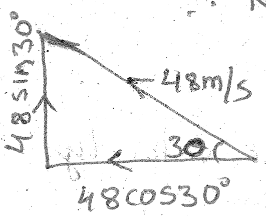
$$P_x = 313.81\ N$$

Applying impulse momentum equation in vertical direction

$$P_y * 0.02 = \frac{1}{9.81}\left[48\sin 30 - 0\right]$$

$$P_y = 122.32\ N$$

$\therefore$ Resultant force, $P = \sqrt{P_x^2 + P_y^2}$



$$= \sqrt{(313.81)^2 + (122.32)^2}$$

$$P = 336.81\ N$$

$$\theta = \tan^{-1}\left(\frac{P_y}{P_x}\right) = \tan^{-1}\left(\frac{122.32}{313.81}\right)$$

$$\theta = 21.30°$$

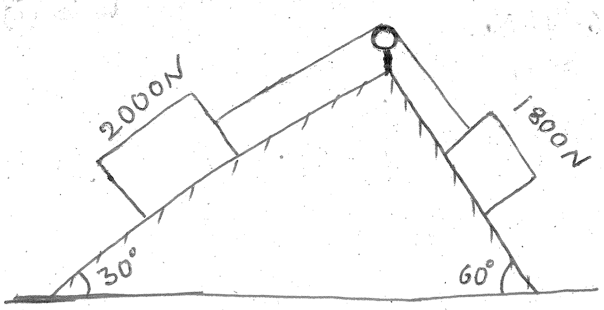
Connected Bodies

The problems involving connected bodies may be solved by any one of the following two methods

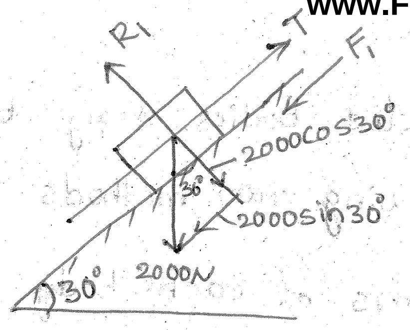
First Method: Free body diagrams of each body is drawn separately. Impulse-momentum equation for each body in the direction of its motion is written and then the equations are solved to get the required values.

Second Method: If the connected bodies have same displacement in the same time, the impulse of internal tension in connecting chords will get cancelled. Hence free body diagram of combined bodies may be considered and impulse momentum equation applied in the direction of motion of combined bodies. This method is applicable only if displacement of each body is the same in given time.

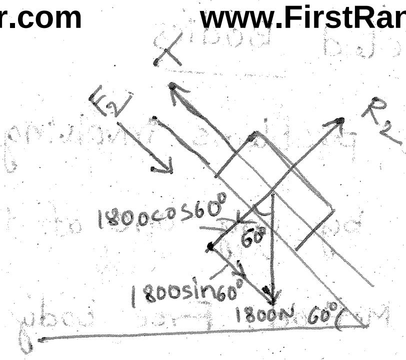
(P) Determine the time required for the weights shown in Fig. to attain a velocity of 9.81 m/s. What is the tension in the chord? Take $\mu = 0.2$ for both planes. Assume the pulleys as frictionless.



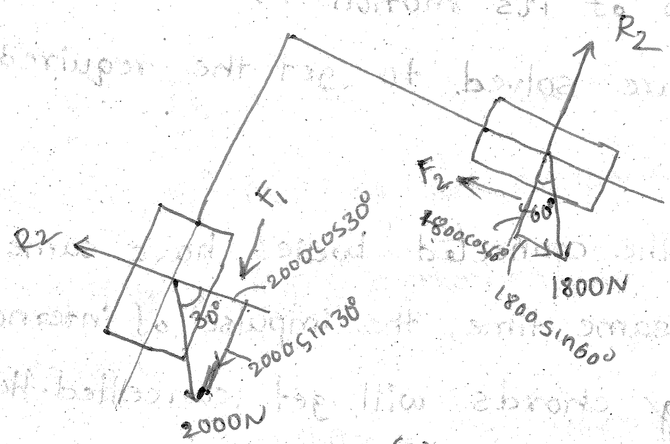
Sol:-



(a)



(b)



(c)

Impulse momentum Equation

$Rt = m(v-u)$

First Method

Let T be the tension in the chord.

Initial velocity, $u=0$

Final velocity, $v = 9.81\ m/s$

Applying impulse momentum equation for the 2000 N block in upward direction parallel to the plane

$$(T - 2000\sin 30^\circ - F_1)t = \frac{2000}{9.81}(v-u)$$

$$(T - 1000 - 346.41)t = \frac{2000}{9.81}(9.81-0)$$

$$(T - 1346.41)t = 2000 \quad \text{---} (1)$$

$F_1 = \mu R_1$

$= 0.2 \times 2000\cos 30^\circ$

$F_1 = 1732.05 \times 0.2 = 346.41\ N$

$F_2 = \mu R_2$

$= 0.2 \times 1800\cos 60^\circ$

$= 0.2 \times 900 = 180\ N$

Applying impulse momentum equation for 1800 N block in the direction parallel to 60° incline plane

$$(1800 \sin 60^\circ - T - F_2)t = \frac{1800}{9.81}(v-u)$$

$$(1800 \sin 60^\circ - T - 180)t = \frac{1800}{9.81}(9.81-0)$$

$$(1378.85 - T)t = 1800 \rightarrow (2)$$

Dividing equation (1) by (2)

$$\frac{(T - 1346.41)t}{(1378.85 - T)t} = \frac{2000}{1800}$$

$$T = 1363.48 \text{ N}$$

#

Equation (2) $\Rightarrow (1378.85 - T)t = 1800$

$$t = 117.11 \text{ sec}$$

#

Second Method:

Since the displacement of both bodies are same in given time, consider combined FBD of the blocks as shown in Fig. Writing impulse momentum equation in the direction of motion

$$(1800 \sin 60^\circ - F_2 - 2000 \sin 30^\circ - F_1)t = \frac{2000+1800}{9.81}(v-u)$$

$$(1800 \sin 60^\circ - 180 - 2000 \sin 30^\circ - 346.41)t = \frac{3800}{9.81}(9.81-0)$$

$$t = 117.11 \text{ sec}$$

#

To find tension in the chord, consider the impulse momentum equation of any ~~two~~ one block, say 2000 N block

$$\left(T - 2000\sin 30^\circ - F_1\right)t = \frac{2000}{9.81}(v - u)$$

$$\left(T - 2000\sin 30^\circ - 346.41\right)117.11 = \frac{2000}{9.81}(9.81 - 0)$$

$$T = 1363.48\ N$$

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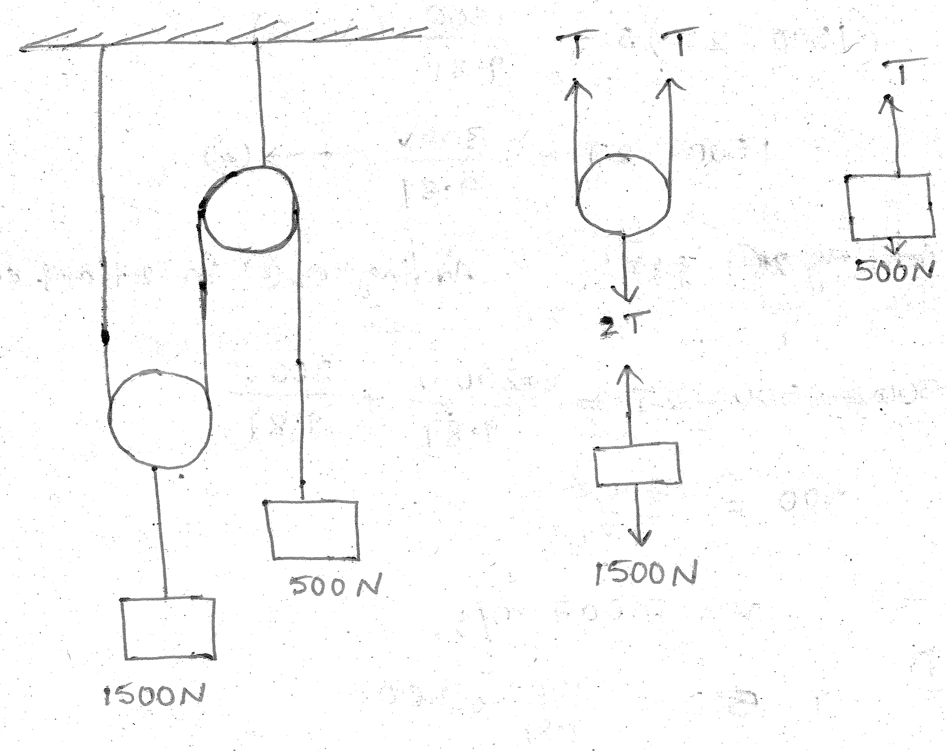
(P) When 150N block moves a distance s, in the same time 500N block moves a distance 2s. Hence if velocity of 1500N block is v m/s that of 500N block will be 2v m/s. Let T be the tension in the chord connecting 500N block. Hence tension in the wire connecting 1500N block will be 2T. Since the velocities of two blockes are different only first method is to be used.



Sol:- Determine the tension in the strings and the velocity of 1500N block shown in Fig. 5 sec after starting from

(a) rest

(b) starting with a downward velocity of 3 m/s

Assume pulleys as weightless & frictionless

Apply impulse momentum equation to 500 N

Case (a): $u = 0, \quad t = 5\,sec$

$$Rt = m(v-u)$$

$$(T-500)t = \frac{500}{9.81}(2V-u)$$

$$(T-500)5 = \frac{500}{9.81}(2V-0)$$

$$T-500 = \frac{200V}{9.81} \rightarrow (1)$$

Apply impulse momentum equation to 1500 N

$$(1500-2T)t = \frac{1500}{9.81}(V-u)$$

$$(1500-2T)5 = \frac{1500}{9.81}(V-0)$$

$$1500-2T = \frac{300V}{9.81} \rightarrow (2)$$

~~Adding 2×(1) & (2)~~ Adding eq (2) in 2 times eq (1)

$$2T-1000+1500-2T = \frac{2\times200V}{9.81} + \frac{300V}{9.81}$$

$$500 = \frac{700V}{9.81}$$

$$V = 7.007\,m/s$$

(1) $$T-500 = \frac{200}{9.81} \times 7.007$$

$$T = 642.86\,N$$

Case (b): $u = 3\,m/s \qquad v = 2V \qquad t = 5\,sec$

Impulse momentum eq to 500 N

$$(T-500)t = \frac{500}{9.81}(2V-3)$$

$$T-500 = \frac{100(2V-3)}{9.81} \rightarrow (3)$$

Impulse momentum eq for 1500 N body

$V = V$

$u = 3\,m/s$

$$(1500 - 2T)\,5 = \frac{1500}{9.81}(v-3)$$

$$1500 - 2T = \frac{300}{9.81}(v-3) \quad \text{— (4)}$$

Adding eq (4) & 2 times eq (1)

$$1500 - 1000 = \frac{100}{9.81}(7V - 15)$$

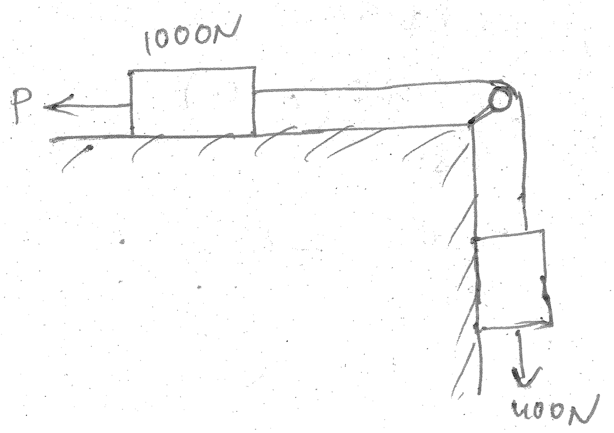
$$V = 9.15\ m/s$$

$$(3) \Rightarrow T - 500 = \frac{100}{9.81}(2 * 9.15 - 3)$$

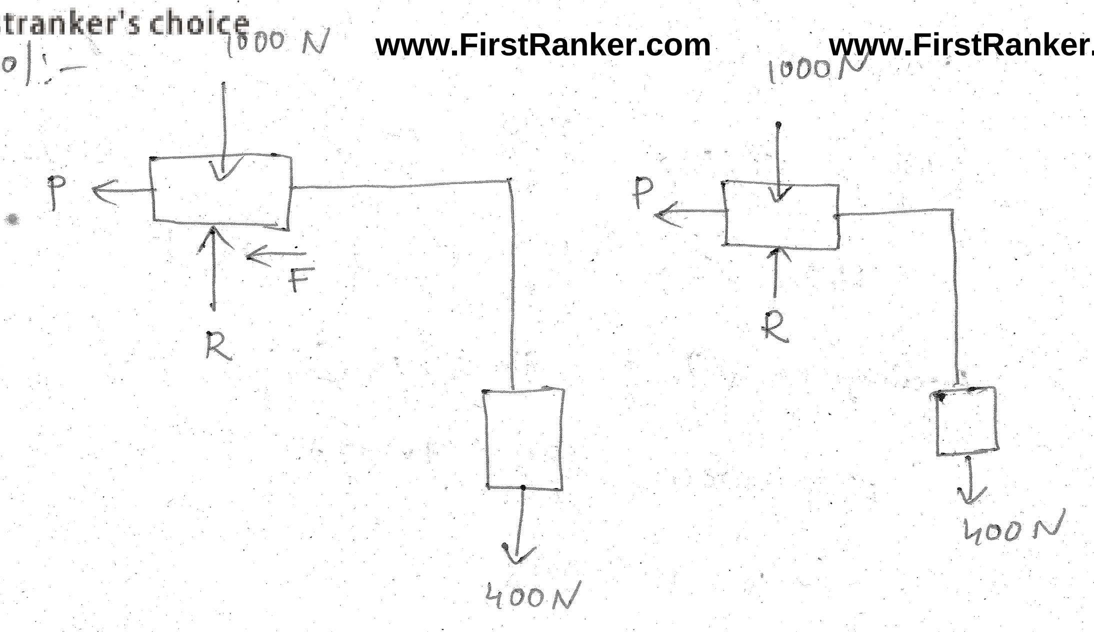
$$T = 655.96\ N$$

#

(P) The system shown in Fig has a rightward velocity of 4 m/s, just before a force P is applied. Determine the value of P that will give a leftward velocity of 6 m/sec in a time interval of 20 sec. Take coefficient of friction = 0.2 & assume ideal pulley.



Sol:-



$$F = \mu R = 0.2 \times 1000 = 200\,N$$

when the system is moving right ward, frictional force acts leftward as shown in Fig. Force P first brings the system to stationary position, then starts moving leftward. At this stage, the frictional force acts rightward as shown in Fig.

Let t_1 be the time required to bring the system to the stationary condition.

Applying the impulse momentum eq for the motion upto stationary condition

$$(400 - 200 - P)t_1 = \frac{400+1000}{9.81}(0-4)$$

$$(P-200)\,t_1 = \frac{5600}{9.81} \quad \text{—} (1)$$

Applying impulse momentum equation for the motion from stationary position to left-ward motion, after total time of 20 sec, we have

$$(P-F-400)(20-t_1) = \frac{1000+400}{9.81}(v-0)$$

$$(P-600)(20-t_1) = \frac{1400}{9.81} * 6 \qquad \text{—} (2)$$

simultaneous equations (1) & (2) may be solved to get t_1 & P.

Trial & error method may be advantageously used here. Looking at Eqn (2) the value of P should be more than 600. Let us take a trial value of P as 700. From Eqn (1)

$$t_1 = 1.142 \text{ sec}$$

substituting it in Eqn (2)

$$P = 645.4 \text{ N}$$

substituting this value of P in Eqn (1)

$$t_1 = 1.28 \text{ sec}$$

substituting it in Eqn (2)

$$P = 645.7 \text{ N}$$

This value is almost same as trial value 646.41 N.

Hence $P = 645.7$ N