ELECTRICAL AND MECHANICAL TECHNOLOGY

I B.TECH II SEM ECE

SYALLABUS:

UNIT-I: DC Machines:

Principle of operation of DC generator – emf equation – types of DC machine – torque equation of DC motor –applications – three point starter, speed control methods – OCC of DC generator.

Transformers: Principle of operation of single phase transformers – e.m.f equation – losses –efficiency and regulation.

UNIT – II AC Rotating Machines:

Principle of operation of alternators – regulation by synchronous impedance method – principle of operation of 3- Phase induction motor – slip-torque characteristics - efficiency – applications.

UNIT III Measuring Instruments:

Classification – Deflection, controlling, damping torque, ammeter, voltmeter, wattmeter, MI, MC instruments –Energy meter – Construction of CRO.

UNIT - I

DC Machines

Conductor :

when the valence electrons in the atoms are 1, 2, or 3 they are bound to the nucleus with very low amount of force of attraction. Hence they can be easily removed from the periphery of the mother atom. such electrons are called free electrons. The materials made up of such atoms will conduct electricity. Hence they are called conductors or conducting materials.

Eg: silver, copper, Aluminium etc.

Voltage (V) :

In electric circuits one terminal will be electrically at a higher charge or potential or pressure. when compare the other terminal will be electrically at a lower charge or potential or pressure. Hence there exists electric potential difference or electric pressure difference between two terminals. It is in general called voltage.

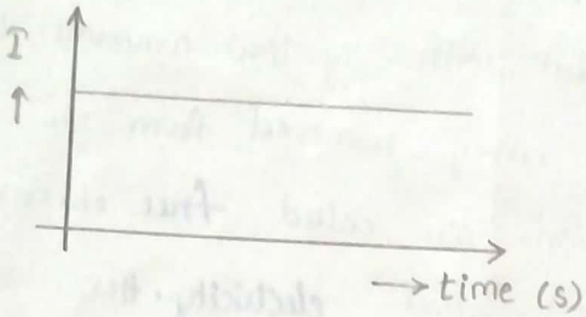
Infact it is the force that drives electrons through an electric circuit. Hence it is called electro motive force (e.m.f). It is represented by 'V' or 'E'. It is measured in volts (V).

current (I) :

Due to electromotive force (e.m.f) between the two terminals of an electric circuit, electric current will flow from positive

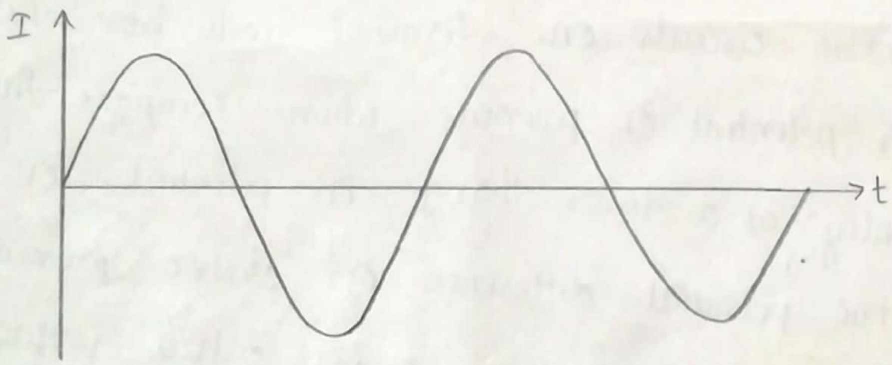
...minal to negative terminal. In general electric current is represented by 'I'. It is measured in amperes '(A)'

DC (Direct current):



A current which is having continuously same direction of flow and constant magnetude is known as Direct current.

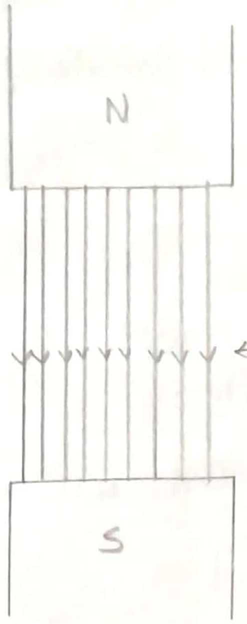
AC (Alternating current):



Alternating current is varying with time in a cyclic manner and current direction polarity have been reversing periodically

Magnetic flux (ϕ):

The total number of lines of force that emit from a north pole or enter into the south pole is called magnetic flux of that pole. It is simply called flux. It is represented by the greek letter ϕ and is measured in webers



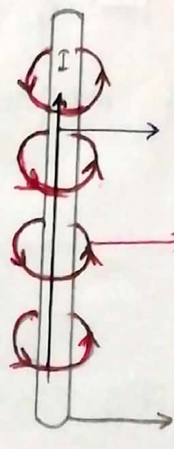
A pole of m webers means it emits lines of force of 'm' webers

In C.G.S system flux is measured in lines of force.

where 1 weber = 10^8 lines of force.

Magnetic effect of electric current:

whenever electric current flows through a conductor, magnetic field is set up around the conductor through out its length as an ...umour. This fact was first identified by "oersted".



The magnetic field established will have the following nature

... spreads over the entire length

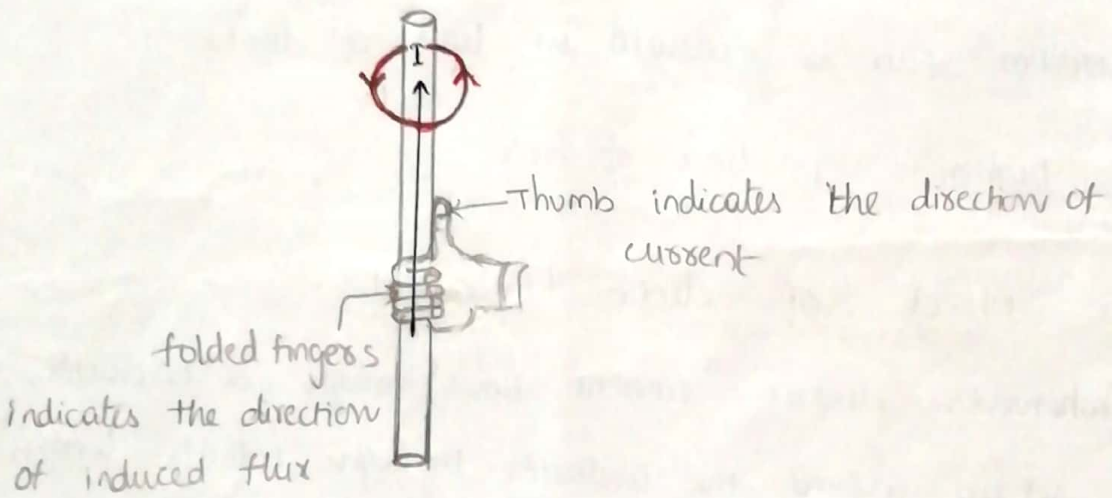
2. Its strength depends upon the strength of the current, flowing through conductor

ie, $H \propto I$

3. The magnetic field is stronger nearer to the conductor weaker as we go away from the conductor.

4. The direction of the magnetic field will depend upon the direction of the current flowing through it

Right hand thumb rule:



Hold the conductor in your right hand keeping the thumb indicating the direction of current flowing through it. Then the folded fingers will denote the direction of magnetic field established around that conductor.

Flux linkages:

when a magnetic line-of-force is cut by a conductor it is called flux linkage. flux linkages may happen in the two ways.

1. A static magnetic line of force is cut by a moving (dynamic) conductor
2. A static conductor is linked by a moving line of force.

... magnetic Induction :

These laws are used to determine either the ... of motion or the magnitude and direction of electric/ magnetic field.

1. faraday's laws of electromagnetic induction
2. lenzs law
3. flemings left and right hand rules.

1. faradays laws of electro magnetic Induction :

Electricity and magnetism are closely associated with each other. oersted proved that magnetism is produced due to electric current. later faraday established the fact that magnetic energy can be converted back into electric energy.

faradays laws of electromagnetic Induction explain how an emf is induced across a conductor and the amount of emf induced.

Faraday first law :

whenever a conductor cuts a magnetic flux an emf is induced across that conductor.

(or)

As and when flux linkages happen an emf will induce in that conductor.

faraday second law :

This law states that the magnitude of emf induced in the conductor is directly proportional to the rate of change of flux.

$$\text{ie, } e \propto \frac{d\phi}{dt} \text{ wb/sec}$$

Let N be the no of turns in the coil

ϕ_1 be the flux links with the coil at time t_1

and ϕ_2 be the flux lines with the coil at time t_2

$\therefore$ flux linkages at time $t_1 = N\phi_1$

and flux linkages at time $t_2 = N\phi_2$

change of flux linkages in time $t_2 - t_1 = N\phi_2 - N\phi_1$

According to faradays second law of electromagnetic induction

The magnitude of induced emf $e = \dfrac{\text{change of flux linkages}}{\text{change of time}}$

$$= \frac{N\phi_2 - N\phi_1}{t_2 - t_1} \text{ wb-turns/s or volts}$$

$$= \frac{N(\phi_2 - \phi_1)}{t_2 - t_1} \text{ wb-turns/s or volts}$$

$$= N\frac{d\phi}{dt} \text{ volts}$$

But induced emf $e = -N\dfrac{d\phi}{dt}$ volts

Here minus sign indicates that the induced emf opposes the voltage which generates the magnetic effect.

2. LENZ'S LAW:

This laws states that "always Induced emf" or the "induced current" will oppose the cause of its creation.

Emf can be produced by means of electro magnetic
Induction in the two ways.

1. Dynamically induced emf

2. Statically induced emf

Statically induced emf can further be divided into

a) self induced emf

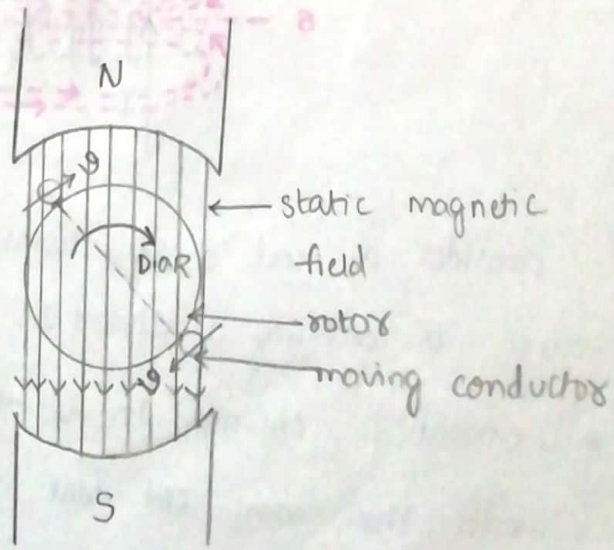
b) Mutually induced emf

Dynamically induced emf:

when a conductor is moving in a static magnetic field causing flux-linkages an emf is induced in that conductor. As the conductor is dynamic, the emf induced is called dynamically induced emf. The magnitude of the emf is directly proportional to the rate of change of flux

i.e.) $e \propto \frac{d\phi}{dt}$ .. volts

Example: Generator.



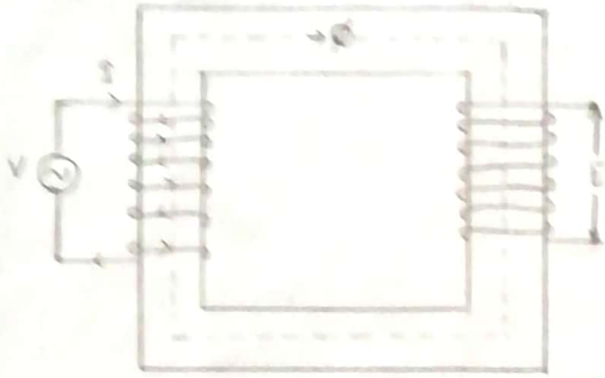
v - velocity

D.O.R - Direction of rotation

Statically induced emf:

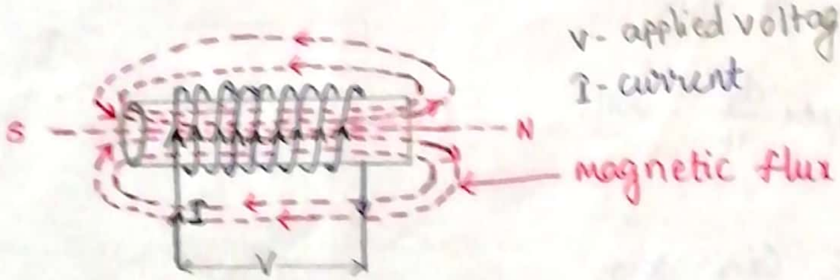
when moving magnetic lines of force are linking with a static conductor. this emf is called statically induced emf

Example: Transformer



a) self induced e.m.f:

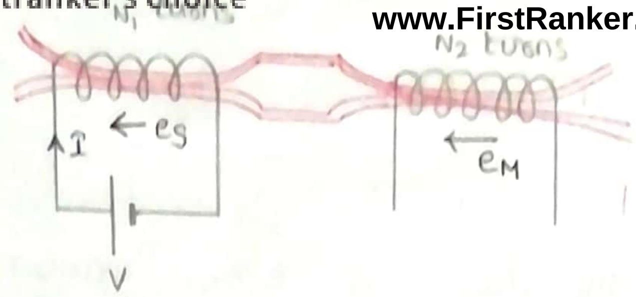
"This is an emf induced in the coil due to magnetic flux caused by the current passing through the same coil"



consider a coil of N turns wound on a solenoid as shown in figure is carrying a current I that produces magnetic flux in the coil If the current I change then the flux will change. the changing flux links with the same coil will induce an emf called self induced emf

b) Mutual induced emf:

This is an emf induced in the coil due to magnetic flux caused by the current passing through the neighbouring coil



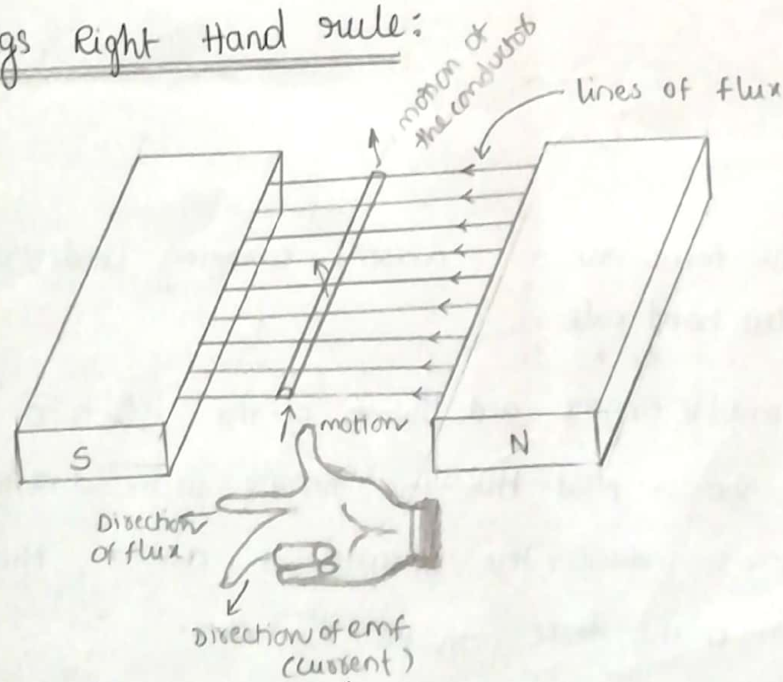
V - applied voltage

e_s - self induced emf

e_m - mutually induced emf

consider two coils as shown. If supply voltage is given to the first coil then flux produced in that coil. This flux linking with the neighbour coil then the emf will induce proportional to the number of turns N_2 of the second coil. This emf is also opposite in direction with respect to the applied emf 'V'. Hence it is also negative. It is called mutually induced emf (e_m).

8. Flemings Right Hand rule:



keep the first three fingers of the right hand at right angles to each other. Then

thuMb indicates the direction of "Motion of the conductor

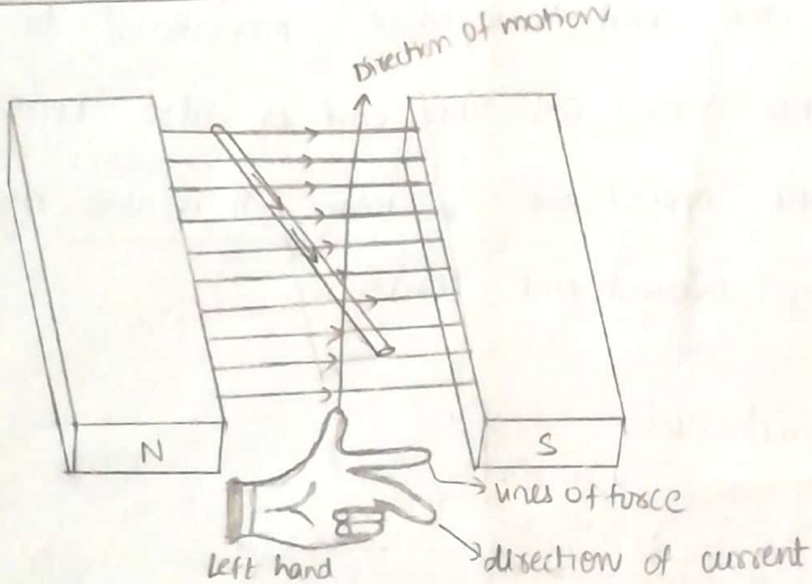
"Forefinger indicates the direction of "flux

"MIddle finger indicates the direction of induced emf or current

If the directions of any two quantities are known the direction of third quantity can be identified with the help of Flemings right hand rule.

It helps in identifying the direction of e.m.f induced in a generator.

Flemings Left hand rule:

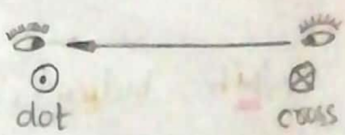


* The direction of the force on a current carrying conductor can be obtained by flemings left hand rule.

keep the fore and middle finger and thumb of the left hand at right angles to each other as shown point the fore finger in the direction of the field and the middle finger indicates the direction of current, then the thumb gives the direction of the force on the conductor.

To indicate the direction of quantities leaving or entering into plane:

Dot ⊙: It indicates that the quantity is coming perpendicularly towards the observer



cross ⊕: It indicates that the quantity is going away perpendicularly from the observer.

Introduction:

The study of the electrical engineering basically involves the analysis of the energy transfer from one form to another. An electrical machine deals with the energy transfer either from mechanical to electrical form or viceversa. This process is called "electromechanical energy conversion."

An electrical machine which converts mechanical energy into an electrical energy is called an "electric generator."

An electrical machine which converts electrical energy into the mechanical energy is called an "electric motor."

The electric machines are of two types

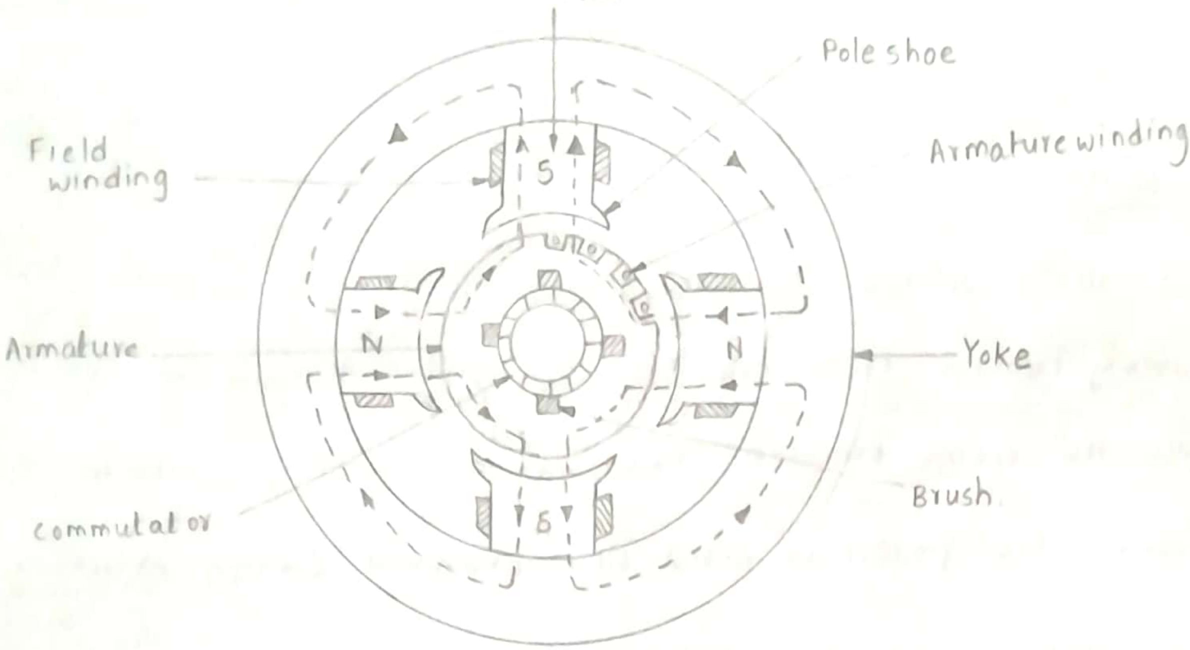
1. AC Machines
2. DC Machines

The dc machines are classified as dc generators and dc motors. The construction of a dc machine basically remains same whether it is generator or motor.

DC Generator:

"A DC generator is a machine which converts mechanical energy into an electrical energy." In this the energy conversion is based on the principle of the production of dynamically induced emf i.e, "whenever a moving conductor cuts the magnetic flux lines an e.m.f is induced across the conductor terminals."

Construction of dc generator:



The dc generators and dc motors have the same general construction. Any dc generator can be run as a dc motor and vice versa.

The dc machine consists of following parts.

1. Yoke:

The outer frame or yoke serves double purpose

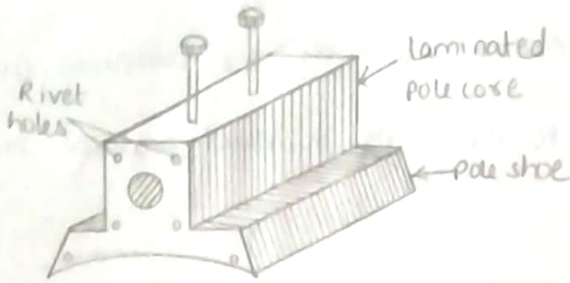
(i) It provides mechanical support for the poles and acts as a protecting cover for the whole machine and

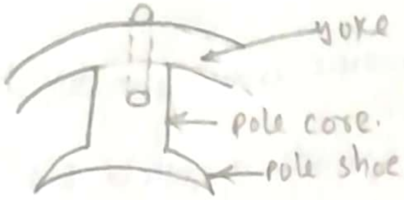
(ii) It carries the magnetic flux produced by the poles.

In small generators yokes are made of cast iron. and for large machines cast steel or rolled steel is used. The feet and terminal box etc are welded to the frame

Each pole is divided into two parts namely

1. pole core
2. pole shoe





Pole structure

a) Functions of pole core and pole shoe:-

1. Pole core basically carries a field winding which is necessary to produce flux
2. It directs the flux produced through airgap to armature core, to the next pole.
3. Pole shoe enlarges the area of armature core to come across the flux, which is necessary to produce larger induced emf. To achieve this poleshoe has been given a particular shape

b) choice of material:- It is made up of magnetic material like cast iron or cast steel.

As it requires a definite shape and size, laminated construction is used. The laminations of required size and shape are stamped together to get a pole which is then bolted to the yoke.

field winding:

The field wdg is wound on the pole core with a definite direction

functions: To carry current due to which pole core on which the field winding is placed behaves as an electromagnet, producing necessary flux.

→ As it helps in producing the magnetic field i.e., exciting the pole as an electromagnet it is called field winding or Exciting winding.

→ field winding is made up of aluminium or copper. But field coils are required to take any type of shape and bend about pole core and copper has good pliability i.e., it can bend easily. so copper is the proper choice.

Armature core:

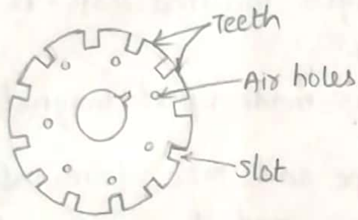
→ Armature core is cylindrical in shape mounted on the shaft. It consists of slots on its periphery and the air ducts to permit the air flow through armature which serves cooling purpose.

→ Armature core provides house for armature winding ie, armature conductors

→ To provide a path of low reluctance to the magnetic flux produced by the field winding.

→ It is made up of cast iron or cast steel.

→ It is made up of laminated construction to keep eddy current loss as low as possible.



Armature winding:

→ Armature winding is nothing but the interconnection of the armature conductors placed in the slots provided on the armature core periphery

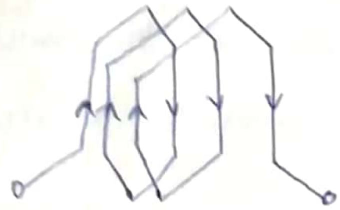
→ It is made up of copper material

→ The number of armature conductors which are connected in specific manner as per the requirement so is called armature winding.

→ According to the way of connecting the conductors, armature winding has basically two types namely

a) lap winding

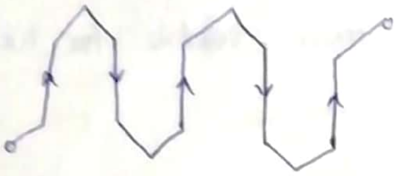
b) wave winding



for lap winding the number of parallel paths $A = P$ = number of poles in the machine.

large number of parallel paths indicate high current capacity of machine hence lap winding is preferred for high current rating generators.

b) wave winding:



In this type of connection $A = 2$ ie, number of parallel paths always irrespective of no of poles of the machine.

As number of parallel paths are less it is preferrable for low current, high voltage capacity generators.

commutator:

→ The function of the commutator is to facilitate collection of current from the armature conductors and it converts the alternating current induced in the armature conductors into unidirectional current.

→ It is made up of copper segments

→ It is cylindrical in shape and is made up of wedge shaped segments of hard drawn high conductivity copper.

→ these segments are insulated from each other by thin layer of mica

→ Each commutator segment is connected to the armature conductor by means of copper strip.

Brushes:

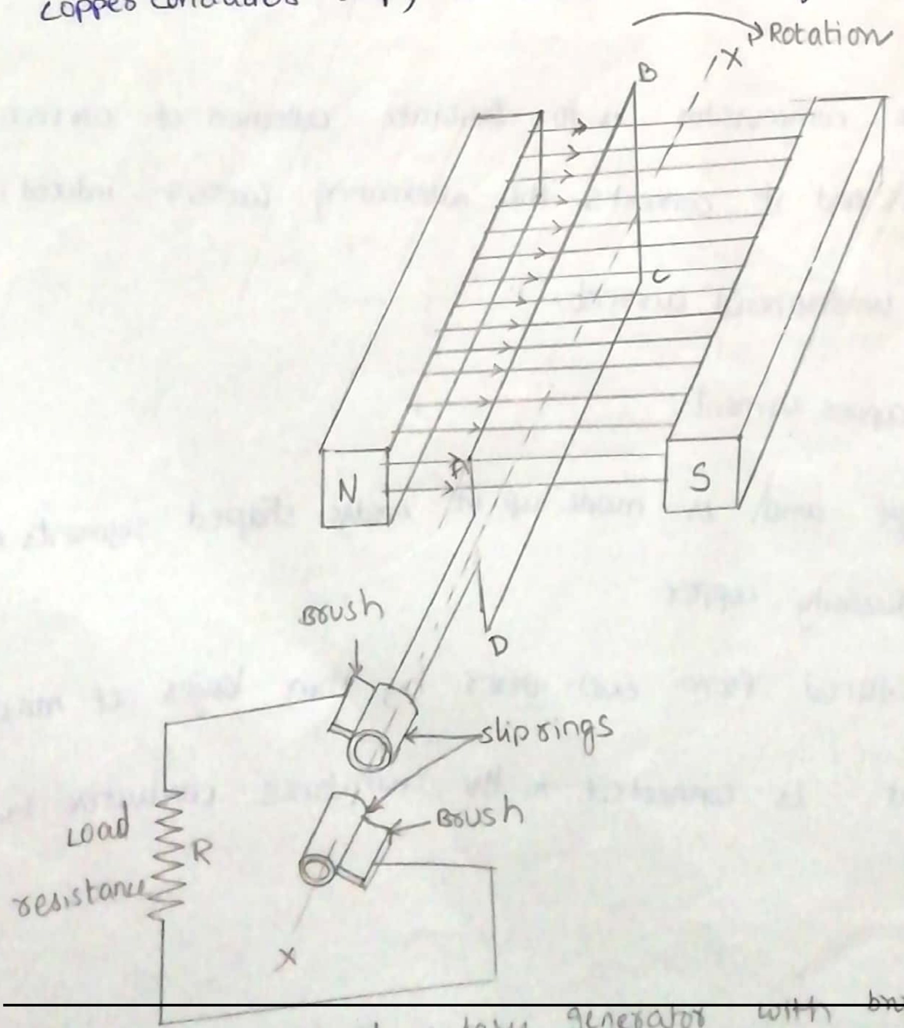
→ Brushes are stationary and resting on the surface of the commutator
→ It collects current from commutator and make it available to the stationary external circuit.
→ Brushes are normally made up of soft material like carbon
→ Brushes are rectangular in shape.

Bearings:

Ball bearings are usually used as they are more reliable for heavy duty machines roller bearings are preferred.

Principle of operation of a DC generator:

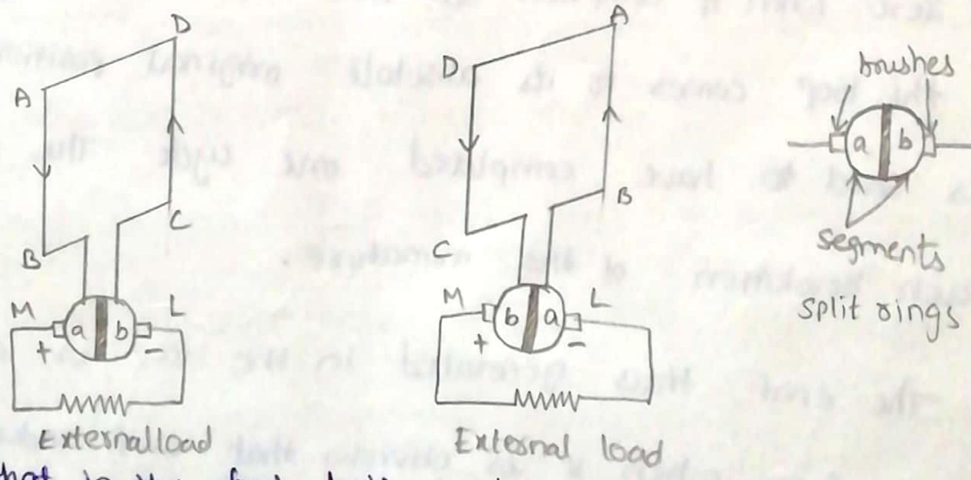
Consider a simple loop generator (A single turn rectangular copper conductor loop) as shown in figure(1)



fig(1) Elementary generator with brushes and collector rings

...insulated rings known as slip rings or collector rings mounted on the generator shaft and making the two stationary brushes pressing against the sliprings one brush bearing on each ring shown in fig (1). when the coil is rotated the generated alternating emf causes a current to flow first in one direction and then in the other through the coil and external circuit. such a current is called an alternating current.

To obtain the uni-directional or direct current in the external circuit the collecting arrangement is modified as shown in figure. In this arrangement the slip rings are replaced by split ring made of a conducting material and splitted into two halves separated by from each other by insulation and brushes are placed diametrically opposite instead of being side by side.



It will be observed that in the first half revolution current flows along ABMLCDA ie, the brush M in contact with segment 'a' acts as the +ve pole of the supply and brush L in contact with segment 'b' acts as the −ve pole. In the next half revolution the direction of induced current in the coil is reversed but at the same time segments 'a' and 'b' are also reversed with the result that brushes M and L again come in contact with +ve segment 'b' gradually with the angular movement of the loop and becomes zero at the instant the loop becomes again parallel to the faces of the field magnets but with the sides AB and CD's position interchanged with respect to zero position as shown in fig (c)

In the third quarter of the revolution of loop ie;) between 180 to 270° radians the rate at which the conductors cut across the magnetic field hence induced emf gradually increases as the loop moves and becomes maximum at the instant the loop assumes the position shown in fig (d). However the direction of emf in the loop is now from A to B and from C to D ie, opposite to that in the first two quarters.

In the fourth quarter of the revolution of the loop ie;) between 270° to 360° the induced emf decreases as the coil moves and become zero when it completes 360° from the starting instant. At this instant the loop comes to its absolute original position and hence the loop is said to have completed one cycle. The cycle then repeats for each revolution of the armature.

The emf thus generated in the loop is of the form shown in fig(3) from which it is obvious that emf induced in armature conductors is of pulsating nature. Such an emf is known as alternating emf.

The current induced in the coil is collected and conveyed to the external load circuit by connecting the coil terminals to two

Imagine the coil to be rotating in clockwise direction assumes successive positions in the field, the flux linked with it changes. Hence an emf is induced in it which is proportional to the rate of change of flux linkages.

While rotating at the instant the loop of wire ABCD assumes the position shown in fig(a) i.e., when the loop ABCD is just parallel to the faces of field magnets N and S the flux linking with the loop is maximum but the rate of change of linking flux is zero as at this instant no flux is cut by the coil sides AB and CD which are just moving parallel to them. Hence induced emf is zero when the loop is in this position.

As the coil is turned in clockwise direction at a constant speed the coil sides begin to cut across the field slowly at first but at gradually increasing rate. Thus the magnitude of induced emf gradually increases as the loop moves and becomes maximum when the loop assumes the position shown in fig (b). The direction of emf induced in the coil as given by Flemings right hand rule. is from B to A and from D to C. In fig (b) the coil sides are moving at right angles to the field and are therefore cutting across the field at maximum rate consequently the emf induced at this instant is of max value.

In the next quarter of the revolution of the loop ie, between 90 to 180° the rate at which the conductors cut across the magnetic field gradually decreases causing the magnitude of induced emf

In fig (1) a single turn rectangular copper coil ABCD rotating about its own axis in a magnetic field provided by either permagnent magnet is or electromagnets. The two ends of the coil are joined to two slip rings 'a' and b which are insulated from each other and from central shaft. Two collecting brushes press against slip rings. Their function is to collect the current induced in the coil and to convey it to external load resistance R.

working:

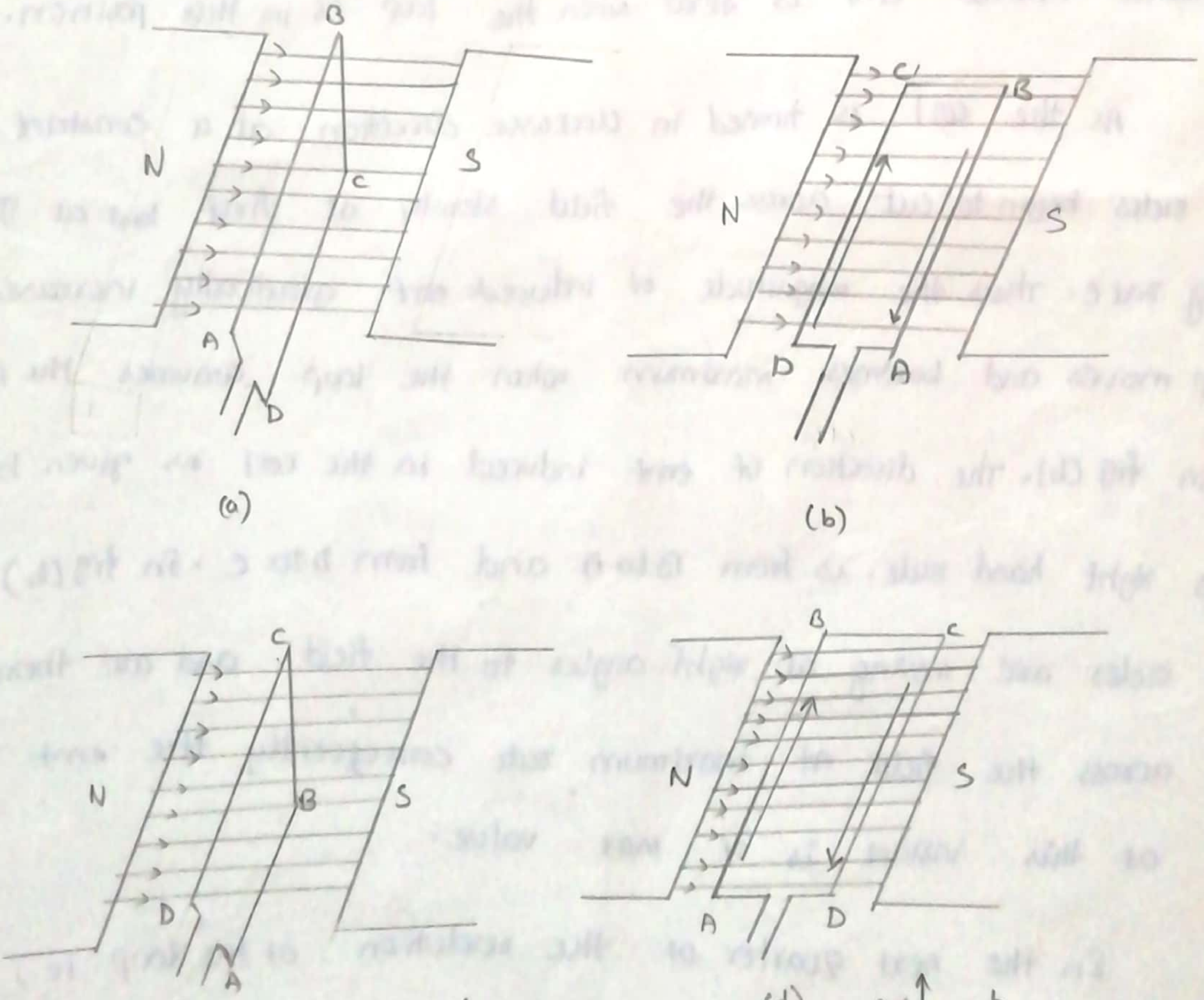


fig (2)

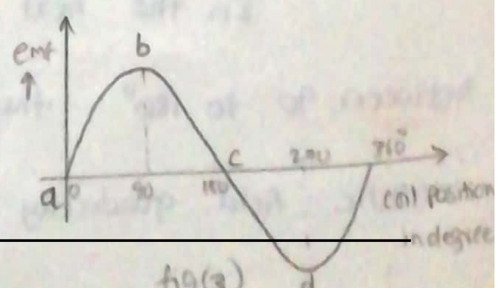
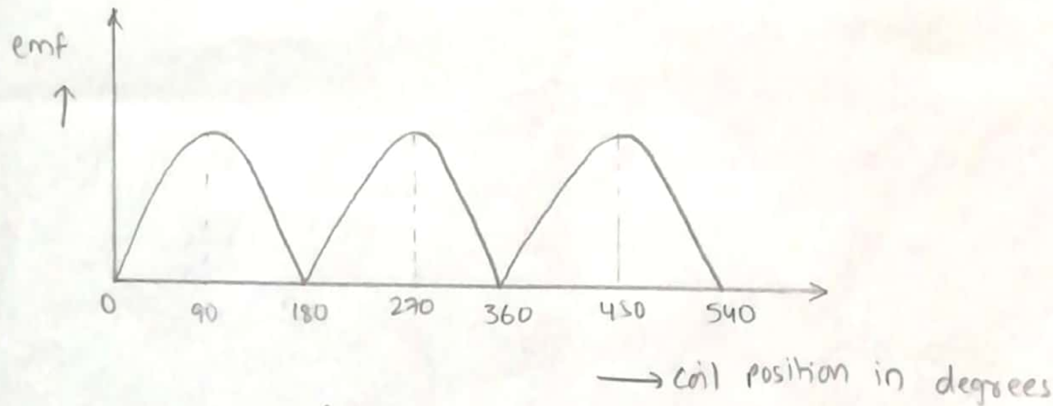


fig (3)

... a' respectively. Thus the direction of current in the external load circuits remains the same.

Thus the emf or current induced in the armature conductors of a dc generator is alternating which is rectified by the split ring known as commutator.

Although after rectification the current through the load circuit is always in the same direction but is not stead current since the emf generated in the armature coil and applied to the brushes varies from zero to maximum and back to zero twice each revolution. The variation is shown in fig (4).



fig(4)

EMF Equation:

let ϕ = flux per pole in weber

Z = Total no. of armature conductors

= No. of slots × No. of conductors per slot

P = No. of poles

A = No. of parallel paths in armature

N = No of revolutions per minute or

speed of the armature in revolutions per minute (rpm)

E_g = Emf generated in any parallel path in armature.

According to the faradays laws of electro magnetic induction the induced emf is proportional to the rate of change of the flux linkages.

ie.,
$$e = \frac{d\phi}{dt}$$

During one revolution of armature in a P-pole generator each armature conductor cuts the magnetic flux P times. So flux cut by one conductor in one revolution $d\phi = P\phi$ webers

No. of revolutions made by the armature per minute = N

∴ No. of revolutions made per second = $\frac{N}{60}$

∴ Time taken for revolution $dt = \frac{60}{N}$ second

∴ Emf generated per conductor $= \frac{d\phi}{dt} = \frac{P\phi}{\frac{60}{N}} = \frac{\phi NP}{60}$ volts

No. of conductors in series per parallel path = $\frac{Z}{A}$

$\therefore$ Emf generated per parallel path = emf generated / conductor $\times$ No. of conductors in series per parallel path

$$= \frac{\phi P N}{60} \times \frac{Z}{A}$$

$$E_g = \frac{\phi P N Z}{60 A} \text{ ... volts}$$

where $A = 2$ for wave winding

$A = P$ for lap winding

Problems:

1. A 6-pole dc generator has a wave wound armature having 45 slots with 18 conductors. The flux per pole is 0.025 wb. Determine the generated e.m.f at a speed of 1000 rpm.

Sol: given data

No. of poles $P = 6$

No. of conductors / slot = 18 conductors

No. of slots = 45

flux per pole = 0.025 wb

speed N = 1000 rpm

$A = 2$

Generated Emf E_g = ?

Generated emf $E_g = \frac{\phi Z N P}{60 A}$ volts

No. of conductors $Z = 45 \times 18 = 810$

$\therefore E_g = \frac{0.025 \times 810 \times 1000 \times 6}{60 \times 2}$

$E_g = 1012.5$ volts //

→ A 4-pole generator having wave wound armature winding has 51 slots each slot contains 20 conductors. what will be the voltage generated in the machine when driven at 1500 rpm assuming the flux per pole to be 7.0 mwb?

Sol: given data

No. of poles = 4

No. of slots = 51

No. of conductors per slot = 20

Speed N = 1500 rpm

flux per pole $\phi = 7 \times 10^{-3}$ Wb

A = 2

Generated emf = ?

No. of conductors $Z = 20 \times 51 = 1020$

$E_g = \frac{\phi Z N P}{60 A} = \frac{7 \times 10^{-3} \times 1020 \times 1500 \times 4}{60 \times 2}$

$E_g = 357$ volts

3 Calculate the emf generated by a 6 pole lap wound armature with 65 slots and 12 conductor per slot when driven at 1000 rpm The flux /pole is 0.02 wb.

Sol: given data

No. of poles $P = 6$

for lap winding $A = P = 6$

No. of slots $= 65$

No. of conductors / slot $= 12$

$\therefore$ Total No. of conductors $Z = 65 \times 12 = 780$

Speed $N = 1000$ rpm

flux per pole $\phi = 0.02$ wb

Emf generated $E_g = \dfrac{780 \times 0.02 \times 1000 \times 6}{60 \times 6}$

$E_g = 260$ V //

4. A 4-pole DC generator has an armature with 60 slots each carrying 24 lap connected conductors. If the machine runs at 1000 rpm and generates 432 V what is the useful flux per pole If the armature is wave connected other condition remaining same what would be the emf generated.

given data

No. of poles $P = 4$

No. of slots $= 60$

conductor / slot $= 24$

Total No. of conductors Z = 1440

Emf generated $E_g = 432\,V$

$A = P = 4$

useful flux per pole ϕ = ?

Emf generated when wave is connected = ?

$$E_g = \frac{\phi Z N P}{60 A}$$

$$\phi = \frac{60 A \times E_g}{Z N P}$$

$$= \frac{60 \times 4 \times 432}{1440 \times 1000 \times 4} = 0.018\ wb$$

when wave connected

$$E_g = \frac{\phi Z N P}{60 A}$$

Here A = 2

$$E_g = \frac{0.018 \times 4 \times 1000 \times 1440}{60 \times 2}$$

$= 864\ V$

5. A 4 pole dc generator runs at 750 rpm and generates an emf of 240V. The armature is wave wound and has 792 conductors. If the total flux per pole is 0.0145 wb what is the leakage coefficient?

sol:- given data

No. of poles = 4

No of conductors Z = 792

E = 240 V

N = 750 rpm

A = 2

$\phi_t = 0.0145$ wb

leakage coefficient = ?

$$E_g = \frac{\phi Z N P}{60 A}$$

$$\text{useful flux } \phi = \frac{60 A \times E_g}{Z N P}$$

$$= \frac{240 \times 60 \times 2}{792 \times 750 \times 4}$$

$$= 0.01212 \text{ wb}$$

$$\text{leakage co-efficient} = \frac{\phi_t}{\phi} = \frac{0.0145}{0.01212} = 1.196$$

6. A 4-pole lap wound generator has 56 coils with 6 turns per coil. The speed is 1150 rpm. what must be the flux per pole in order to generate 265 V ? How many commutator bars are required for this generator.

sol: given data

P = 4

N = 1150 rpm

A = 4

Z = 56 × 6 × 2 = 672

E = 265 V

$\phi = \frac{[illegible]}{ZNP} = \frac{[illegible]}{1150 \times 672 \times 4} = 0.0205 \text{ wb}$

No. of commutator boxs = No. of cells = 56

7. The armature of a 6 pole DC generator has a wave wdg containing 664 conductors. calculate the generated emf when the flux per pole is 0.06 weber and the speed is 250 rpm. At what speed must the armature be driven to generate an emf of 250V It the flux per pole is reduced to 0.058 wb

given data

$P = 6$

$Z = 664$

$N_1 = 250 \text{ rpm}$

$\phi_1 = 0.06 \text{ wb}$

$A = 2$

$N_2 = ?$

$$E_{g_1} = \frac{\phi Z N_1 P}{60 A} = \frac{0.06 \times 664 \times 250 \times 6}{60 \times 2}$$

$= 498 \text{ V}$

given that

$E_{g_2} = 250 \text{V}$

$\phi_2 = 0.058 \text{ wb}$

$$\frac{E_1}{\phi_1 N_1} = \frac{E_2}{\phi_2 N_2}$$

Speed $N_2 = \frac{E_2 \times \phi_1 N_1}{\phi_2 E_1}$

$= \frac{250 \times 0.06 \times 250}{0.058 \times 498}$

$N_2 = 129.83$ rpm.

$E_2 = \frac{\phi_2 Z N_2 P}{60 A}$

$250 = \frac{0.058 \times 664 \times N_2 \times 6}{60 \times 2}$

$N_2 = \frac{250 \times 60 \times 2}{0.058 \times 664 \times 6}$

$N_2 = 129.83$ rpm

Types of D.C generators:

The magnetic field required for the operation of a d.c generator is produced by an electromagnet. this electromagnet carries a field winding which produces required magnetic flux when current is passed through it.

Thus supplying current to the field winding is called "excitation" and the way of supplying the exciting current is called "method of excitation".

There are two methods of excitation used for dc generator

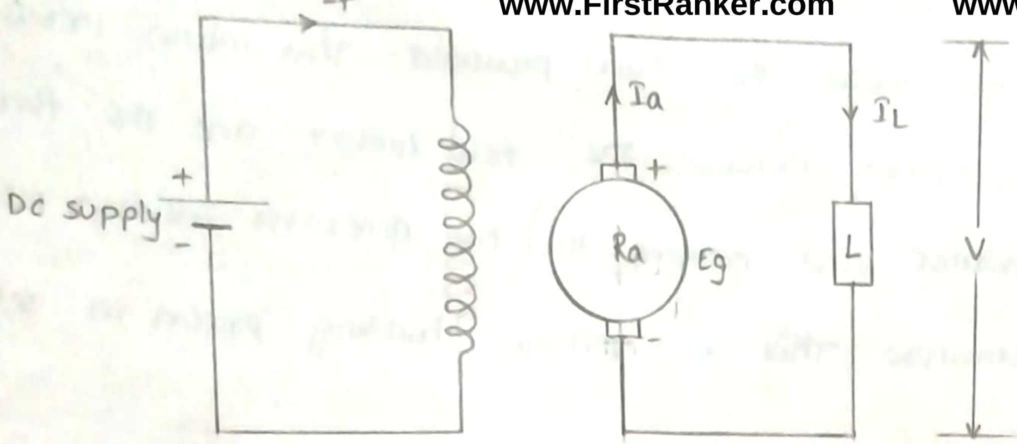
1. separate excitation
2. self excitation

Depending on the method of excitation used, the dc generators are classified as,

1. separately excited generator
2. self excited generator.

1. separately excited generator:

when the field winding is supplied from external, separate dc supply then the generator is called separately excited generator. Schematic representation of this type is shown in figure.



let I_a = current flowing through the armature in amperes

I_L = current flowing through the load in amperes

V = terminal voltage or voltage across the load in volts

E_g = generated emf in volts

R_a = armature resistance in ohms

V_b = voltage drop across brushes

let $I_a = I_L = I$

Terminal voltage $V = E_g - I_a R_a - V_b$

power developed $P_g = E_g I_a = E_g I$

power delivered to external load $P_L = V I_L = VI$

Self-Excited DC generators:

"when the field winding is supplied from the armature of the generator itself then it is said to be self excited generator."

pratically though the generator is not working, without any current through field winding, the field poles posses some magnetic flux. This is called "residual flux" and the property is called residual magnetism

~~Thus when the generator is started~~ due to such residual flux, it develops

...ual emf which now drives a small current through the field winding. This tends to increase the flux produced. This inturn increases the induced emf. This further increases the field current and the flux. The process is cumulative and continues till the generator develops rated voltage across its armature. This is voltage building process in self excited generators.

Based on how field winding is connected to the armature to derive its excitation, this type is further divided into following three types:

(i) shunt generator

(ii) series generator

(iii) compound generator.

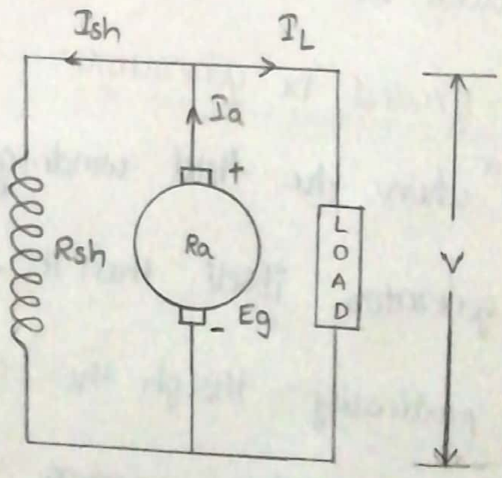
(i) shunt generator:

when the field winding is connected in parallel with the armature and the combination across the load then the generator is called shunt generator. The field winding has large number of turns of thin wire so it has high resistance.

Let Rsh be the resistance of the field winding

from the figure

$I_a = I_L + I_{sh}$ ... amperes



Now voltage across load is V which is same across field winding as both are in parallel with each other

$\therefore I_{sh} = \frac{V}{R_{sh}}$ .... amperes

Terminal voltage $V = E_g - I_a R_a - V_b$

In practice brush contact drop can be neglected

power developed $P_g = E_g I_a$

power delivered $P_d = V I_L$

Series Generator:

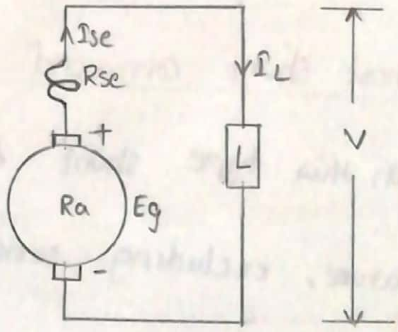
"when the field winding is connected in series with the armature winding while supplying the load then the generator is called series generator." the resistance of series field winding is very small and hence naturally it has less no. of turns of thick cross section wire

Let R_{se} be the resistance of the series field winding

I_{se} be the current flowing through the field winding

As all armature field and load are in series they carry the same current

$\therefore I_a = I_{se} = I_L$



terminal voltage $V = E_g - I_a R_a - I_{se} R_{se} - V_b$

$V = E_g - I_a (R_a + R_{se}) - V_b$

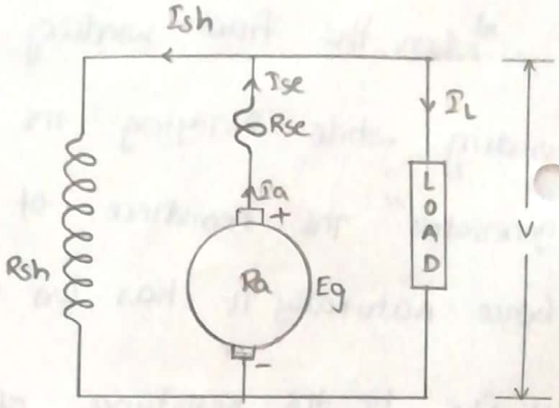
Compound generator:

In this type the part of the field winding is connected in parallel with armature and part in series with the armature both series and shunt field windings are mounted on the same poles. Depending upon the connection of shunt and series field winding compound generator is further classified as

a) long shunt compound generator
b) short shunt compound generator

a) Long shunt compound generator:

In this type shunt field winding is connected across the series combination of armature and series field winding as shown in figure.



from the figure

$I_a = I_{se}$

and $I_a = I_{sh} + I_L$ ---- A

voltage across shunt fied winding is V

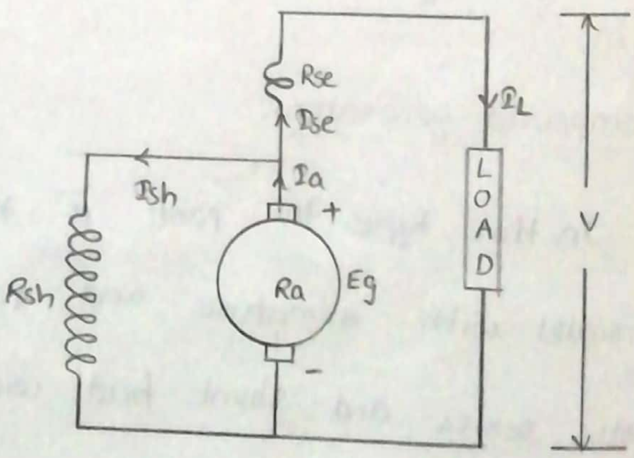
$I_{sh} = \frac{V}{R_{sh}}$ --- A

Terminal voltage

$V = E_g - I_a(R_a + R_{se}) - V_b$ --- volts

b) short shunt compound generator:

In this type shunt field winding is connected only across the armature, excluding series field winding as shown in figure.



from the figure

$I_{se} = I_L$

$I_a = I_{se} + I_{sh}$ -- amperes

(or)

$I_a = I_L + I_{sh}$ --- amperes

The drop across shunt field winding is drop across the armature.

so drop across shunt field winding $= E - I_a R_a$

$\therefore I_{sh} = \frac{E_g - I_a R_a}{R_{sh}}$ . . . amperes —— ①

Terminal voltage

$V = E_g - I_a R_a - I_{se} R_{se} - V_b$ . . . volts

, neglect V_b then

$V = E_g - I_a R_a - I_{se} R_{se}$

$E_g - I_a R_a = V + I_{se} R_{se}$

From equation ①

$E_g - I_a R_a = I_{sh} R_{sh}$

$\therefore I_{sh} R_{sh} = V + I_{se} R_{se}$

$I_{sh} = \frac{V + I_{se} R_{se}}{R_{sh}}$ . . . . . amperes

(or)

$I_{sh} = \frac{V + I_L R_{se}}{R_{sh}}$ $(\because I_L = I_{se})$

Any of the two above expressions of I_{sh} can be used, depending on the quantities known while solving the problems.

Cumulative and differential compound generator:

The two windings shunt and series field are wound on the same poles. depending on the direction of winding on the pole, two fluxes produced by shunt and series field may help or may oppose each other. This fact decides whether generator is cumulative or [illegible] If the two fluxes help each other as shown in figure (a) the generator is called cumulative compound generator.

$$\phi_T = \phi_{sh} + \phi_{se}$$

where ϕ_{sh} = flux produced by shunt

ϕ_{se} = flux produced by series, field windings

If the two windings are wound in such a direction that the fluxes produced by them oppose each other then the generator is called differential compound generator. This is shown in fig (b).

$$\phi_T = \phi_{sh} - \phi_{se}$$

where ϕ_{sh} = flux produced by shunt field winding

ϕ_{se} = flux produced by series field winding.

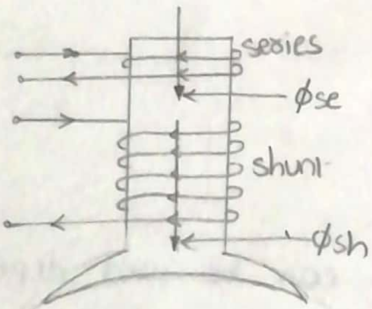


fig (a)

cumulative compound generator

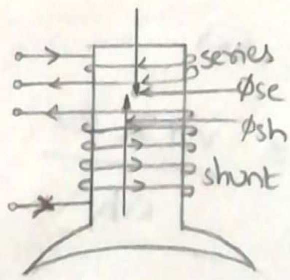


fig (b)

differential compound generator

problems:

1. A separately excited dc generator when running at 1200 rpm supplies a current of 200A at 125 volts to a circuit of constant resistance. What will be the current when speed drops to 1000 rpm if the field current is unaltered armature resistance is 0.04Ω

the brush voltage drop at the brushes is 2 volts. ignore the change in armature reaction.

Sol: given data

speed $N_1 = 1200$ rpm

$I_{L_1} = 200$ A

$V = 125$ V

speed $N_2 = 1000$ rpm

armature resistance $R_a = 0.04\,\Omega$

Voltage drop at brushes $V_b = 2$ V

current $I_{L_2} = ?$

In separately excited dc generator $I_a = I_L$

$\therefore I_{a_1} = I_{L_1} = 200$ A

$V = E_g - I_a R_a - V_b$ ... volts

$E_g = V + I_a R_a + V_b$ ... volts

$E_{g_1} = 125 + 200 \times 0.04 + 2$

$E_{g_1} = 135$ volts //

we know that $E_g = \frac{\phi Z N P}{60 A}$

from this $E_g \propto N$

$\therefore \frac{E_{g_1}}{E_{g_2}} = \frac{N_1}{N_2}$

$\Rightarrow \frac{135}{E_{g_2}} = \frac{1200}{1000} = 1.2$

$E_{g_2} = \frac{135}{1.2} = 112.5$ volts

$E_{g_2} = V_2 + I_{a_2} R_a + V_b$

$112.5 = V_2 + 0.04\, I_{a_2} + 2$

$V_2 = 110.5 - 0.04\, I_{a_2}$ . . . volts

Load resistance $R_L = \frac{V}{I_{a_1}} = \frac{125}{200} = 0.625\ \Omega$

Load current $I_{a_2} = I_{L_2} = \frac{V_2}{R_L} = \frac{110.5 - 0.04\, I_{a_2}}{0.625}$

$0.625\, I_{a_2} = 110.5 - 0.04\, I_{a_2}$

$0.625\, I_{a_2} + 0.04\, I_{a_2} = 110.5$

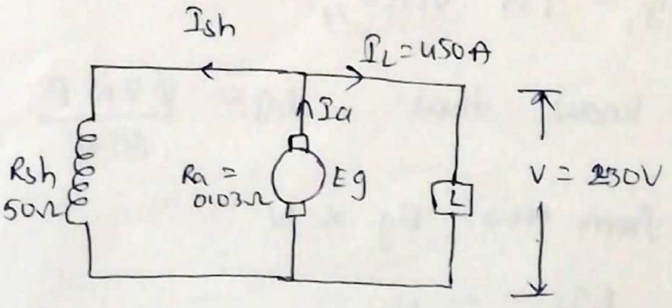
~~0.665~~ I_{a_2} = ~~110.5~~ $0.665\, I_{a_2} = 110.5$

$I_{a_2} = \frac{110.5}{0.665} = 166.16\ A$

$\therefore\ I_{a_2} = I_{L_2} = 166.16\ A$

2. A shunt generator delivers 450 A at 230V and the resistances of the shunt field and armature are 50Ω and 0.03Ω respectively calculate the generated emf.

given data



$V = 230V$

$I_L = 450A$

$R_{sh} = 50\ \Omega$

$R_a = 0.03\ \Omega$

$E_g = ?$

$E_g = V + I_a R_a$

$I_a = I_L + I_{sh}$

$I_{sh} = \frac{V}{R_{sh}}$

$I_{sh} = \frac{230}{50} = 4.6$ A

$I_a = I_L + I_{sh} = 450 + 4.6 = 454.6$ A

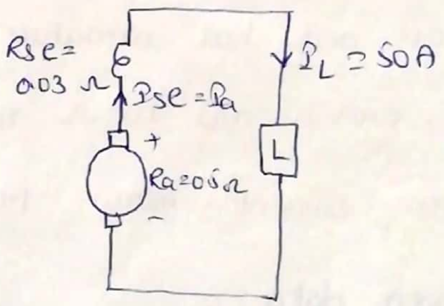
$E_g = V + I_a R_a$

$= 230 + 454.6 \times 0.03$

$E_g = 243.638$ V //

3. A dc series generator has armature resistance of 0.5Ω and series field resistance of 0.03Ω. It drives a load of 50A. If it has 6 turns/coil and total 540 coils on the armature and is driven at 1500 rpm calculate the terminal voltage at the load. Assume 4 poles, lap type winding, flux per pole as 2 mWb and total brush drop as 2V

Sol: given data



$R_a = 0.5\,\Omega$

$R_{se} = 0.03\,\Omega$

$I_L = 50A$

$N = 1500$ rpm

$P = 4$

$A = P = 4$ for lap winding

$\phi = 2 \times 10^{-3}$ Wb

$V_b = 2V$

$V = ?$

[1 turn = 2 conductors]

Total coils are 540 with 6 turns/coil

Total turns = 540 × 6 = 3240

∴ Total conductors Z = 2 × turns

= 2 × 3240 = 6480

Generated emf $E_g = \frac{\phi Z N P}{60 A}$

$$= \frac{2 \times 10^{-3} \times 6480 \times 1500 \times 4}{60 \times 4}$$

$= 324 V$

for series generator:

we know that $E_g = V + I_a(R_a + R_{se}) + V_b$ .... volts

and $I_a = I_{se} = I_L = 50 A$

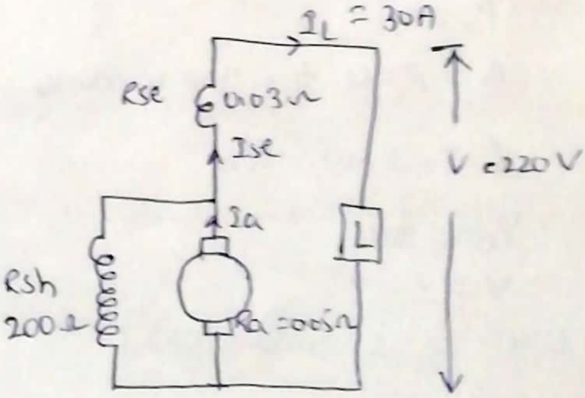
$\therefore V = E_g - I_a(R_a + R_{se}) - V_b$

$= 324 - 50(0.5 + 0.03) - 2$

$V = 295.5 V$

4. A short shunt compound dc generator delivers a load current of 30A at 220V and has armature, series field and shunt field resistances of 0.05 Ω, 0.03 Ω and 200 Ω respectively. calculate the induced emf and the armature current. Allow 1.0V per brush for contact drop.

sol: given data



$I_L = 30 A$

$V = 220 V$

$R_a = 0.05 Ω$

$R_{se} = 0.03 Ω$

$R_{sh} = 200 Ω$

$V_b = 1 \times 2 = 2 V$

($\because V_b = 1$ volt per brush so for two brushes $V_b = 2 \times 1 = 2V$)

In short shunt compound generator

$I_L = I_{se} = 30A$

for long shunt field $V_{sh} = E - I_a R_a - V_b$ (or) $V + I_L R_{se}$

$$V_{sh} = V + I_L R_{se}$$
$$= 220 + 30 \times 0.03$$
$$= 220.9 V$$

we know that $I_{sh} = \frac{V_{sh}}{R_{sh}} = \frac{220.9}{200} = 1.1045 A$

$I_a = I_{sh} + I_L = 1.1045 + 30 = 31.1045 A$

$V = E_g - I_a R_a - I_{se} R_{se} - V_b$

$\Rightarrow E_g = V + I_a R_a + I_{se} R_{se} + V_b$

$= 220 + 31.1045 \times 0.05 + 30 \times 0.03 + 2$

$= 224.455 V$ //

5. A 4-pole long shunt lap wound generator supplies 25kW at a terminal voltage of 500V. The armature resistance is 0.03 Ω series field resistance is 0.04 Ω and shunt field resistance is 200 Ω. The brush drop may be taken as 1V. Determine the emf generated.

sol: given data

$P = 4$

$P_L = 25kW$

$V = 500V$

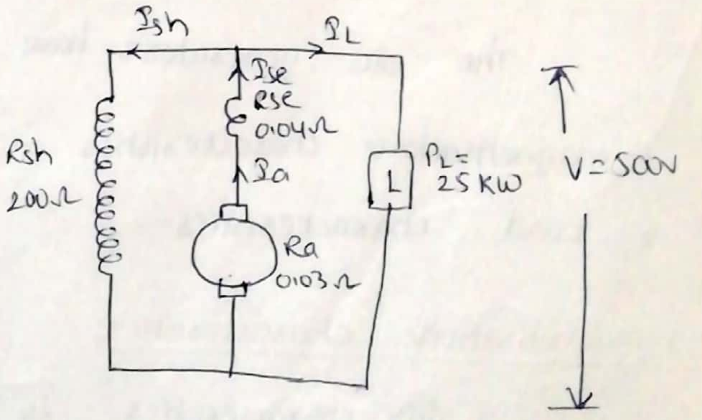
$R_a = 0.03 \Omega$

$R_{se} = 0.04 \Omega$

$R_{sh} = 200 \Omega$

$V_b = 1V$ for brush $\Rightarrow$ total $V_b = 2 \times 1 = 2V$

$E_g = ?$



for lap winding $A = P = 4$

$P_L = V I_L$

$I_L = \frac{P_L}{V}$

$I_L = \frac{25 \times 10^3}{500} = 50A$

for long shunt:

$I_a = I_{se} = I_L + I_{sh}$

$I_{sh} = \frac{V}{R_{sh}} = \frac{500}{200} = 2.5A$

$I_a = 50 + 2.5 = 52.5 A$

$V = E_g - I_a(R_a + R_{se}) - V_b$

$E_g = V + I_a(R_a + R_{se}) + V_b$

$= 500 + 52.5 (0.03 + 0.04) + 2$

$E_g = 505.675 V$ //

characteristics of DC generators:

The dc generators have following characteristics in general

1. magnetisation characteristics
2. Load characteristics.

1. magnetisation characteristics:

This characteristics is the graph of generated no load voltage E against the field current I_f when speed of generator is maintained constant. As it is plotted without load with open output terminals it is also called "No load characteristics" or "open circuit characteristics".

E_0 Vs I_f is magnetisation characteristics

where

E_0 = no load induced emf

I_f = field current

But for generator

$$E = \frac{\phi P N Z}{60 A}$$

$E \propto \phi$ with $\frac{PNZ}{60A}$ constant

$E \propto \phi$

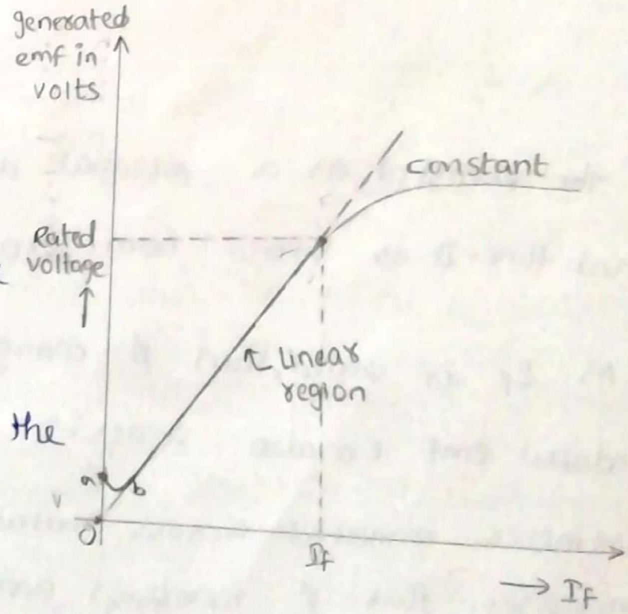
$\phi \propto I_f$

$E \propto I_f \uparrow$

$\boxed{E \propto I_f}$ as $\phi \propto I_f$

Thus induced emf increases directly as I_f increases. But after certain I_f core gets saturated and flux ϕ also remains constant though I_f increases. Hence after saturation voltage also remains constant. This is shown in figure.

The curve starts from point 'a' instead of 'o' when the field current is zero which is due to residual magnetism. The curve from point b to point c is practically a straight line. At point c saturation of the magnetic circuit begins.



Load characteristics:

These are further divided into two categories

1) External characteristics
2) Internal characteristics

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characteristics is the graph of the terminal voltage V against load current I_L.

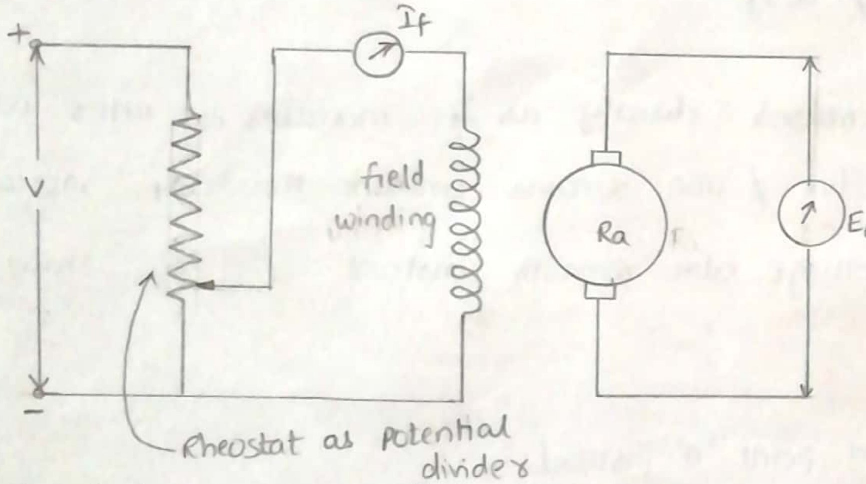
The internal characteristics is the graph of the generated induced emf E_g against the armature current I_a.

Note: while plotting both the characteristics, the speed N of the generator maintained constant.

characteristics of separately Excited DC generators:

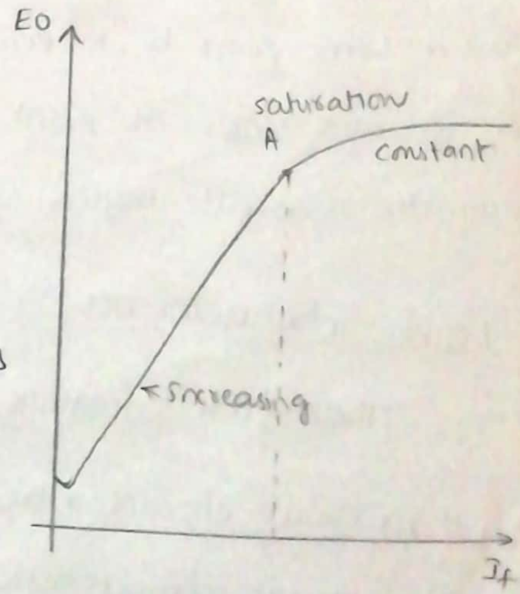
1. magnetisation or open circuit or no load characteristics:

The arrangement to obtain this characteristics is shown in figure



The rheostat as a potential divider is used to control the field current and flux. It is varied from zero and is measured on ammeter connected

As I_f is varied, then ϕ changes and hence induced emf E_0 also varies. It is measured on voltmeter connected across armature. As I_f increases flux ϕ increases and E_0 increases. After point A saturation occurs when ϕ becomes constant and hence E_0 saturates.



open circuit characteristics

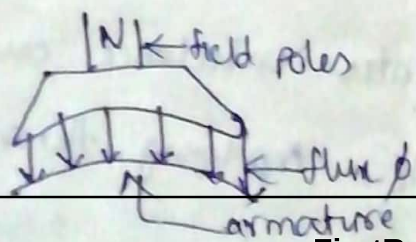
DC Motor:

It is a device which converts electrical energy into Mechanical energy.

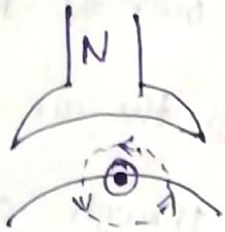
working of DC motor:

The field poles are of electromagnetic type placed inside the yoke or frame and distributed around the periphery of the armature conductors. The field poles when gets excited by the field current produce uniform magnetic field over the pole face completing its circuit from North pole through the air gap to armature and then to south pole again through air gap. for the purpose of understanding the principle consider a single pole with the armature conductors around the periphery of the armature and under pole in three stages.

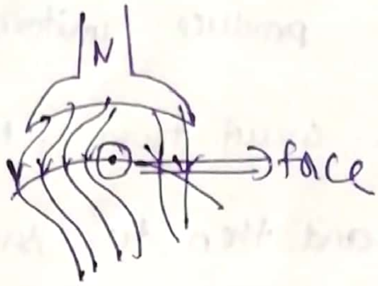
In stage one the pole is excited due to which it produces a uniform magnetic field of ϕ webers. for the sake of convenience a salient north pole is taken. The flux leaves the north pole and enters the armature through the airgap, the conductor in the armature remaining stationary. as shown in fig (a)



conductor is shown with a current in it. The directions of current is shown in ... the cross section of the conductor showing that the current is flowing from the inside of the conductor towards the dot. This current produces a magnetic flux around it and the direction of magnetic flux can be found out by the holding the conductor with right hand. The direction of magnetic field in case of the conductor is anti clock wise.



(b)



(c)

Stage three gives the combined effect of magnetic flux due to main pole and current carrying armature conductor, The flux towards the left of the conductor is aiding the main flux while the direction of flux towards right oppose each other. The resultant flux pattern is shown in fig (c). It is evident from the figure that the magnetic flux of the main pole get strecthed as an elastic band towards the left of the conductor to complete the magnetic circuit while towards the right the flux density is least. This creates unbalance condition, and the conductor is positioned unstable. Applying flemings left hand rule.

The direction of force ... on the conductor ... right since the conductor is free to move about the axle. it moves towards right and in doing so the conductor adjacent to it towards left occupies the position of the first conductor. It also experiences the same effect of the previous conductor and get pushed towards right giving way to the third conductor. As the conductor moves the shaft also rotates since whole of the periphery of the armature is full of conductors, the force on the conductors under the pole is continuous as long as current exists in the conductor. A continuous torque is therefore developed which produces mechanical rotation.

Significance of Back emf:

As soon as the armature starts rotating inside a magnetic field due to the current in the conductor the following conditions are set in

a) the conductor is in motion

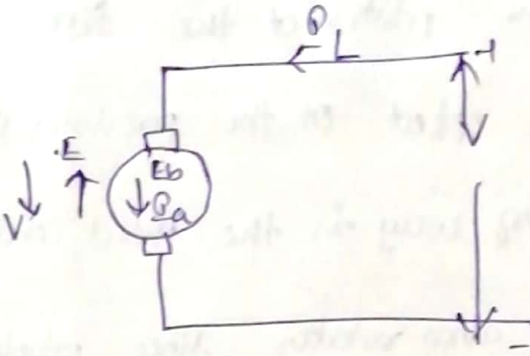
b) The flux of the main pole exists

c) while the conductor is in motion, it also cuts the main pole flux. this condition creates a situation for inducing emf in the conductor. The direction of induced emf as found by the Flemings right hand rule. as in direct opposition to the applied voltage across the armature conductors. It is termed as back emf (E_b).

The magnitude of back emf is given by

$$E_b = \frac{\phi 2NP}{60A}$$

This back emf acts in opposite direction to the applied voltage in the armature as shown in figure.



Net voltage across armature $= (V - E_b)$

If the resistance of the armature is R_a the armature current is given by

$$I_a = \frac{V - E_b}{R_a} \text{ .. amps}$$

$$I_a R_a = V - E_b$$

$$\Rightarrow V = E_b + I_a R_a$$

$$P = V I_a = E_b I_a + I_a^2 R_a$$

The gross mechanical power developed by the motor is

$$P_m = V I_a - I_a^2 R_a.$$

$$\frac{dP_m}{dI_a} = V - 2 I_a R_a = 0$$

∴ The gross mechanical power developed by the dc motor will be maximum when the back emf is equal to the half of the applied voltage.

Classification of DC motor:

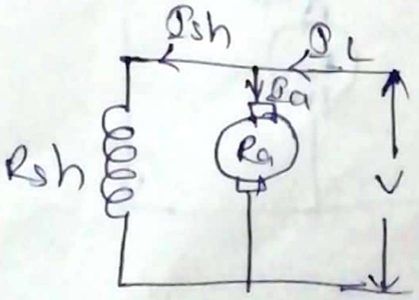
- DC motor
  - separately excited DC motor
  - self excited DC motor
    - DC shunt motor
    - DC series motor
    - DC compound motor
      - Long shunt DC compund motor
      - short shunt DC compound motor

DC shunt motor:

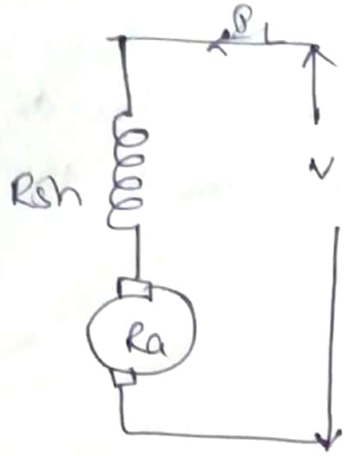
$I_a = I_L - I_{sh}$

$I_{sh} = \frac{V}{R_{sh}}$

$E_b = V - I_a R_a$



DC series motor:



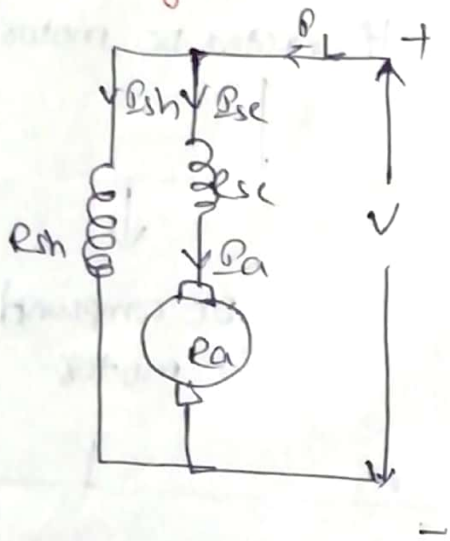
$$V = E_b + I_a R_a + I_{se} R_{se}$$

$$I_L = I_a = I_{se}$$

$$V = E_b + I_a (R_a + R_{se})$$

DC compound motor:

a) long shunt:



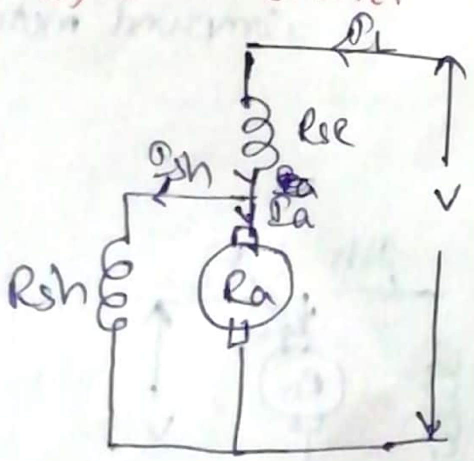
$$V = E_b + I_a R_a + I_{se} R_{se}$$

$$I_a = I_{se}$$

$$V = E_b + I_a (R_a + R_{se})$$

$$I_L = I_{sh} + I_a$$

b) short shunt:



$$V = E_b + I_a R_a + I_{se} R_{se}$$

$$I_L = I_{se} = I_a + I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{E_b - I_a R_a}{R_{sh}}$$

The torque developed by the armature $= T_a 2\pi N$ .. watts

The mechanical power developed in the armature $= E_b I_a$ .. watts

But $E_b I_a = \frac{2\pi N}{60} T_a$ where N is the rpm and $\frac{N_{rpm}}{60} = N_{rps}$

$$E_b = \frac{\phi Z N P}{60 A}$$

$$\frac{\phi Z N P}{60 A} \times I_a = T_a \frac{2\pi N}{60}$$

$$T_a = \frac{\phi Z N P}{60 A} \times \frac{60 I_a}{2\pi N} = \frac{\phi Z I_a P}{60 A}$$

$$= \frac{1}{2\pi} \phi Z I_a \frac{P}{A}$$

$$= 0.159 \phi Z I_a \frac{P}{A} \text{ kg-m}$$

$$= 0.0162 \phi Z I_a \times (P/A) \text{ .. kg.m}$$

this is also known as the gross torque. The torque developed at the shaft is the useful torque and is known as shaft torque.

$$BHP = \frac{T_{sh} \times 2\pi N}{735.5}$$ where N is in r.p.s

$$T_{sh} = \frac{(BHP)_m \times 735.5}{2\pi N}$$

The motor output is given by $T_{sh} \times 2\pi N$ watt

$$T_{sh} = \frac{\text{output in watts}}{2\pi N}$$

$$= \frac{60}{2\pi} \times \frac{\text{output}}{N} = 9.55 \times \frac{o/p}{N} \quad N \cdot m$$

where N is in rps.

The difference b/w armature torque and shaft torque is the lost torque.

$$\text{Torque lost} = 0.159 \times \frac{\text{iron and frictional loss}}{N}$$

Applications:

1) shunt motor
- → milling and lathe machine
- → electric fan
- → blowers
- → pump

2) series motor
- → Conveyor pump, grinder
- → Elevators
- → Small crane and hoist
- → Electric traction.

3) compound motor
- → flour mill conveyors
- → milling work
- → Rubber, paper mills and rolling mills

Necessity of speed control of dc motor:

In many industrial applications like crane, hoist and electric traction etc, the speed has to be varied smoothly either in the direction of acceleration or deceleration. Besides it should also be possible to have the speed constant at any desired value.

$$E_b = V - I_a R_a = \frac{\phi Z N P}{60 A} = K_1 \phi N$$

$$N = \frac{V - I_a R_a}{K_1 \phi},$$

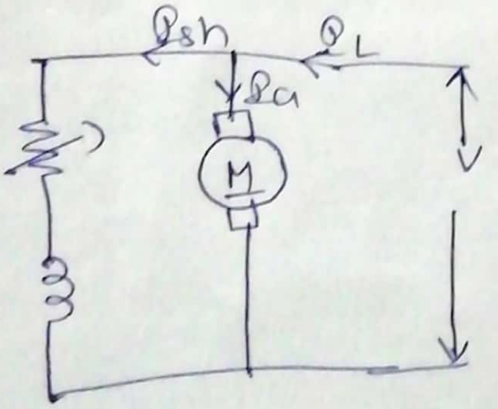
The speed can be varied by varying

a) V the terminal voltage or

b) I_a the armature current or

c) ϕ the shunt field flux

Speed control by flux variation:



$$N = \frac{V - I_a R_a}{K_1 \phi}$$

$$N \propto \frac{1}{\phi}$$

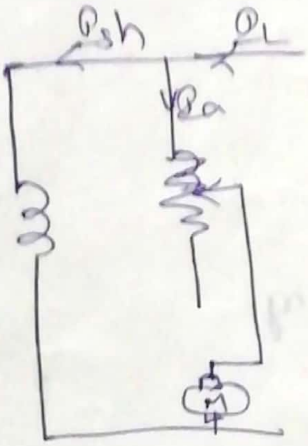
If flux decreases speed increases and vice versa. The flux of a DC shunt motor can be changed by changing I_{sh} with the help of shunt field rheostat

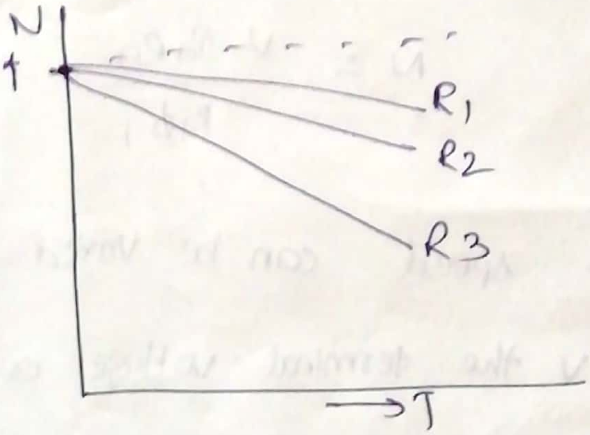
Disadvantages:

a) poor commutation at high speeds

b) Armature gets overheated.

Speed control by varying armature current:





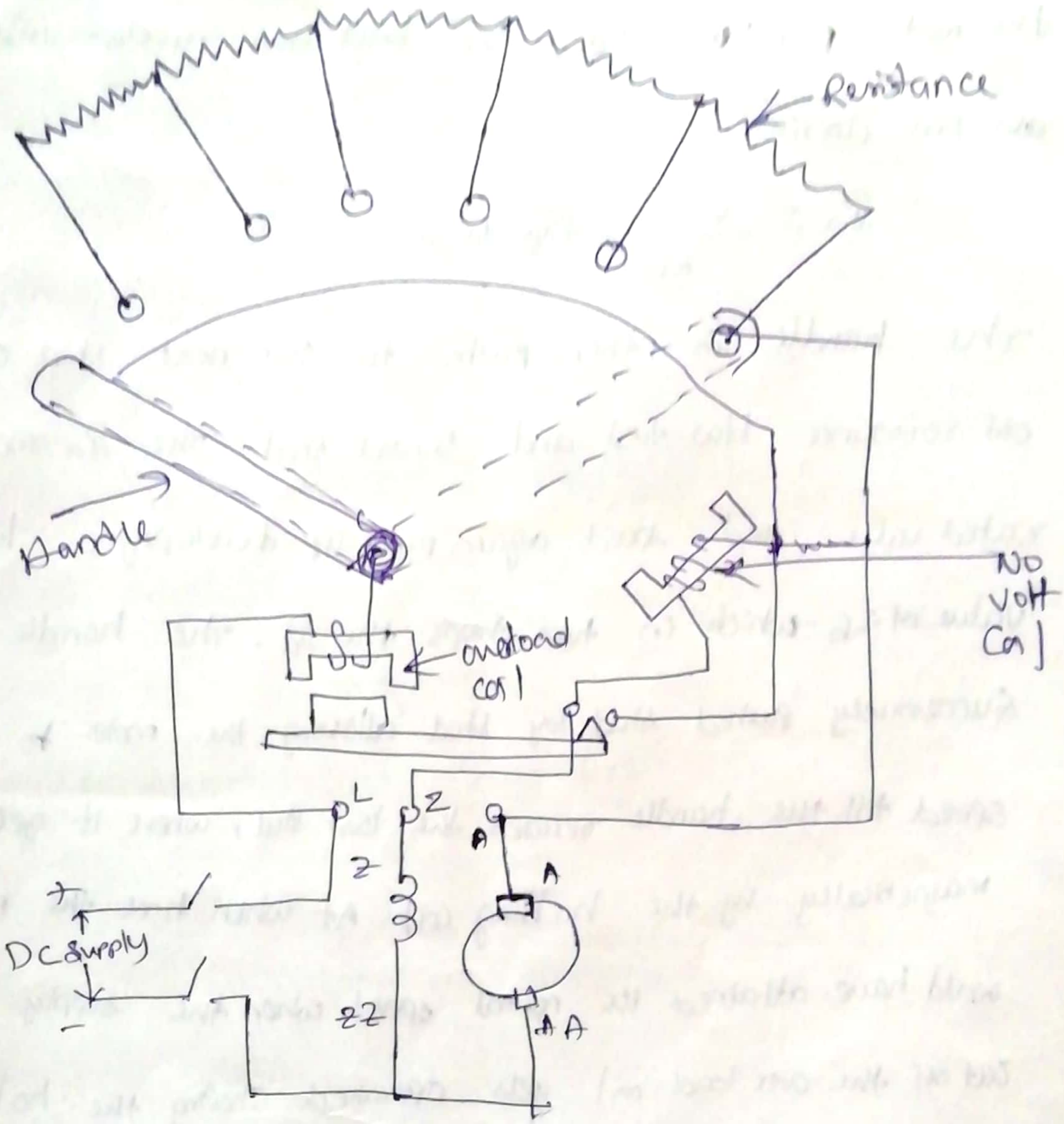
$$I_a = \frac{V - E_b}{R_a}$$

$$I_a R_a = V - E_b$$

$$= V - K_1 N$$

$$N = \frac{V - I_a R_a}{K_1}$$

Three point starter:



At the time of starting the handle is pushed to the first stud, whereby the total resistance from the first to last stud becomes active and comes in series to the armature limiting the current to its rated value. The handle also touches a copper strip in all the positions and full supply voltage is made available to the shunt field. As the motor picks up speed, the back emf develops and the armature current

$$I_a = \frac{V - E_b}{R_a}$$ drops down.

The handle is then pushed to the next stud cutting off resistance b/w first and second stud. The I_a rises to its rated value and speed again picks up developing a higher value of E_b which in turn drops the I_a. The handle is successively pushed stud by stud allowing the motor to pick up speed till the handle reaches the last stud, where it gets locked magnetically by the holding coil. At which time the motor would have attained its normal speed when the supply is cut off the over load coil gets operated shorting the holding coil ends which in turn releases the handle. The handle comes back to its initial starting position with the help of helical spring.

Transformer: A transformer is a static machine which transfers the electrical power from one circuit to another with the desired change in voltage or current and without any change in frequency.

A transformer used to increase the voltage is called a "step up transformer", while that used to decrease the voltage is called a "step down transformer".

Working principle of a transformer:

A transformer works on the principle of electromagnetic induction and mutual inductance between two coils. The elementary transformer is shown in figure. It consists of two windings electrically separated but linked by a common magnetic circuit of low reluctance formed by a laminated soft iron core. The winding which is connected to the supply is known as "primary winding" and the winding connected to the load circuit is called "secondary winding".

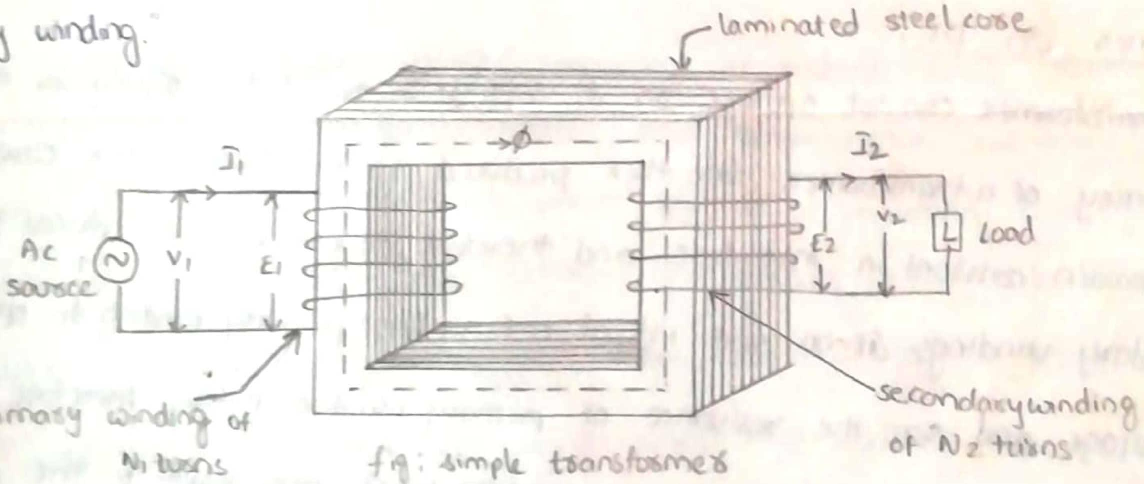


fig: simple transformer

When the primary winding is connected to an ac supply mains a current flows through it. Since this winding links with an iron core so current flowing through this winding produces an alternating flux ϕ in the core. Since this flux is alternating and links with the secondary winding also, so induces an emf in the secondary winding. The frequency of induced emf in secondary winding is the same as that of the flux or that the supply voltage. The induced emf in the secondary winding enables it to deliver current to an external load connected across it. Thus the energy is transformed from primary winding to the secondary winding by means of electro magnetic induction without any change in frequency.

The whole of the flux of the iron core links not only with the secondary winding but also with the primary winding so produces self induced emf in the primary winding called
This induced emf in the primary winding opposes the applied voltage and therefore it limits the primary current.

HV and LV windings:

The winding connected to higher voltage circuit is called the high voltage (HV) winding while that connected to the lower voltage circuit is called the low voltage (LV) winding. In case of a step up transformer low voltage winding is the primary and high voltage winding is the secondary while in case of a step down transformer, the high voltage winding is the primary and low voltage winding is the secondary.

Transformer on DC:

A transformer can not operate on dc supply. If rated dc supply is applied to the primary of a transformer the flux produced in the transformer core will not vary but remain constant in magnitude and therefore no emf will be induced in primary and secondary windings. If no self induced emf in the primary winding to oppose the applied voltage and since the resistance of primary winding is low, therefore a heavy current will flow through the primary winding which may result in the damage of the winding. This is the reason that "dc is never applied to a transformer".

Construction of a transformer:

A transformer is a static device and its construction is simple as there are no moving parts. The main components of a transformer are shown in figure (1)

1. core: The core material and its construction should be such that the maximum flux is created with minimum magnetizing current and minimum core loss. core is divided into two parts.

or legs: The vertical position on which windings or coils are wound is called limbs.

Yokes: The top and bottom horizontal positions are called as Yokes of the core, which connect the legs and serve for closing the magnetic circuit

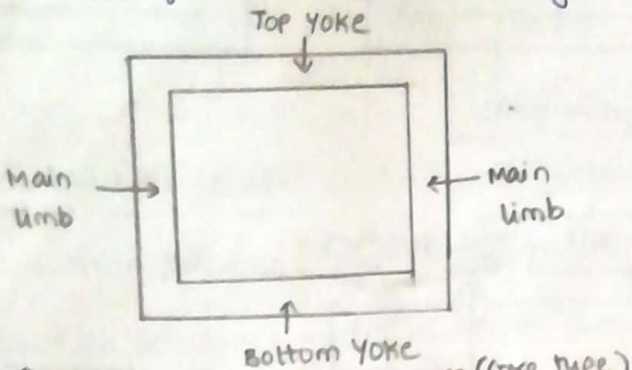


fig 2 (a) 1-ϕ Two limbed core (core type)

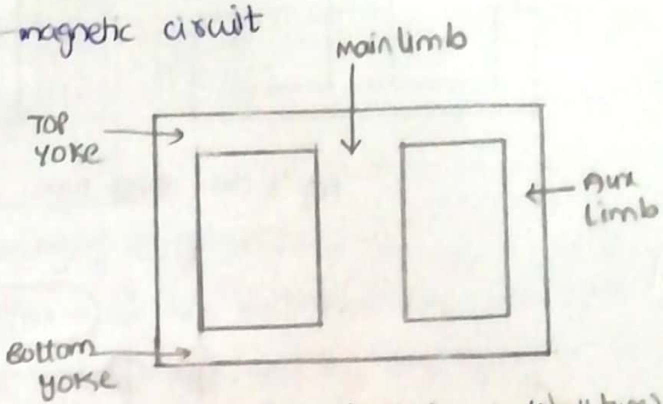


fig 2 (b) 1-ϕ Three limbed core (shell type)

the two limbed and three limbed cores are shown in figure (2)(a) and figure 2(b)

In two limbed core the cross sectional area of the limbs and the Yokes are identical

In three limbed core windings are placed around the central limb also known as main limb.

Core is made up of laminated stampings to reduce eddy current losses. Generally thin sheets of high grade silicon steel laminations are used. Various laminations are insulated from each other by using insulation like varnish

In addition to eddy current loss hysteresis loss occurs in the core. Hysteresis loss depends upon the area of hysteresis loop of the core material. So special silicon steel is used for laminations. this has a small hysteresis loop area and high resistivity to eddy currents. thus the hysteresis loss and eddy current loss will be minimised.

The laminated stampings are of different shapes like L, E, U etc are used which are shown in fig (3) a

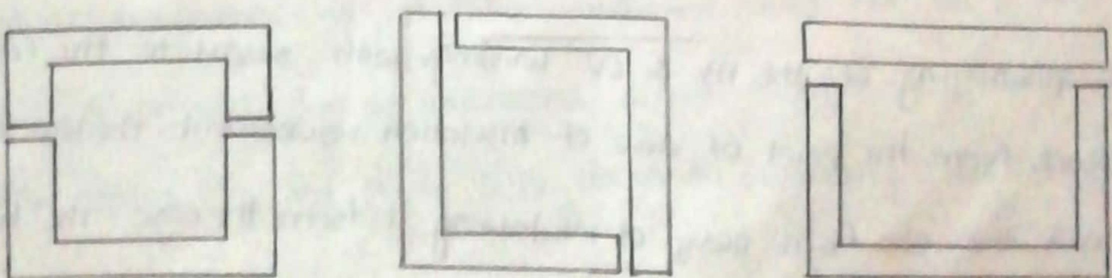


fig 3(a) core type laminations

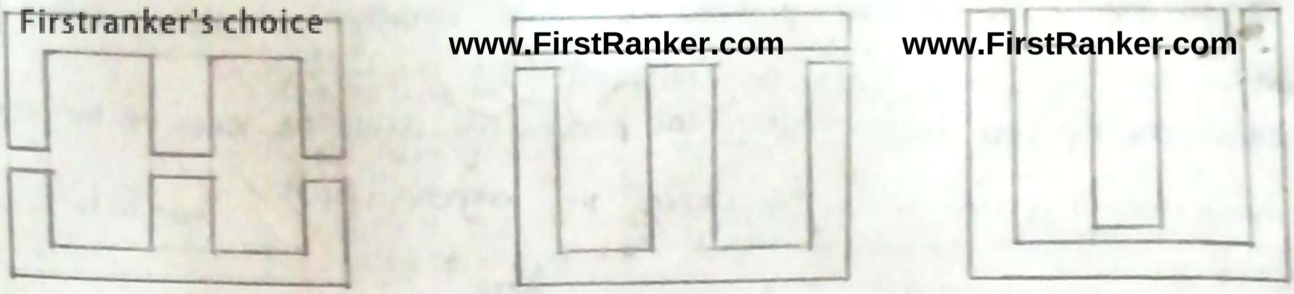


fig 3 (b) shell type laminations

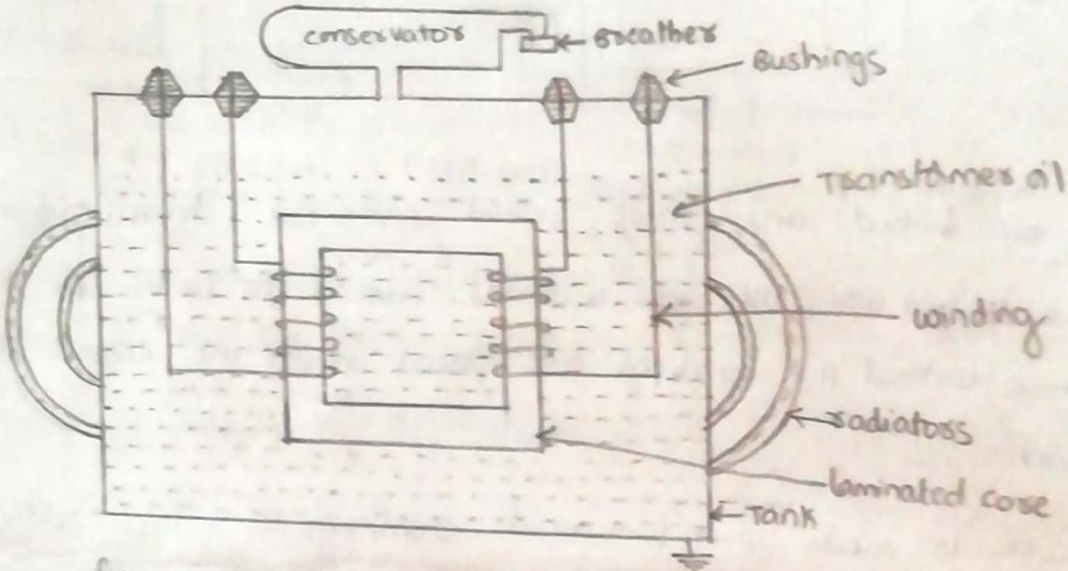


fig 1 constructional details of a transformer

2. Windings or coils:

The transformer has high voltage and low voltage coils for each phase. The coils may be either cylindrical concentric or sandwiched type.

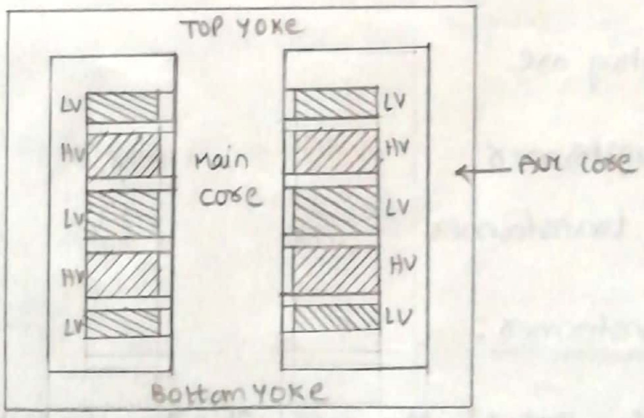
concentric coils: concentric coils are used in core type transformers. In core type transformers two windings are wound on two different limbs due to this leakage flux increases and hence performance of transformer gets affected badly. To reduce this leakage flux, the two windings are wound on the same limb in the transformer. A very common arrangement is cylindrical concentric windings as shown in figure.

The positioning of the HV & LV windings with respect to the core is also very important from the point of view of insulation requirement. The low voltage is placed near the core from easy of insulating it from the core. The high voltage is placed after it.

concentric winding (core type)

Sand witched coils:

Sand witched coils are commonly employed for shell type transformers. The leakage reactance of the windings can be easily controlled by employing sand witched winding. The nearer the high voltage and low voltage coils, the less is the leakage flux leakage can be further reduced by subdividing the HV & LV coils. The high voltage sections lie between two consecutive low voltage sections. The two end sections are LV sections and contain half the turns of other LV sections lower values of reactance can be obtained by increasing the number of subdivisions. The schematic diagram of a sandwitched winding is shown in figure



Sand witched windings(shell type)

3. Conservator tank:

when a transformer is oil filled and self cooled the oil in the tank is heated and will expand due to variations in load current. The conservator tank provides the means for the oil to settle down by expanding under heavy loads.

4. Bushings:

The purpose of the bushings is to provide proper insulation for the

...ing and emerging leads.

5. Breather:

The breather completely prevents the moisture and dust from coming into contact with the oil in the conservator tank when it expands or contracts depending on the variations in the load.

methods of cooling of transformers:

There are three methods which are commonly used for cooling of the transformer windings and the core.

a) natural radiation: This method is used for low voltage transformer.

b) oil filled and self cooled: This method is used for large sized transformers with high ratings.

c) forced cooling with air blast: this method is used for machines with higher ratings.

Types of transformer:

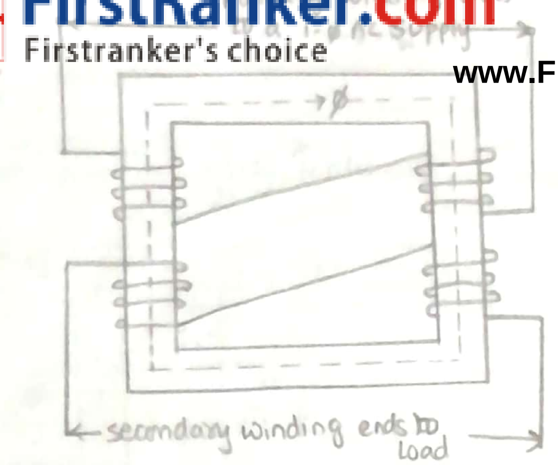
Depending up on the type of ~~transfor~~ construction, the transformers are classified into two types. They are

(i) core type transformer

(ii) shell type transformer.

(i) core type transformer:

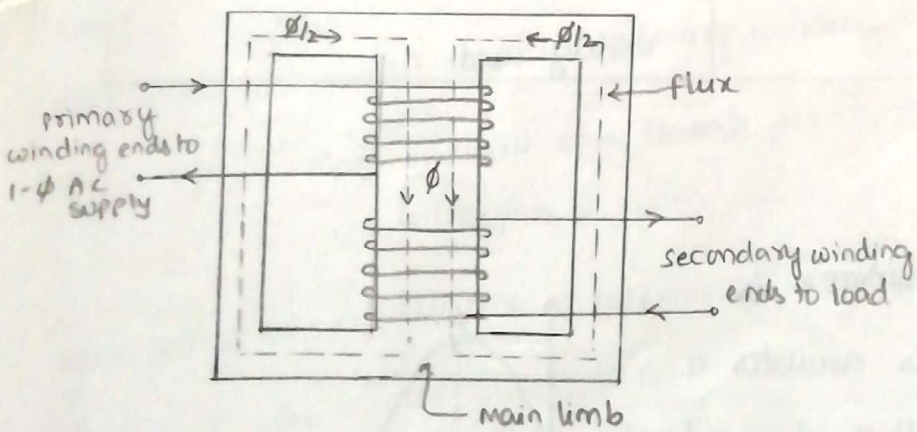
A considerable part of the core of a transformer is surrounded by a coil is known as core type transformer. In this type of transformer, the windings are wound around the two limbs. The flux is same in both the limbs. It has only one magnetic path or circuit. The primary and secondary windings are split into two parts. Half the primary and half the secondary windings are placed side by side concentrically on each limb to reduce the leakage flux. coils used are of cylindrical type such coils are wound in helical layers with different layers insulated from each other by paper, cloth, mica etc



In small size core type transformers a simple rectangular core is used with cylindrical coils which are either circular or rectangular in form. But for large size core type transformers round or circular cylindrical coils are used.

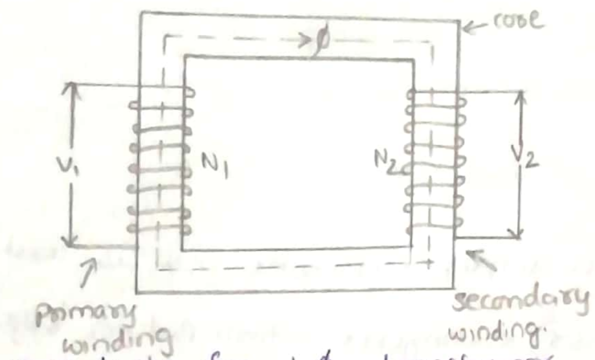
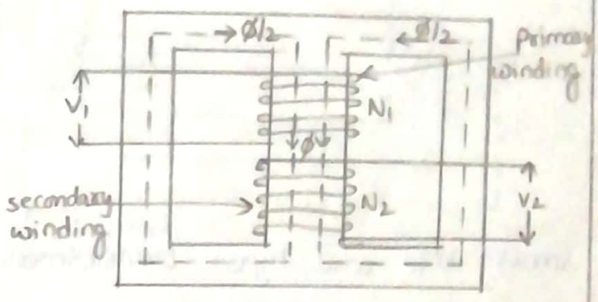
2) Shell type transformer:

A considerable part of the winding of a transformer is surrounded by a core is known as shell type transformer. In this type of transformer the windings are wound on the central limb of a three limbed core. The central limb has flux ϕ while the other two limbs have flux ϕ/2. It has double magnetic circuit. The low voltage winding is wound deep near the core and high voltage winding is done on it. The coils are sandwitch type.



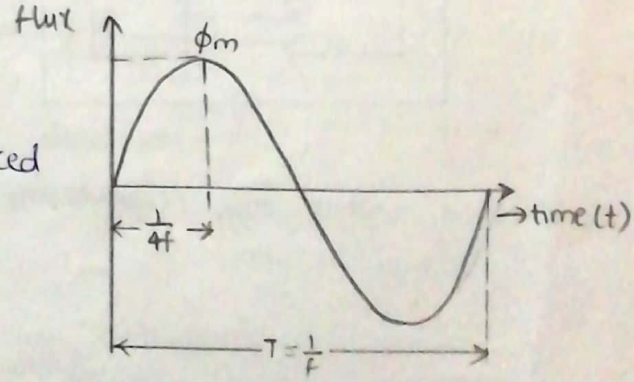
Shell type Transformer

Comparison between core type and shell type transformers:

S.No	core type	shell type
		
1.	Two limbs for 1-ϕ transformer	Three limbs for 1-ϕ transformer
2.	windings are wound around the two limbs	windings are wound on the central limb only
3.	flux ϕ is same in both the limbs	central limb has flux ϕ while the other two limbs have flux $\phi/2$.
4.	single magnetic circuit	Double magnetic circuit
5.	windings surround a considerable part of the core	core surrounds a considerable portion of the windings
6.	cylindrical concentric coils are used	sandwitched or multilayered disc type coils are used.
7.	construction is difficult	Easy construction
8.	Rarely used	widely used.

EMF Equation:

The primary winding of a transformer is excited by alternating voltage. This circulates a current and hence the alternating flux is produced as shown in figure



let ϕ_m = maximum flux in core in webers

$= B_m \times A$

 ... ac input in Hz

N_1 = no of turns in primary

N_2 = no of turns in secondary

from faradays law of electromagnetic induction

Average emf induced per turn = Average rate of change of flux $= \frac{d\phi}{dt}$

As shown in figure flux increases from its zero value to maximum value ϕ_m in one quarter of the cycle ie, in $\frac{1}{4f}$ second.

$\therefore$ Average rate of change of flux $= \frac{\phi_m}{1/4f} = 4f\phi_m$ webers/sec or volts

$\therefore$ Average emf induced per turn $= 4f\phi_m$ volts

Since flux ϕ is varying sinusoidally with time so emf induced will be sinusoidal.

for sinusoidal wave

form factor $= \frac{\text{Rms value}}{\text{Average value}} = 1.11$

$\therefore$ Rms value of induced emf per turn $= 1.11 \times 4f\phi_m = 4.44f\phi_m$ volts

If the number of turns on primary and secondary windings are N_1 and N_2 respectively then

Rms value of induced emf in primary winding

Rms value of induced emf / turn × No of primary turns

E_1 = induced emf / turn × No of primary turns

$= 4.44f\phi_m N_1$ volts

Rms value of induced emf in secondary winding

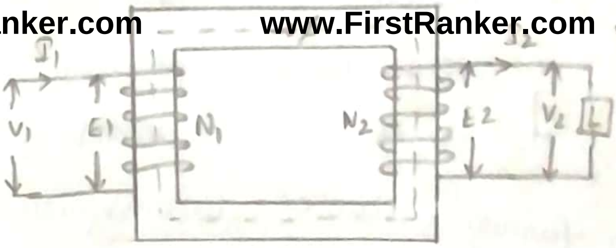
$E_2 = 4.44f\phi_m N_2$ volts

Voltage and current transformation ratios:

Voltage ratio:

Emf induced in primary winding $E_1 = 4.44f\phi_m N_1$ volts —— ①

...nding $E_2 = 4.44 f \phi_m N_2$ volts —②



from equations ① and ②

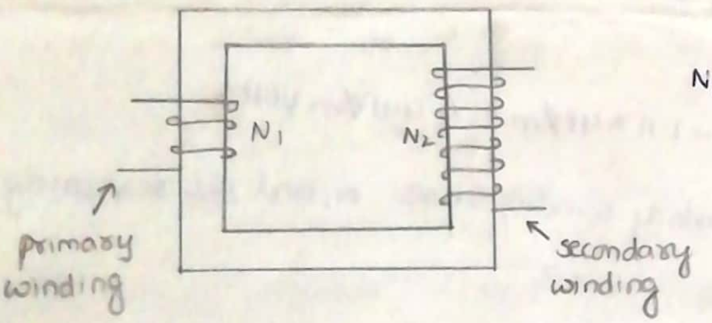
$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44 f \phi_m$$

It means that emf/turn is the same for both primary and secondary windings. In an ideal transformer the voltage drops in primary and secondary windings are negligible so $E_1 = V_1$ supply voltage and $E_2 = V_2$ terminal voltage.

The ratio of secondary voltage to primary voltage is same as the ratio of secondary turns to primary turns. This ratio is known as transformation ratio K

$$\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

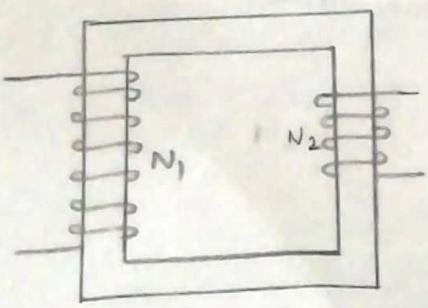
for step up transformer:



for step up transformer

$N_2 > N_1$ or $V_2 > V_1$ ie, $K > 1$

step down transformer:



for step down transformer

$N_1 > N_2$ or $V_1 > V_2$ ie, $K < 1$

current ratio: In an ideal transformer the losses are negligible so that volt ampere input to the primary is approximately equal to volt ampere output from secondary

Input VA = output VA

$$V_1 I_1 = V_2 I_2$$

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

(or)

$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{1}{K}$$

Hence current are in the inverse ratio of the voltage transformation ratio.

Ideal transformer:

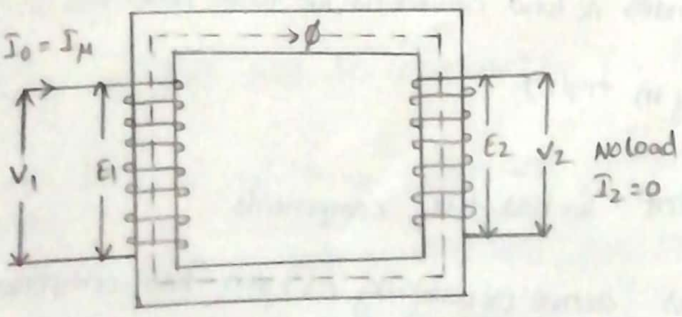
An ideal transformer is one whose input is equal to output. Transformer is called as ideal if it satisfies the following properties.

1. It has no iron losses ie, hysteresis and eddy current losses in transformer core are zero
2. Its windings have zero resistance. It means that there is no ohmic power loss and no resistive voltage drop in an ideal transformer.
3. leakage flux is zero ie, 100% flux produced by primary links with the secondary
4. permeability of core is so high that means negligible current is required to establish the flux in it.
5. Efficiency is 100%. ie, power output = power input.

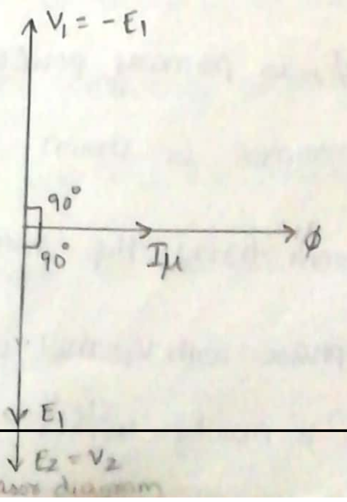
practically no transformer is ideal having above properties. ~~but the discussion of ideal transformer.~~

Ideal transformer on No load:

consider an Ideal transformer shown in figure on No load as it is no load $I_2 = 0$ ie, secondary is open.



(a) circuit diagram



(b) phasor diagram

... current is required. As winding is purely reactive
this current lags V_1 by 90°. This current is called as magnetising current I_μ or I_m

to magnetise the core. The magnetising current I_μ is always in phase with the flux ϕ. The varying flux is linked with both of the windings and so induced emf in both primary and secondary E_1 and E_2 will lag behind the flux producing them by 90°, so these emfs E_1 and E_2 are in phase with each other. The magnitude of induced emf in the primary winding E_1 will be approximately equal but opposite to the applied voltage V_1 and is usually known as counter emf or back emf. The secondary voltage $V_2 = E_2$ as there is no voltage drop in secondary. The phasor diagram is shown in fig (b)

Transformer with losses but no magnetic leakage:

Transformer on No load:

Actually in practical transformer ~~core causes~~ iron losses ~~and~~ in core and copper loss in the windings. When the transformer is on no load the primary input current is not wholly reactive. The primary input current under no load conditions has to supply (i) iron losses in the core ie, hysteresis loss and eddy current loss and (ii) a very small amount of copper loss in primary, there being no cu loss in secondary as it is open. Hence no load primary input current I_0 is not at 90° behind V_1 but lags it by an angle $\phi_0 < 90°$.

No load input power $W_0 = V_1 I_0 \cos\phi_0$

where $\cos\phi_0$ is primary power factor under no load conditions. No load condition of an actual transformer is shown vectorially in fig(2).

As seen from fig(2) the primary current I_0 has two components

(i) one in phase with V_1. This is known as active or working or iron loss component I_w because it mainly supplies the iron loss plus small quantity of primary cu loss.

$I_w = I_0 \cos\phi_0$

(ii) The other component is in quadrature with V_1 and is known as magnetising component I_μ because its function is to sustain the alternating flux in the core

$$I_\mu = I_0 \sin\phi_0$$

I_0 is the vector sum of I_w and I_μ

$$\therefore \bar{I_0} = \bar{I_\mu} + \bar{I_w}$$

$$I_0 = \sqrt{I_\mu^2 + I_w^2}$$

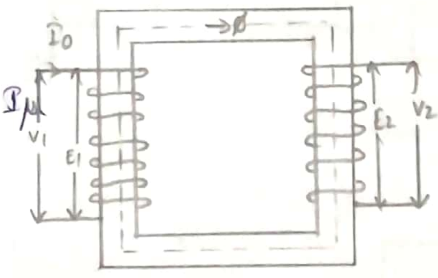


fig (1)
circuit diagram

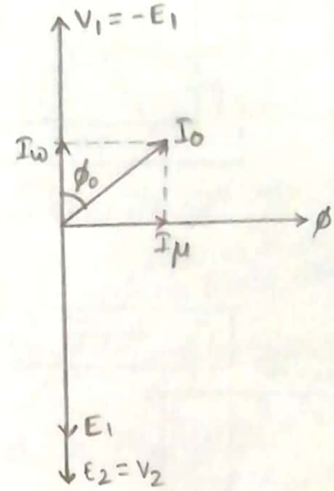


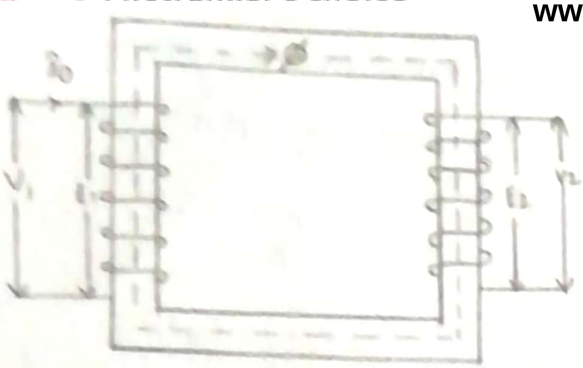
fig (2) phasor diagram

The following points should be noted carefully:

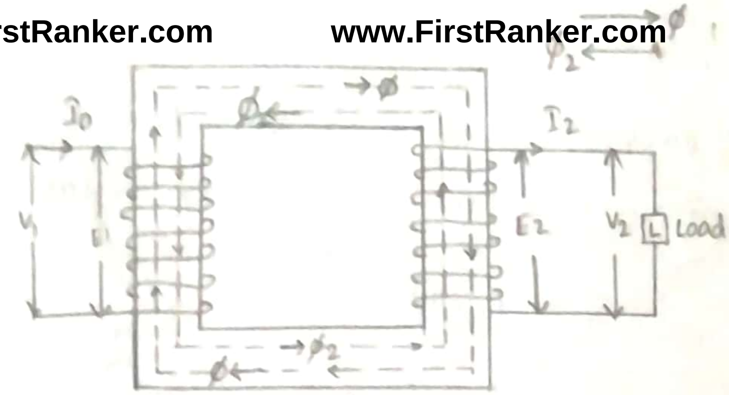
1. The no load primary current I_0 is very small as compared to the full load primary current. It is about 2 to 5% of the full load current.
2. As I_0 is very small the no load primary cu loss is negligible. Thus the no load primary input power is equal to iron loss occuring in the core of the transformer.

Transformer on load:

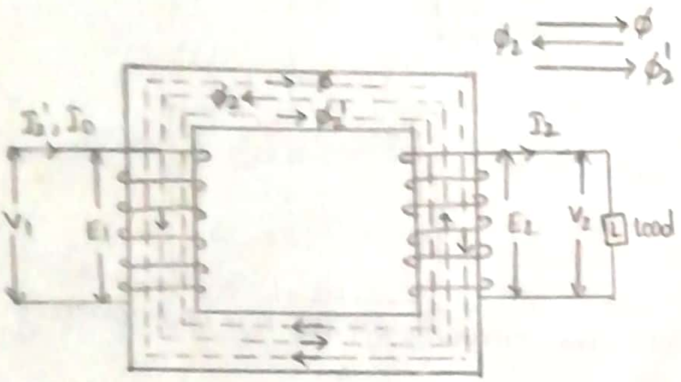
when load is connected to the transformer the current I_2 flows through secondary of the transformer. This current produces flux ϕ_2 in the core which opposes the main flux ϕ produced by the primary current. This reduces the total flux and there by the emf induced in the winding E_1. Due to the decrease of E_1 the current in the primary increases. The increase of current in the primary winding is such that it neutralizes the flux of the secondary winding i.e., it sets up of a flux ϕ_2' which is equal and opposite to ϕ_2 and thus the total flux remains the same from no load to full load. Due to this reason the core loss is practically the same under all load conditions. The whole process is illustrated in figure shown.



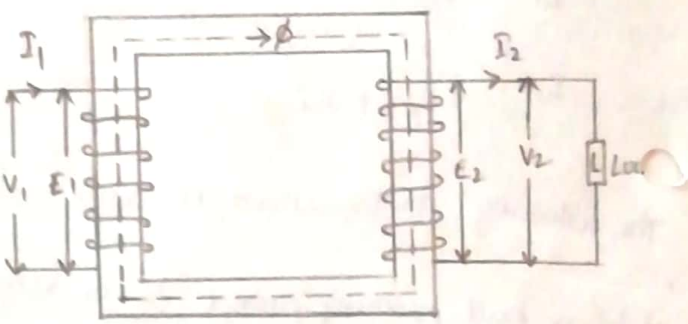
(a) No load condition



(b) Load condition flux ϕ_2 produced



(c) flux ϕ_2' neutralises the ϕ_2



(d) net flux ϕ is same

The increased current which flows in the primary winding due to the load on the secondary winding is equal and opposite to the secondary current I_2 and is known as primary balancing current or load component of primary current I_2'

$$N_2 I_2 = N_1 I_2'$$

$$I_2' = \frac{N_2}{N_1} I_2 = K I_2$$

where K is transformation ratio

As ampere turns are balanced, core flux ϕ is maintained at constant value

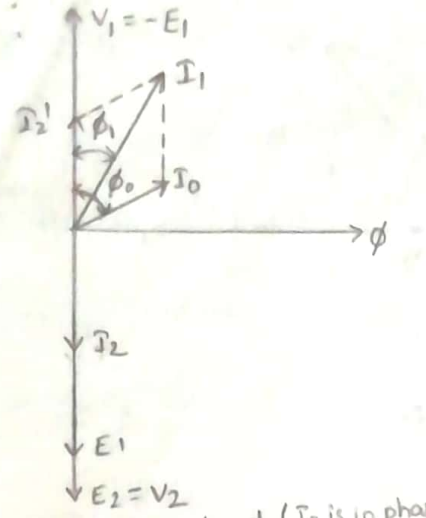
The total primary current I_1 has two components

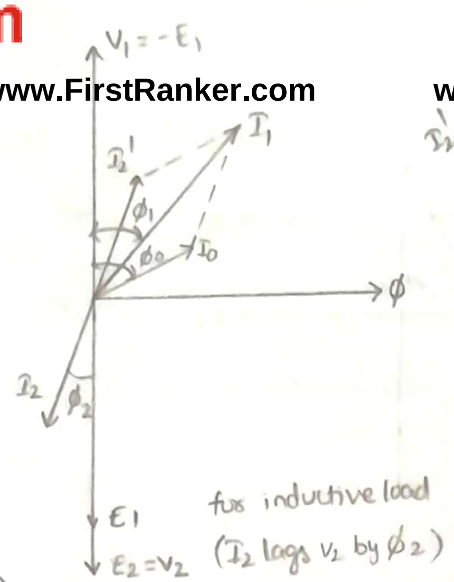
(i) No load current I_0

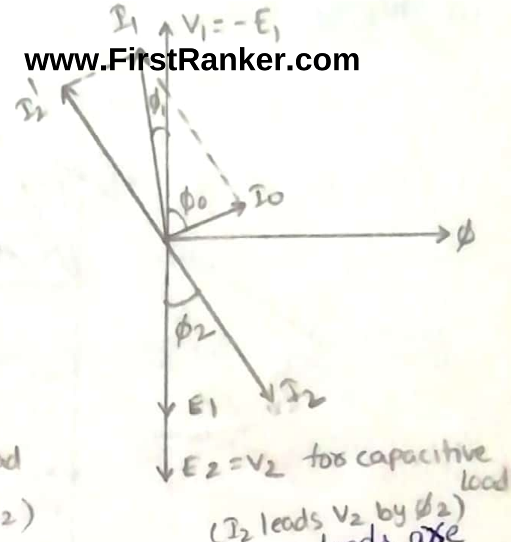
(ii) Load component of primary current I_2'

The vector sum of I_0 and I_2' is the total primary current I_1 lags behind V_1 by angle ϕ_1

$$\bar{I}_1 = \bar{I}_0 + \bar{I}_2'$$







the vector diagrams for transformer on Resistive, inductive and capacitive loads are shown in above figure. since the voltage drops in both of the windings of the transform-er are assumed to be negligible therefore $V_2 = E_2$ and $V_1 = -E_1$ ϕ_0 is no load power factor angle and ϕ_2 is load power factor angle.

Effect of resistance of the transformer windings:

An ideal transformer was supposed to posses no resistance in the windings, but in a practical transformer both the primary and secondary windings possess some resistance R_1 and R_2 respectively. Due to this resistance there is some voltage drop in two winding windings. the result is that:

1. The counter emf of primary $-E_1$ is equal to the vector difference of voltage applied to the primary winding V_1 and voltage drop in the primary winding I_1R_1

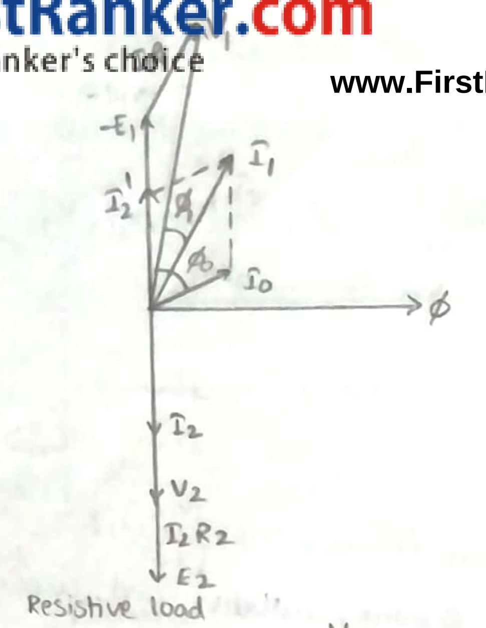
$$-E_1 = V_1 - I_1R_1$$

$$V_1 = -E_1 + I_1R_1$$

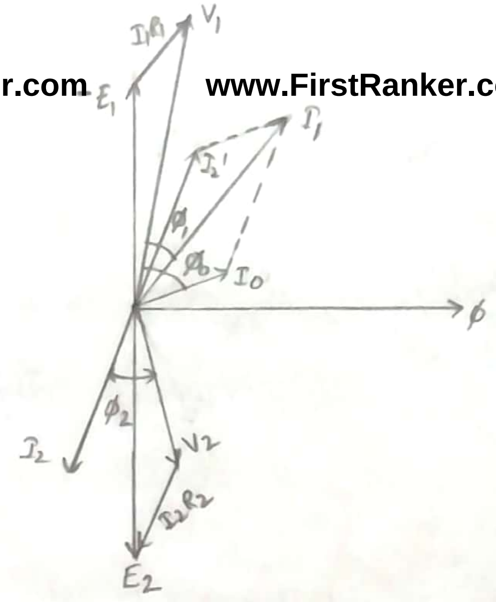
2- The secondary terminal voltage V_2 is less than the secondary induced emf E_2 and is equal to vector difference of E_2 and voltage drop in the secondary winding I_2R_2.

$$V_2 = E_2 - I_2R_2$$

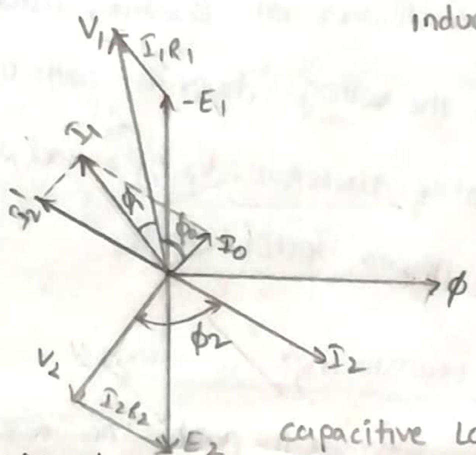
~~The vector diagrams for pure resistive~~, inductive and capacitive loads are shown



Resistive load



inductive load



capacitive Load

Equivalent resistance as referred to primary and secondary:

The primary and secondary windings of transformer have resistances R_1 and R_2 respectively are shown externally. in fig (a). Some times it is more convenient for calculation purposes to assume the resistance of the two windings is transferred to one winding only.

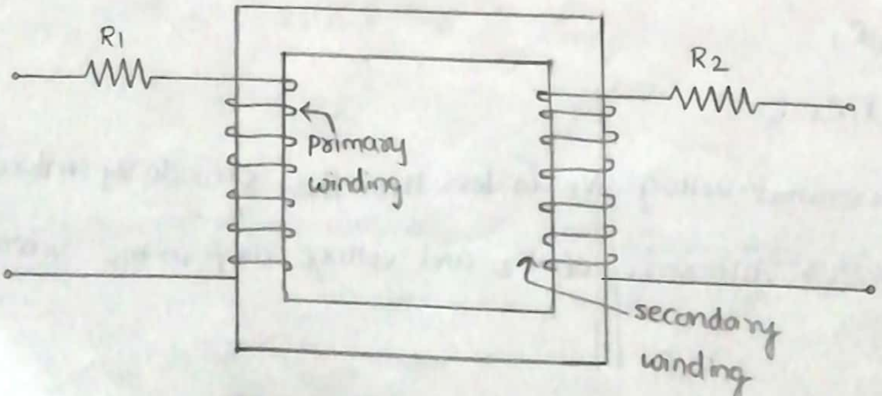


fig (a) Transformer with winding resistances

... R_2 can be transferred to the primary side without disturbing the circuit conditions as follows:

R_2 of the secondary can be transferred to the primary side by introducing an additional resistance R_2' in the primary such that the power absorbed in R_2' when carrying primary current I_1 is equal to the power absorbed by R_2 when carrying secondary current I_2

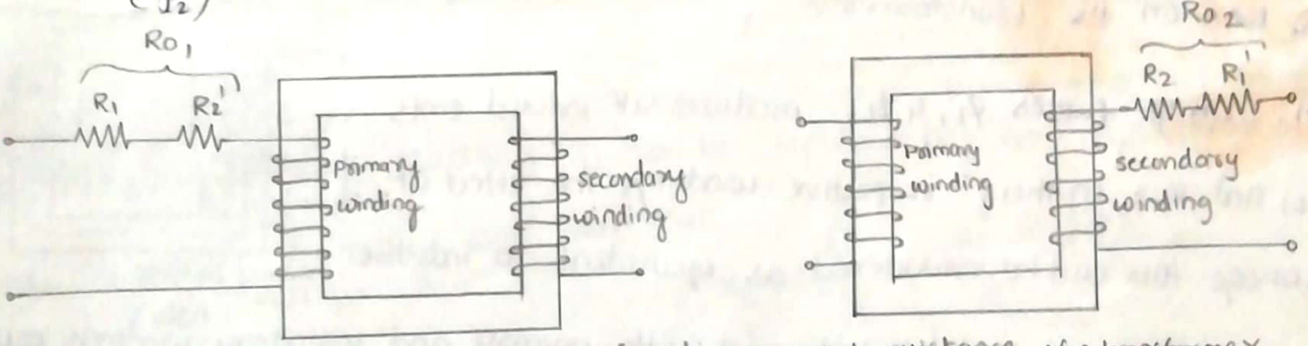
$$I_1^2 R_2' = I_2^2 R_2$$

$$R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2 = \frac{R_2}{K^2} \quad \left(\because K = \frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{E_2}{E_1}\right)$$

Similarly the primary winding resistance R_1 can be transferred to the secondary side is

$$I_1^2 R_1 = I_2^2 R_1'$$

$$R_1' = \left(\frac{I_1}{I_2}\right)^2 R_1 \Rightarrow R_1' = K^2 R_1$$



Total resistance of a transformer referred to primary

Total resistance of a transformer referred to secondary

fig (b)

The total or equivalent or effective resistance of a transformer as referred to the primary is $Ro_1 = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$

The equivalent resistance of a transformer as referred to secondary is

$$Ro_2 = R_2 + R_1' = R_2 + K^2 R_1$$

Magnetic leakage in transformer:

It was assumed earlier that the total flux ϕ produced by the primary

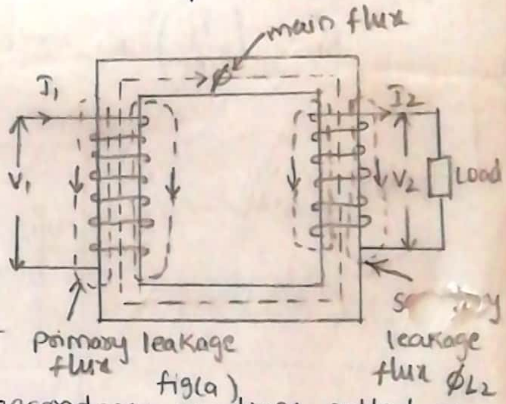
... pass through the core and they will link both the windings. But in practical transformer the total flux produced can be divided into three components as shown in fig(a) namely.

(i) useful flux (ϕ): The flux which links both the windings of the transformer which remains constant at all values of load.

(ii) primary leakage flux (ϕ_{L1}): The flux that links with the primary but not with the secondary winding.

(iii) secondary leakage flux (ϕ_{L2}): The flux that links with the secondary but not with the primary winding.

When the transformer is operating at no load the leakage flux is neglected on loading the transformer leakage fluxes are increased because both the windings carry appreciable current. Therefore the value of leakage flux is proportional to the load on the transformer.



fig(a)

The leakage fluxes ϕ_{L1} & ϕ_{L2} produces self induced emfs e_{L1} and e_{L2} in their respective windings. The effect of leakage flux can be considered as equivalent to inductive reactors X_1 and X_2 connected in series with primary and secondary windings called as leakage reactance of the windings as shown in figure (b) such that the internal emf in each inductive coil is equal to that due to corresponding leakage flux in the actual transformer

$$X_1 = \frac{e_{L1}}{I_1} \quad \text{and} \quad X_2 = \frac{e_{L2}}{I_2}$$

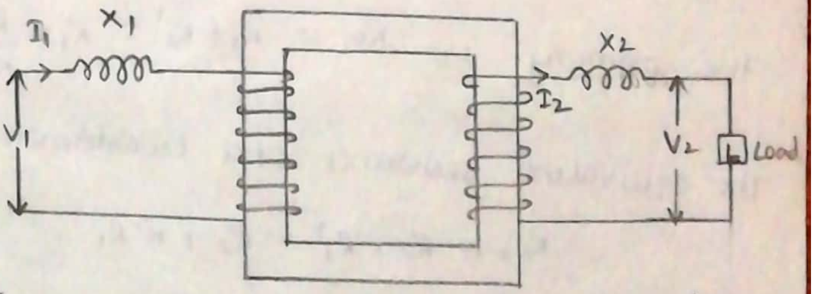


Fig (b) Transformer with leakage reactances

primary leakage reactance $X_1 = 2\pi f L_1$

secondary leakage reactance $X_2 = 2\pi f L_2$

where L_1, L_2 are self inductance of primary and secondary windings.

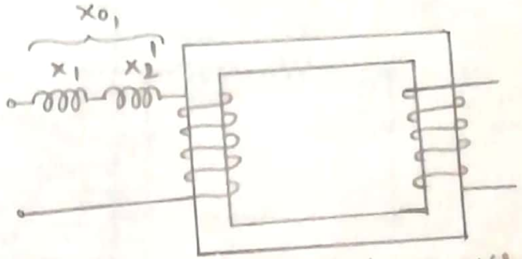
magnitude of leakage flux and hence of leakage reactance can be reduced by increasing the magnetic coupling between the primary and secondary windings.

Due to the leakage reactances there is some voltage drop in the two windings. The result is that the primary voltage V_1 will have to supply reactive drop I_1X_1 in addition to I_1R_1. Similarly E_2 will have to supply I_2R_2 and I_2X_2.

Equivalent leakage reactance as referred to primary and secondary:

Inductance $L = \frac{N d\phi}{dt}$

$$L = \frac{N d\phi}{dt} = \frac{\mu N^2 A}{l}$$



Total reactance of a Transformer referred to primary

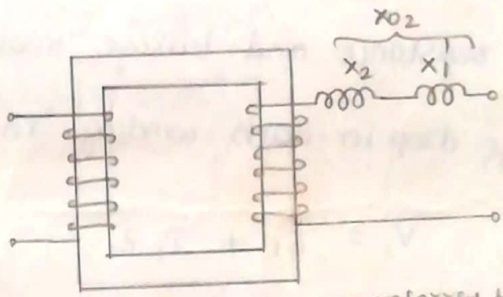
where μ = permeability of the core material

N = Number of turns of the coil

l = length of the coil

$$\therefore X = 2\pi f L = 2\pi f \frac{\mu N^2 A}{l}$$

$$X \propto N^2$$



Total reactance of a transformer referred to secondary

$\therefore$ The secondary leakage reactance X_2 can be replaced by an equivalent reactance X_2' in the primary side such that

$$X_2 \propto N_2^2 \qquad \frac{X_2'}{X_2} = \left(\frac{N_1}{N_2}\right)^2$$

$$X_2' \propto N_1^2$$

$$X_2' = X_2\left(\frac{N_1}{N_2}\right)^2 \Rightarrow X_2' = \frac{X_2}{K^2} \qquad X_2' = \frac{X_2}{K^2}$$

The primary leakage reactance X_1 can be transferred to the secondary side is $X_1' = X_1\left(\frac{N_2}{N_1}\right)^2 = K^2 X_1$

Total equivalent or effective reactance of the transformer referred to the primary

$$X_{o1} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$$

equivalent reactance of the transformer referred to secondary

$$X_{o2} = X_2 + X_1' = X_2 + X_1K^2$$

Transformer with resistance and leakage reactance:

consider a transformer shown in fig (1) having primary and secondary windings of resistances R_1 and R_2 and reactance x_1 and x_2 respectively.

The impedance of the primary winding $Z_1 = R_1 + jX_1$ and

the impedance of the secondary winding $Z_2 = R_2 + jX_2$

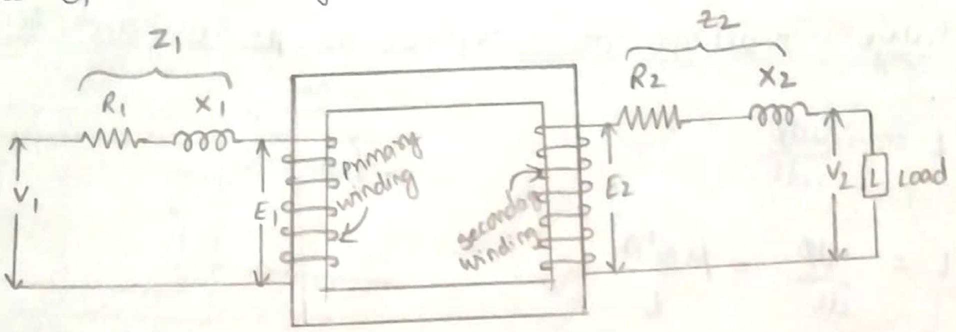


fig (1) Transformer with winding resistances and leakage reactances.

The resistance and leakage reactance of each winding is responsible for some voltage drop in each winding. In primary

$$\bar{V}_1 = \bar{E}_1 + \bar{I}_1 \bar{Z}_1$$

similarly the secondary voltage E_2 is given by

$$\bar{E}_2 = \bar{V}_2 + \bar{I}_2 \bar{Z}_2$$

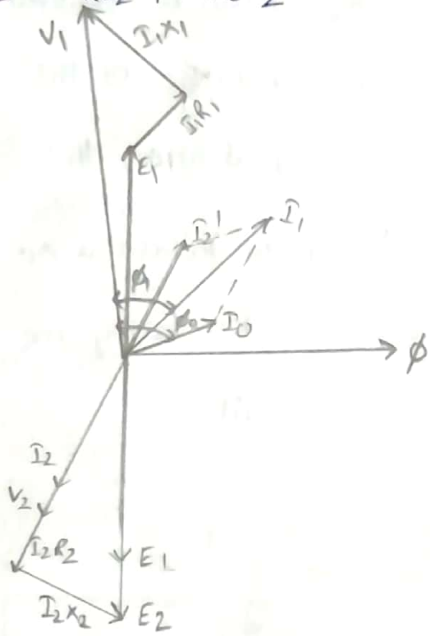
The vector diagram of a transformer for different types of loads are shown in fig (2). The voltage drops in the windings due to resistances are shown in phase with the currents flowing through them and the drops in the reactances are shown at right angles to the currents. The angle ϕ_1 between V_1 and I_1 gives the power factor angle of primary side of the transformer. Angle ϕ_2 is load powerfactor angle.

Total impedance of the transformer as referred to primary

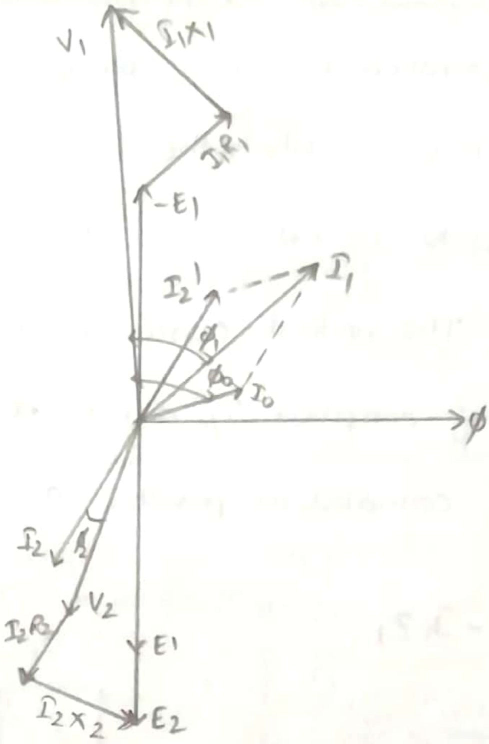
$$Z_{01} = R_{01} + jX_{01}$$

total impedance of the transformer referred to secondary is given by

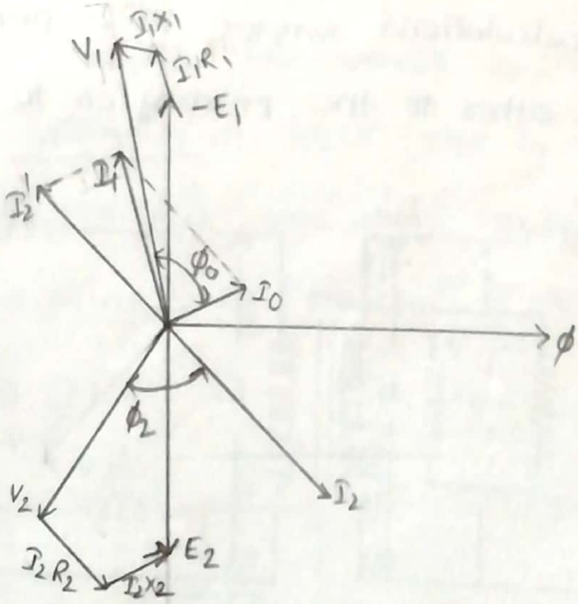
$Z_{O2} = R_{O2} + jX_{O2}$



Resistive load
(I_2 is in phase with V_2)



Inductive load
(I_2 lags V_2 by ϕ_2)



capacitive load
(I_2 leads V_2 by 90°)

Equivalent circuit of a Transformer:

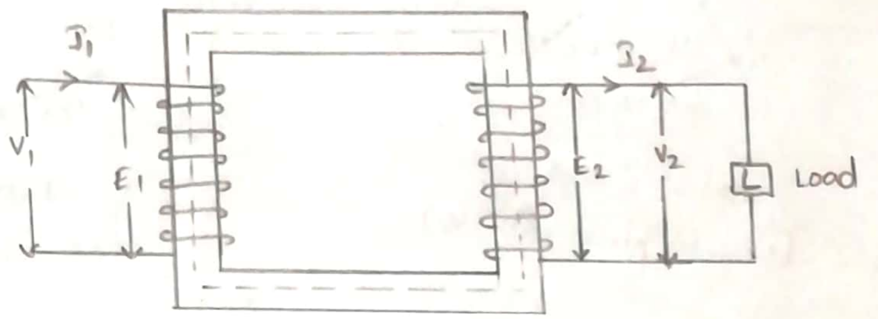
The transformer shown diagrammatically in fig (a) can be resolved into an equivalent circuit in which the resistance and leakage reactance of the transformer are imagined to be external to the winding whose only function then is to transform the voltage. The no load current I_0 is simulated by pure inductance X_0 taking the magnetising component I_μ and a non inductive resistance R_0 taking the working component I_w connected in parallel across the primary circuit.

$$\bar{E}_1 = \bar{V}_1 - \bar{I}_1 \bar{Z}_1$$

$$X_0 = \frac{E_1}{I_\mu}$$

$$R_0 = \frac{E_1}{I_w}$$

$$\bar{I}_0 = \bar{I}_\mu + \bar{I}_w$$



fig(a)

To make transformer calculations simpler it is preferable to transfer voltage, current and impedance either to the primary or to the secondary.

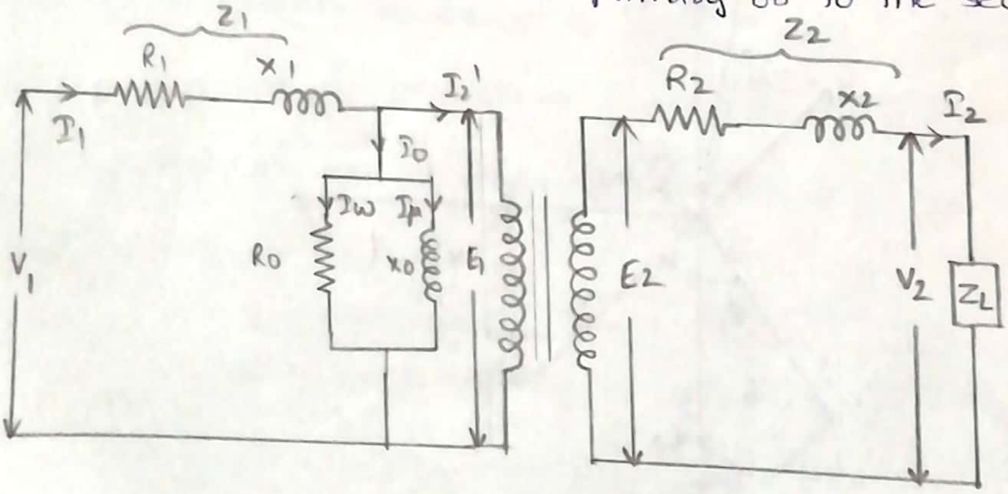


fig (b)

The primary equivalent of the secondary induced emf is $E_2' = \frac{E_2}{K} = E$

Primary equivalent of secondary terminal or output voltage $V_2' = \frac{V_2}{K}$

primary equivalent of the secondary current is $I_2' = K I_2$

for transferring secondary impedance to primary K^2 is used.

Regulation of a transformer

When the transformer is loaded its terminal voltage falls from no load to full load. The change in secondary terminal voltage from no load to full load expressed as secondary no-load voltage is known as 'regulation down'

$$\% \text{ Regulation down} = \frac{V_{20} - V_2}{V_{20}} \times 100$$

$$= V_r \cos\phi - V_x \sin\phi$$

If change in voltage is expressed as secondary full load voltage, then it is known as regulation up.

$$\% \text{ Regulation up} = \frac{V_{20} - V_2}{V_2} \times 100$$

Unless stated otherwise regulation is to be taken as regulation down. Lesser the regulation, better the transformer.

Losses:

Since the transformer is a static machine so there are no friction and windage losses. Hence the losses occurring in a transformer are

(i) Iron or core losses: It consists of hysteresis loss and eddy current loss, this loss remains practically constant at all loads.

Hysteresis loss $W_h = \eta B_m^{1.6} f V$ watt

Eddy current loss $W_e = Q B_m^2 f^2 t^2$ watt

where V = volume

t = lamination thickness.

Iron losses can be minimised by using silicon steel core and by using very thin laminations. Silicon steel reduces the hysteresis loss where as laminations reduces the eddy current loss. This loss is found from no load (oc) test.

(ii) copper loss: These loss occur due to ohmic resistance of the transformer windings. so total copper loss will be $(I_1^2 R_1 + I_2^2 R_2) = I_1^2 R_{01} = I_2^2 R_{02}$. this loss is found from short circuit test.

It is clear that copper loss is proportional to the square of the current i.e., copper loss at half full load is one fourth of that at full load.

Efficiency:

The efficiency of a transformer may be defined as the ratio of output in watt to input in watt and is represented by the letter 'η'

Efficiency $\eta = \frac{\text{output}}{\text{Input}} = \frac{\text{output}}{\text{output + losses}}$

$$= \frac{\text{Input - losses}}{\text{Input}}$$

A transformer is a highly efficient apparatus has very small losses. The efficiency can be calculated by determining iron loss from no load test and copper loss from short circuit test.

Condition for maximum efficiency:

Primary input $= V_1 I_1 \cos\phi_1$

Iron loss $= W_i$

copper loss $= I_1^2 R_{01}$ (or) $I_2^2 R_{02}$

$$\eta = \frac{\text{Input - losses}}{\text{Input}}$$

$$= \frac{V_1 I_1 \cos\phi_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos\phi_1}$$

$$= 1 - \frac{I_1 R_{01}}{V_1 \cos\phi_1} - \frac{W_i}{V_1 I_1 \cos\phi_1}$$

for η to be maximum the above equation should be differentiated w.r.t I_1 and equating it to zero, thus $\frac{d\eta}{dI_1} = 0$

$$\frac{R_{01}}{V_1 \cos\phi_1} + \frac{W_i}{V_1 I_1^2 \cos\phi_1} = 0$$

$$\frac{R_{01}}{V_1 \cos\phi_1} = \frac{W_i}{V_1 I_1^2 \cos\phi_1}$$

$$R_{01} = \frac{W_i}{I_1^2}$$

$$I_1^2 R_{01} = W_i$$

copper loss = Iron loss

Hence the efficiency will be maximum when variable loss are equal to constant loss (W_i)

$$I_1^2 R_{01} = I_2^2 R_{02} = W_i$$

$$I_2 = \sqrt{\frac{W_i}{R_{02}}}$$

Similarly KVA corresponding to maximum efficiency

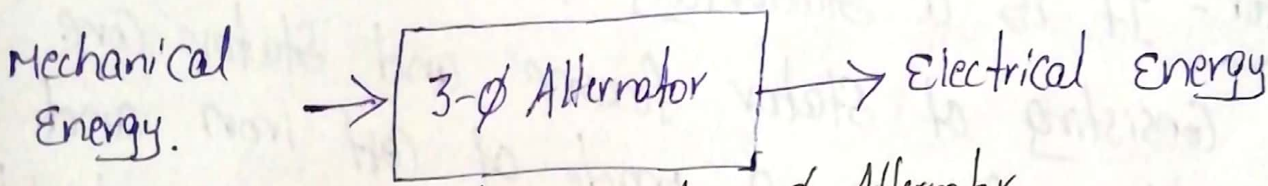
$$= \text{full load KVA} \times \sqrt{\frac{W_i}{\text{F.L. copper loss}}}$$

UNIT 2

Synchronous generators and its operation

Synchronous generator:-

Synchronous generator is a machine, Its function is mechanical Energy to AC Electrical Energy. It is also known as 3-ϕ Alternator (or) 3-ϕ Generator. The Alternator input is given by Primemover.



Block diagram of 3-ϕ Alternator

Classification of Alternators:-

Based on Speed:

a) Low Speed Alternators

b) Medium Speed Alternators

c) High Speed Alternators

Based on field Construction

a) Stationary field type

b) Stationary Armature type (Rotating field)

Based on Rotor Construction:

a) Salient Pole type alternators

b) Non-Salient Pole type alternators

Based on No. of Phases

a) 1-ϕ Alternators

b) 3-ϕ Alternators

Two types of field excitation

a) Self Excitation.

b) Auxilary Excitation.

Constructional details of Synchronous generators (3-ϕ)

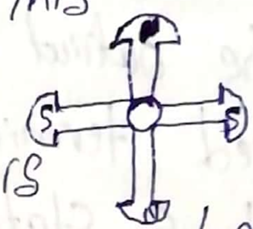
The Synchronous generator mainly having two Parts (i) Stator (ii) Rotor.

Stator:- It is a Stationary Part of an alternator and Consisting of Stator frame and Stator Core. the Stator frame is made of Cast iron and used for holding the armature Stampings and Stator winding in Position. The Stator frame is Provided with Ventilating ducts for the air to Circulate for heat dissipation. The Stator Core is made of Silicon Steel and it is Supported by the Stator frame the Core is Laminated to minimise the Eddy Current Loss due to Eddy Currents the thickness of Laminations is about 0.5 mm and are insulated from Each other by means of Paper (or) Varnish layer (or) Enamel Coating the no. of Slots are Provided the inner Periphery of Core for housing the armature Conductors.

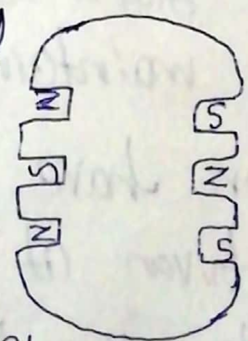
Rotor:- It is a rotating Part of an alternator the rotator is like a flywheel having no. of alternating 'N' and 'S' Poles fixed to its outer rim. the magnetic Poles carries a field winding which is supplied with direct current through two-sliprings by a separate d.c Source. In rotor having two-types

(i) Salient Pole type (ii) Non-Salient Pole type.

Salient Pole type:- In this type the Poles are Projected out from the rotor surface. This Projected Poles are mounted on a large circular steel frame. The field winding is placed around the Poles. The field Pole windings are connected in series and are Excited by a d.c supply. This steel frame is fixed to the shaft of the alternator. It has large diameter and short axial lenght. It has large Number of Poles hence the speed is less.



Non-Salient type rotor:- In this type Poles are not Projected out from the rotor surface. The rotor consist of a smooth solid forged steel cylinder. The slots are made on the outer Periphery. The field winding is Excited by a d.c source through sliprings. These

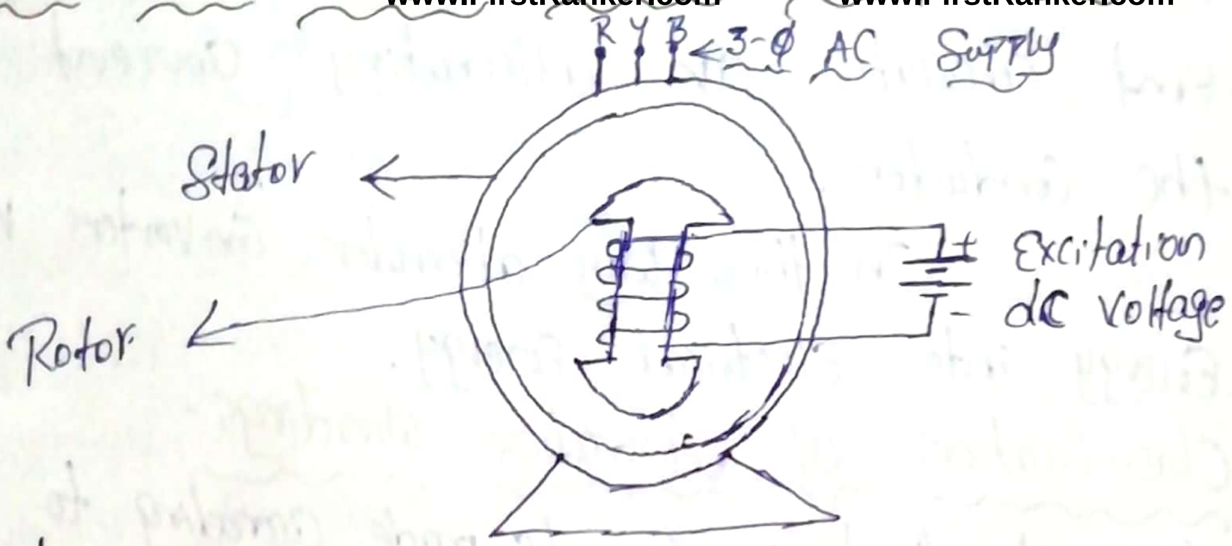


These have less diameter and long axial length
It has less No of Poles 2(or) 4 hence the speed
is high These are used in high speed alternators
This alternators driven by steam turbine i.e
turbo alternators which run at very high speed
have smoth cylindrical type rotor.

Advantages of stationary armature over rotating armature

1) The external current can be controlled directly from fixed terminals on the stator
2) It is stationary armature so easier to insulate.
3) only two sliprings are required for d.c supply.
4) winding of stator is stationary so no possibility of breaking and loosing the winding
5) No-sliding contact between armature and load circuit
6) Armature can have more number of conductors when it is stationary.
7) less maintainance and less damages
8) rotor have lesser weight and inertia so it can be driven at high speed.
9) Stationary Armature construction causes less leakage flux.

Construction and Working Principle of Alternator (3-ϕ)



Construction:-

An alternator has a stator having slots on its inner periphery of housing the armature conductors the rotor is like a flywheel having alternate 'N' and 'S' Poles fixed to its outer rim. The field winding is excited from direct current supplied by a d.c source. because the field winding is rotating the direct current is supplied through two sliprings and brushes

Working:- The alternator is coupled to a suitable Primemover The mechanical energy can be supplied to the alternator by the Primemover. When the rotor is rotated by the Primmover the field system also rotates. The stationary armature conductors are cut by the rotating magnatic field hence according to faraday's laws an emf is induced in the armature conductors As the magnatic

Poles are alternating 'N' and 'S' Poles. This Emf circulates an alternating current in the conductors.

In this way alternator converters mechanical Energy into Electrical Energy.

Classification of Armature windings:-

This classification can be made according to five types of windings.

1) Single Phase and three Phase winding:- In 1-ϕ Alternators the armature having only one winding and is known as single phase winding. In three Phase alternators stator has three separate windings which are displayed by 120° Electrical degrees.

2) Single layer and double layer winding

When one conductor (or) one coil side is placed in one armature slot then the winding is known as single layer winding when two (or) more conductors are placed in one slot (or) then this winding is called as double layer winding. In double layer winding one coil side lies at the top of the slot and the other coil side lies conductors per pole some other slot at a distance of approximately one Pole Pitch.

Lap and wave winding :- In this winding the finishing End of one coil is connected to the Starting End of another coil. This winding is known as lap winding. because the side of adjacent coil overlap each other. The purpose of this winding is increase the No. of Parallel Paths for allowing to carry the large currents

Concentrated and distibuted winding :- when the Number of Slots are Equal to number of Poles then Concentrated winding is obtained. In this winding Concentrated on one slot. This winding Produces more amount of Emf but it is not a Pure Sinsoidal. If the Conductors are Placed Several Slots under one Pole is known as distibuted winding. This type of winding emf is Exactly Sinsoidal form but the heating distibution Provides Perfectly So it can be Cooled Quickly

full Pitch winding and fractional Pitch winding :-
when the two coil sides of a coil are 180° Electrical degrees apart the winding is known as full Pitch winding. when the two coil Sides are less than 180° apart that type of

winding is called as the fractional pitch winding. the main advantages is copper is less, induced emf is improved, harmonics are completely eliminated.

Frequency of induced Emf in Alternator (3-ϕ)

The frequency of induced Emf in the armature conductors depends upon the speed and No. of pole.

let P = Poles N = Speed, F = frequency

one cycle of Emf is produced when a pair of poles passes a conductor. Hence the Number of cycles of Emf produced one revolution is equal to No. of pair of poles

No. of cycles per revolutions $= \frac{P}{2}$

No. of revolution per second $= \frac{N}{60}$

we know that No. of cycles produced per second

Frequency (F) = No. of cycles/revolution × $\frac{\text{No. of revolution}}{\text{sec}}$

$$= \frac{P}{2} \times \frac{N}{120}$$

$$F = \frac{PN}{120} \text{ hz}$$

Winding factors

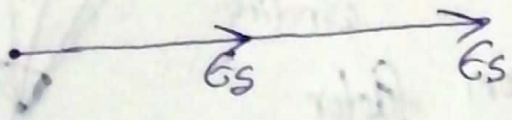
The armature winding of an alternator is uniformly distributed in all slots around the complete circumference of the alternator. The armature winding may use full pitched coils (or) short pitched coils. When the rotating magnetic field cut these winding an emf is induced in the coil by a factor. Hence this two factors which reduce the induced emf of the alternator: (i) Pitch factor (ii) distributed factor.

Pitch factor (or) Chording factor (or) Coil Span factor:-

In full pitched winding induced emf in the both coil sides is equal and inphase hence the resultant emf is the arithmetic sum of emf both coil sides. But to improve the wave form of the induced emf generally the short pitched winding is used. The emf induced in both the coil sides is not in phase,

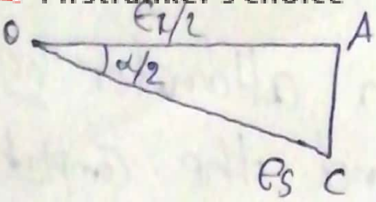
$$\text{Pitch factor } (K_P) = \frac{\text{Vector sum of induced emf}}{\text{Arithmetic sum of induced emf}}$$

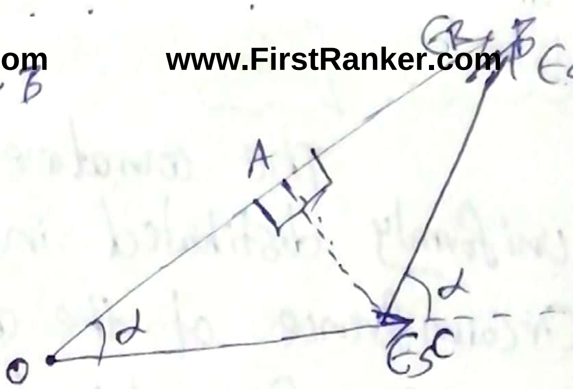
Arithmetic sum of induced emf



Resultant emf $E_R = E_S + E_S$

$E_R = 2E_S$ at Arithmetic sum





from OAC $\quad$ from ACB

$= E_S \cos\frac{\alpha}{2}$ $\quad$ $= E_S \cos\frac{\alpha}{2}$

Vector sum of Induced Emf $= E_S \cos\frac{\alpha}{2} + E_S \cos\frac{\alpha}{2}$

$= 2E_S \cos\frac{\alpha}{2}$

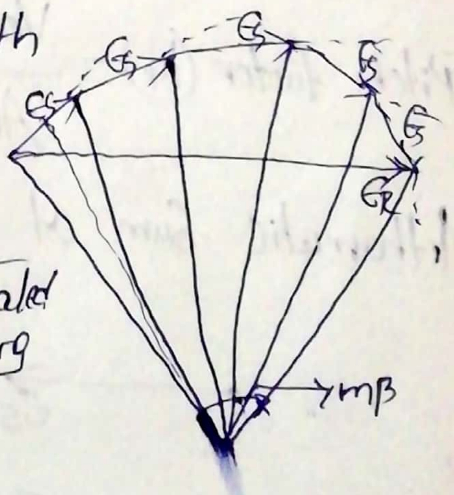
Pitch factor $(K_P) = \frac{\text{Vector Sum}}{\text{Arthamat Sum}}$

$= \frac{2E_S \cos\frac{\alpha}{2}}{2E_S}$

$$K_P = \cos\frac{\alpha}{2}$$

Distibution factor (K_d)

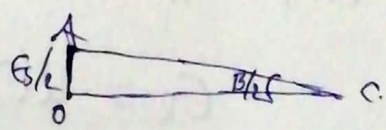
the distibution factor can be defined as the ratio of Emf with distibuted winding to the Emf with Concentrated winding



Distibution factor $= \frac{\text{Emf in distibuted winding}}{\text{Emf in Concentrated winding}}$

Expressions for Distibution factor

Vector Sum $= OA \sin\frac{m\beta}{2}$



Arthmetic Sum of Concentrated winding

$= m \sin \frac{m\beta}{2}$

Distibution factor $(K_d) = \frac{\text{Vector Sum}}{\text{Arthamiti Sum}}$

$$= \frac{2OA \sin \frac{m\beta}{2}}{2OA\, m \sin \beta/2}$$

$$K_d = \frac{\sin \frac{m\beta}{2}}{m \sin \beta/2}$$

* This two winding factors are always less than "unity".

Emf Equation of a 3-ϕ Alternator

Faraday's 1st law

whenever a conductor cuts the magnatic flux an Emf induced in that conductor

Faraday's 2nd law

the induced Emf is directly proposral to the rate of charge of flux linkages.

$$E = \frac{d\phi}{dt} \quad (\because N \cdot 1)$$

flux cut by the Each conductor one revelution of conductor $d\phi = \phi P$ in webars revelations per Second $= \frac{60}{N}$ RPS

Emf induced to the rate of flux $(E) = \frac{d\phi}{dt}$

$$= \frac{\phi P}{60/N}$$

$$E = \frac{\phi P N}{60} \text{ in Volts/Phase} \rightarrow (1)$$

Already we know the speed formula from frequency Expression

$$F = \frac{PN}{120} \Rightarrow N = \frac{120F}{P} \rightarrow (2)$$

Subsitute Equation → (2) in (1)

$$\text{Average Emf Per Phase } (E) = \frac{\phi P}{60} \cdot \frac{120F}{P}$$

$$E = 2F\phi$$

Average Emf Per Phase in 'Z' No. of Conductors

$$E = 2F\phi Z_{ph}$$

one Conductor also having two turns So

$$E = 4F\phi T_{ph} \qquad \because Z_{ph} = \frac{T_{ph}}{2}$$

$$\Rightarrow T_{ph} = 2Z_{ph}$$

Rms Value of Emf = form factor x Average Value of Emf

$\Rightarrow 1.11 \times 4 F \phi T_{ph}$

Rms Value of induced Emf $= 4.44 F \phi T_{ph}$

If kp & kd are distributed and pitch factors so

Rms Value of induced Emf Per Phase

$$E_{ph} = 4.44 F \phi T_{ph} k_p k_d \text{ in Volts}$$

for star Connection

$$E_L = \sqrt{3} E_{ph} \text{ volts}$$

here

Z = No. of Conductors

P = No. of Poles

ϕ = flux Per Pole

f = Supply frequency

T_{ph} = No. of turns Per Phase

k_p = Pitch factor

k_d = distributed factor

E_{ph} = Average Emf Per Phase.

An alternator has two-types of winding namely field winding and armature winding when the alternator is running at No-Load, no-current flows through the armature winding. The flux Produced in the air gap will be only due to the field current when the alternator is Loaded, the Load current will flows through three Phases and a rotating magnetic field is Produced in the air gap.

Definition:- The armature reaction is defined as the Effect of magnetic field Set up by the armature current on the distibution of main flux under the main Poles of generator (or) Alternator.

The Effect of magnetic field not only depends on magnitude of armature current but also depends on the nature of the Load Power factor.

1) when the Power factor unity (resistive Load)
2) When the Power factor lagging (Inductive Load)
3) when the Power factor leading (Capacitive Load)

Regulation: The change in voltage from No-load to full load by the fraction of No-Load is known as 'Regulation'.

$$\%\text{Regulation} = \frac{\text{No-Load voltage} - \text{full Load Voltage}}{\text{No-Load voltage}} \times 100$$

$$\%\text{Regulation} = \frac{E_{g_0} - V_{FL}}{E_{g_0}} \times 100$$

Here

E_{g_0} = No-Load voltage in volts

V = full Load Voltage in Volts

The Regulation is not only depends on voltage difference but also depends on Nature of Load.

→ Resistive Load (unity Power factor)

→ Inductive Load (lagging Power factor)

→ Capacitive Load (Leading Power factor)

The Regulation is determined by three different methods namely:

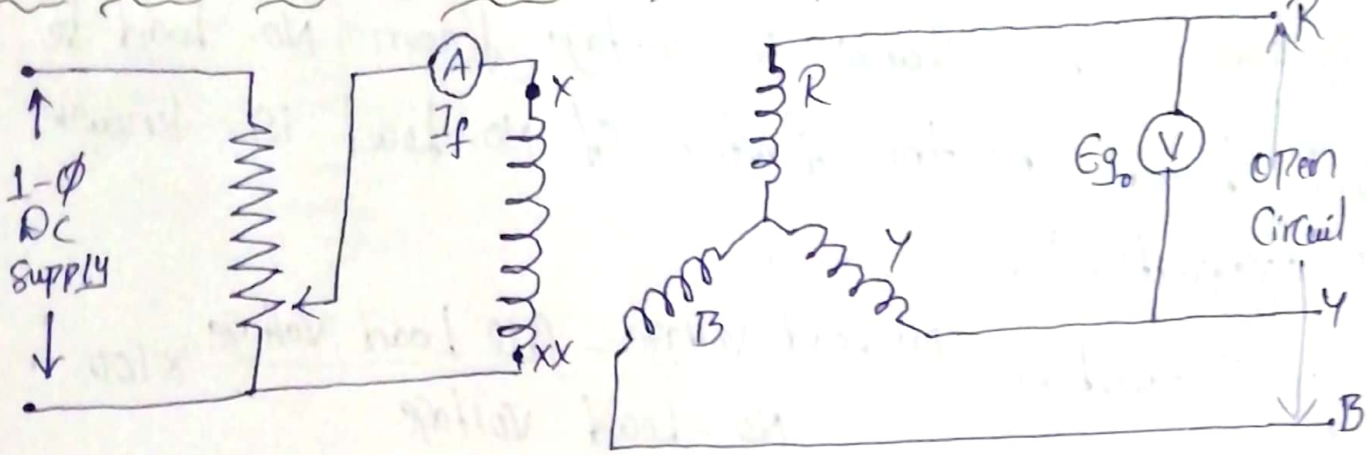
1) Synchronous Impedance method (or) Emf method

2) Ampere-turns method (or) mmf method

3) Zero Power factor method (or) Potier triangle method

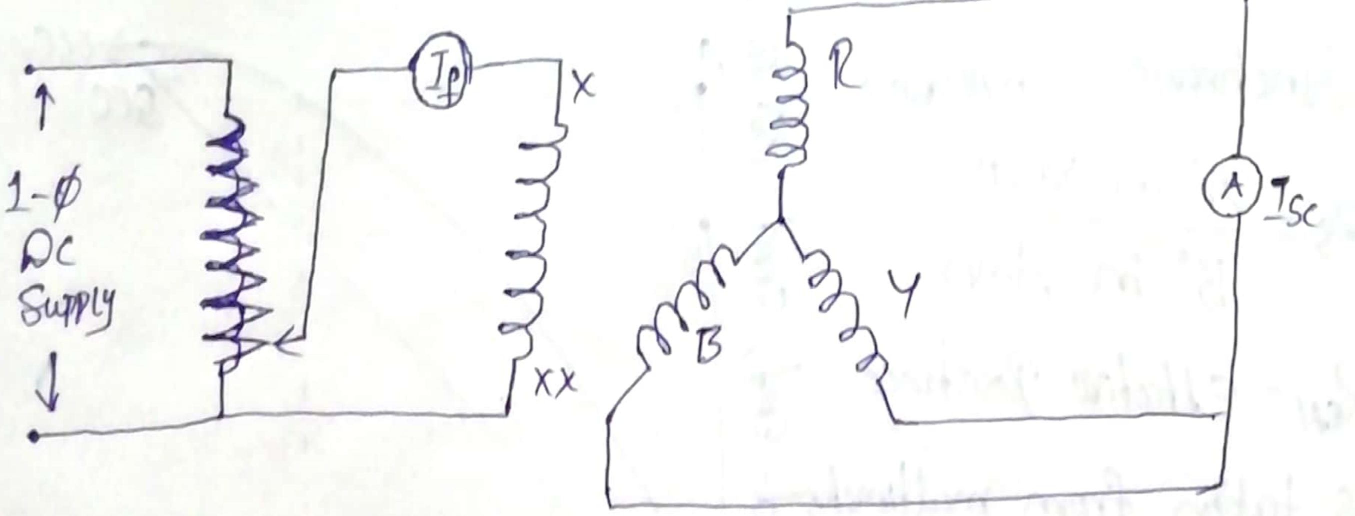
Synchronous Impedance method:-

No- Load test (or) open circuit characteristics test



* the field Excitation is given by separate d.c supply.
* The field supply at kept at minimum position.
* The Primmover is started with the help of starter.
* After that gradually increase the field rheostat, the field current will increases, also field flux is increases.
* So gradually the voltmeter shows No- load voltage value gradually.
* Note down the readings of both field Ammeter and voltmeter and tabulate them.
* The O.C.C curve is drawn between the No-Load generated Emf (vs) field current in Amps.

Short Circuit Charactristics test



* In this the one Ammeter is Connected to field winding and another Ammeter is Connected where the three Phase are Short Circuit

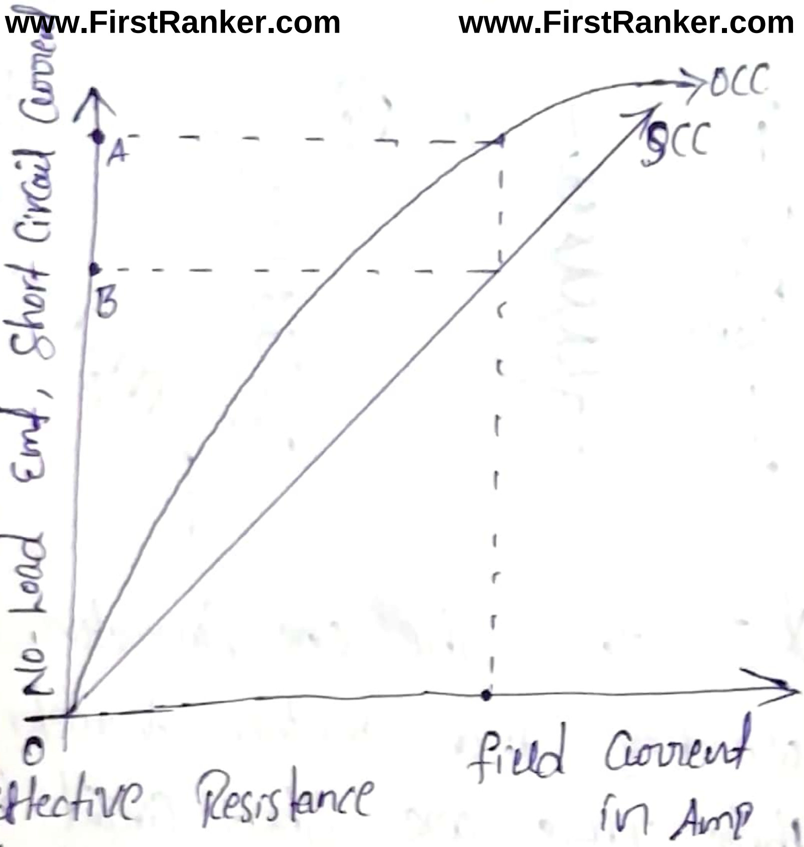
* The mechanical Energy of Alternator is given by Primmover it is Coupled to 3-ϕ Alternator

* gradually vary the field rheostat, when the short circuit Ammeter Shows the rated current of 3-ϕ Alternator

* Note down the field ammeter and Short Circuit ammeter values in tabular Column.

* Draw the SCC Curve between the Short Circuit Current and field Current.

Model graph:



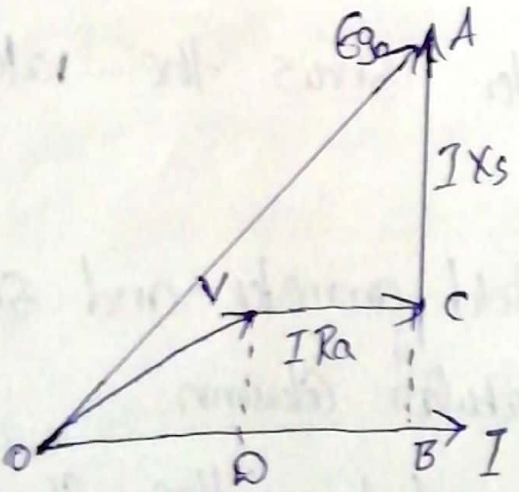
Synchronous Impedance

$Z_s = \frac{\text{'A' in Volts}}{\text{'B' in Amps}}$

R_{eff} = Effective Resistance is taken from multimeter, then value is multiplied with 1.5 then we will get Effective Resistance value = R_{eff}

Synchronous Reactance = $\sqrt{Z_s^2 - R_{eff}^2}$

Vector diagram for Lagging Power factor



According to pythagoras rule

$OA^2 = OB^2 + (AB)^2$

$E_{g_0}^2 = (OD + BD)^2 + (BC + AC)^2$

$$E_{go}^2 = (V\cos\phi + IR_a)^2 + (V\sin\phi \pm IX_s)^2$$

$$E_{go} = \sqrt{(V\cos\phi + IR_a)^2 + (V\sin\phi \pm IX_s)^2} \text{ volts}$$

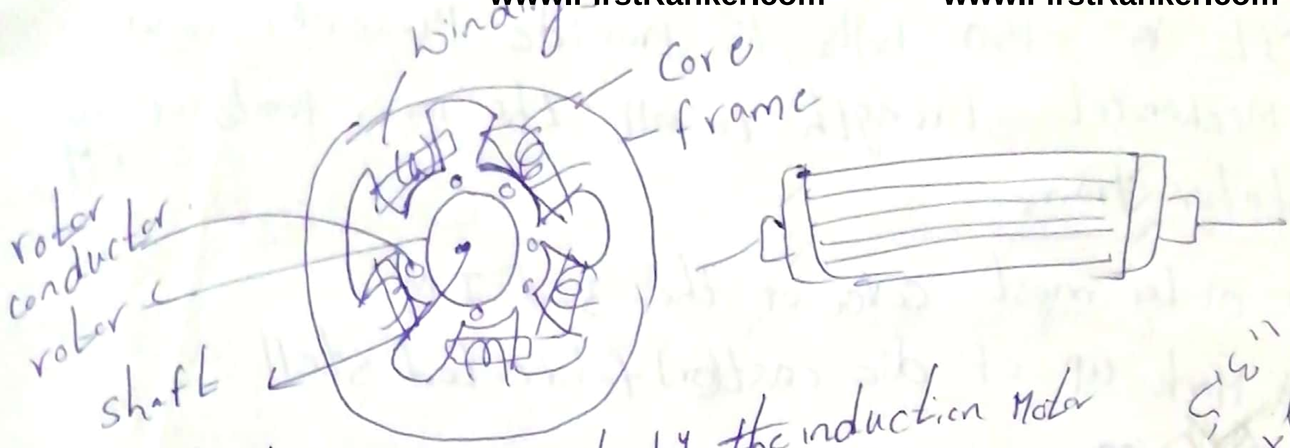
'+' sign for lagging Power factor

'−' sign for Leading Power factor

When unity Power factor

The $\sin\phi$ value is equal to 0 So

$$E_{go} = \sqrt{(V\cos\phi + IR)^2 + (IX_s)^2} \text{ Volts}$$



80% of the power used by the induction Motor

* low cost
* No external Primemover
* maintanence free

Rotation Magnetic field:- Main parts
* stator
* Rotor
* shaft
* Bearings
* Terminal box
* fans

shaft: to transmit the torque to the load

Bearings: to support this rotational shaft

Terminal box:- to Provide electrical connection

fans : One of the Problem associated the machine over heating to Provide fans.

The stator has 3 Main parts:
* stator frame → the frame is the outermost casing for the 3-ϕ IM
* " core → inside the stator frame
* stator winding →

squirrel cage - Y-Δ - stator

slip ring - external Resistance

3-ϕ AC supply is provide to 3-ϕ Induction Motor - RYB windings are provide

Rotor conductor are

* windings are balanced to the special distrib-
* -ution of w/. are arranged when if the D-3-ϕ

R.M.F is Produced.

RMF $- e = -\frac{d\phi}{dt}$

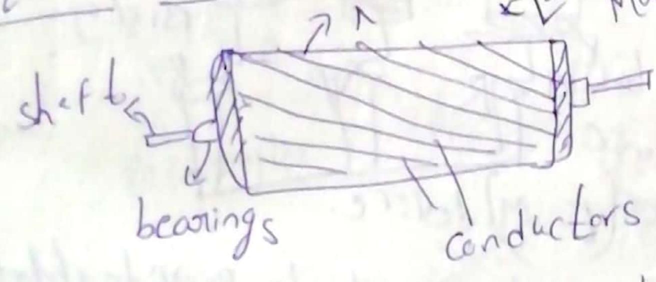
Torque is developed

Rotor: squirrel cage type of
slip ring (or) Phase wound

stator part is same

squirrel cage: cylindrical shape

* to Reduce magnetic hamming noise
* to Reduce the locking tendency



skewing → ||el (or) ⊥'er to the

The slots on the pherephary of the Motor (or) rotor ||ly arranged in skewed to the shaft.

slots are ||el to the each other are speed w.r.to the on's

rotor of shaft. This is done to prevent the magnetic teeth locking of your rotor slots & stator slots

reduce the cogging

Skewing: ||er (or) ⊥er

conductors — Aluminium, brass, copper.

endrings :- Cu, (or) Al rings.

* Rotor conductors are shorted to the end by the endrings.

Resistance is very small

to form a closed ckt

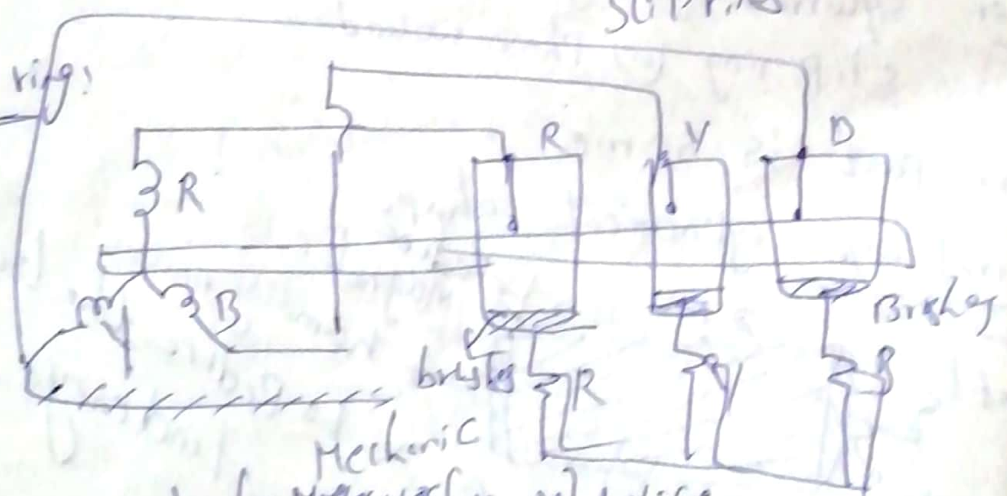
Ad: Maintenance free

commutators & brushes — Not use

cost less

used in :- industries, lathe, fans, printing etc.

slip ring:



Mechanic electromagnetic (E.M) device.

slip ring : transfer power (or) Ele. signal to provide to the rotor

three terminals

Brushes - power transfer to and from the rotor.

3 - Metal collar - 3 are short ckts

Ex. R :- reducing the starting current

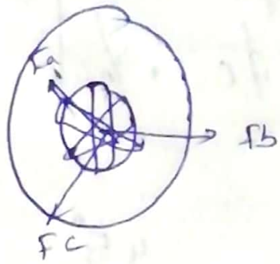
* high starting torque
and speed regulation

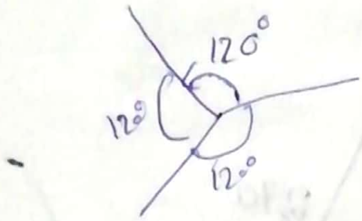
Sliprings - allowing the 'P' to the Brushes. used. I.M device

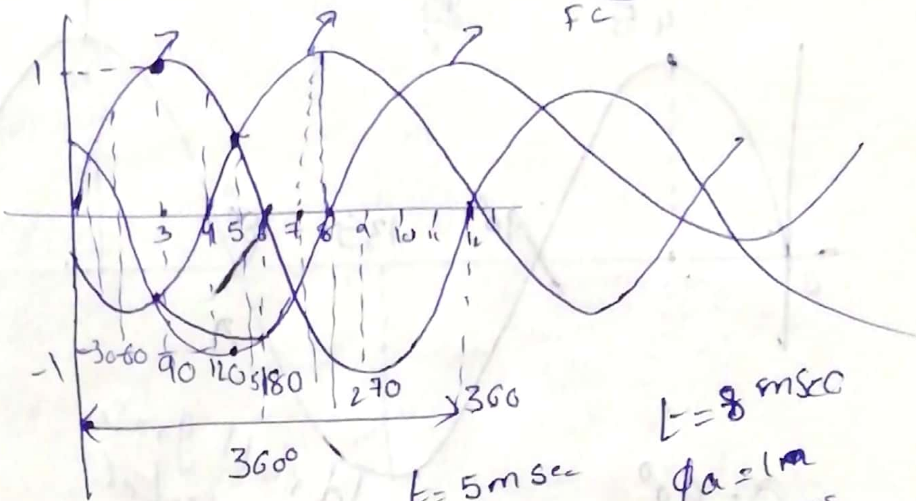
App: hoist cranes, elevators.

sliprings - phosphor - bronze.

Production of Rotating field:-







$\theta = 0°$; $\phi_1 = 0$; $\phi_2 = -\frac{\sqrt{3}}{2}$;

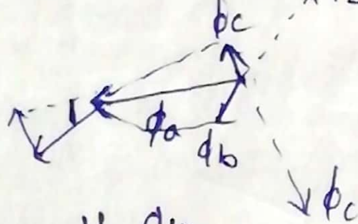
$\phi_3 = \sqrt{3}/2$

$t = 3$ msec
$\phi a = 1$
$\phi b = -0.5$
$\phi c = -0.5$
0°

$t = 5$ msec
$\phi a = 0.5$
$\phi b = 0.5$
$\phi c = -1$

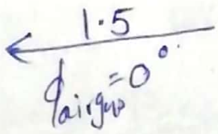
$t = 8$ msec
$\phi a = 1$
$\phi b = -0.5$

$t = 7$ msec. 7 msec
60°
120°
$\phi b = 1$
$\phi a = -0.5$
$\phi c = -0.5$

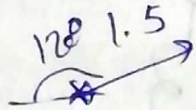


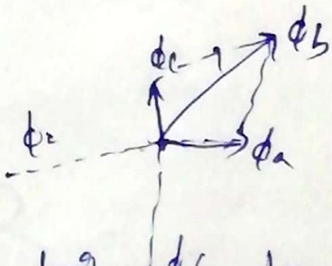
Result-flux

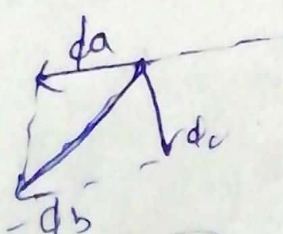
$\phi a = 0.5$
$\phi c = 0.5$
$\phi b = -1$



$\theta = 1$ gimsec.



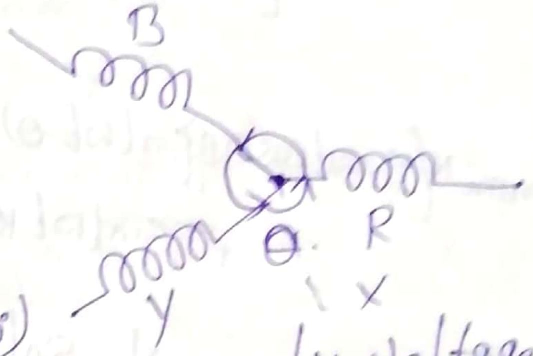




- stator has 3-φ winding wound for particular of poles
- stator is fed from 3-φ source, φ currents flow in the wirg.

Alternate method to obtain the speed and magnitude of the rotating Magnetic field:-

consider three coils R, Y, B, which are located in space such that their axis are 120 electrical degrees apart from each other as shown in fig.



$i_R, i_Y; i_B$

$i_R = I_{max} \sin \omega t$

$i_Y = I_{max} \sin(\omega t - 120°)$

$i_B = I_{max} \sin(\omega t + 120°)$

$\omega = 2\pi f \rightarrow$ frequency of the supply voltage.

Each winding produces a magnetic flux

MMF → No. of turns × current

MMF Produced by each winding is given by the product of the no. of turns and the current in coil. ox is any arbitary axis taken as 'θ'

The Resultant magnetic field (mmf) along ox at any instant of time t,

$M_R = i_R N \cdot \cos\theta$

$M_Y = i_Y N \cos(120 - \theta)$

$M_B = i_B N \cos(120 + \theta)$

The resultant M.M.F $M_F = M_R + M_Y + M_B$

$M_F = i_R N \cos\theta + i_Y N \cos(120-\theta) + i_B N \cos(120+\theta)$

$= N[i_R \cos\theta + i_Y \cos(120-\theta) + i_B \cos(120+\theta)]$

$= N[I_{max} \sin\omega t \cos\theta + I_{max} \sin(\omega t - 120)\cos(120-\theta) + I_{max} \sin(\omega t + 120)\cos(120+\theta)]$

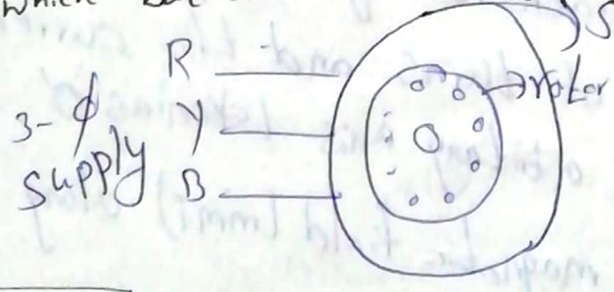
$= I_{max} N[\sin\omega t \cos\theta + \sin(\omega t - 120)\cos(120-\theta) + \sin(\omega t + 120)\cos(120+\theta)]$

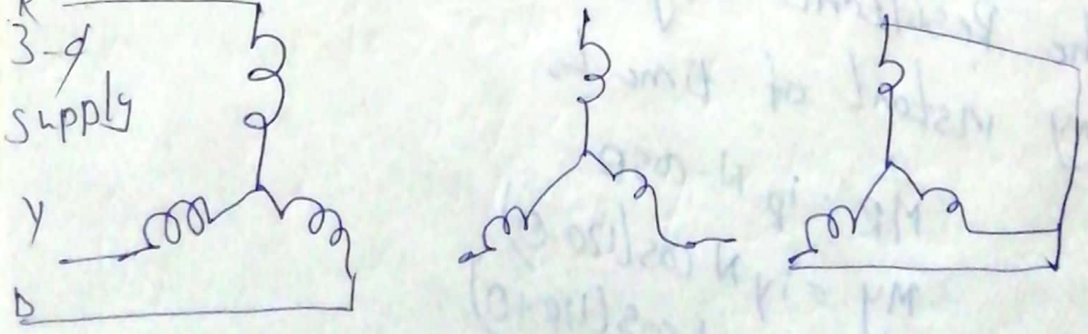
$= I_{max} N[\sin\omega t \cos\theta + \frac{1}{2}\{\sin(\omega t - \theta) + \sin(\omega t + \theta - 240)\} + \sin(\omega t + \theta + 240) + \sin(\omega t + \theta)]$

$M_f = \frac{3}{2} I_{max} N \sin(\omega t - \theta)$

Principle of operation:

* RMF is not only induced emf in the stator winding, which balances the supply voltage but also induces an emf in the rotor winding.





Sin A Sin B

$\sin A \cos B = \frac{1}{2}\left[\sin\left(\frac{A+B}{2}\right) + \sin\left(\frac{A-B}{2}\right)\right]$

$\sin C \sin D = 2\left[\sin\left(\frac{C+D}{2}\right) + \cos\left(\frac{C-D}{2}\right)\right]$

Rotor frequency:

$$\text{frequency} = \frac{\text{Poles} \times \text{relative speed b/w rotating flux and rotor structure in r.p.s}}{2}$$

* frequency f_2 of the rotor emf and current is equal to the line frequency f_1.

rotor → n_r - r.p.s

The relative speed b/w synchronously rotating flux and rotor conductors becomes $(n_s - n_r)$ rps.

$$f = \frac{P \times (n_s - n_r)}{2} \qquad \boxed{\frac{PN_s}{2} = f_1}$$

$$\text{slip} = \frac{N_s - N_r}{N_s}$$

$$\boxed{N_s - N_r = S \cdot N_s}$$

$$f = \frac{P \times S N_s}{2}$$

$$\boxed{f_2 = S \cdot f_1}$$

$\Rightarrow \boxed{n_r + SN_s = N_s}$ speed of rotor field w.r.t. stator.

$N_r = N_s - SN_s$

$\boxed{N_r = N_s(1-S)}$

Rotating Magnetic field set up by pole phase current is common for both stator and rotor winding. This field induced both of the winding. The stator induced emf in per phase is

$$E_1 = 4.44 K W_1 T_1 f \phi_m \quad \text{---} (1)$$

$KW \rightarrow$ winding factor

$E = 4.44 \phi f N$ $\quad T_1 \rightarrow$ No. of turns

$f \rightarrow$ stator frequency ; ϕ_m - Max flux

The rotor induced emf in per phase is

$$E_2 = 4.44 K W_2 T_2 f_2 \phi_m \quad \text{---} (2)$$

under stationary condition

$$f_1 = f_2$$

$$E_2 = 4.44 K W_2 T_2 f \phi_m \quad \text{---} (3)$$

$(3)/(1) \Rightarrow \dfrac{E_2}{E_1} = \dfrac{4.44 K W_2 T_2 f_r \phi_m}{4.44 K W_1 T_1 f \phi_m}$

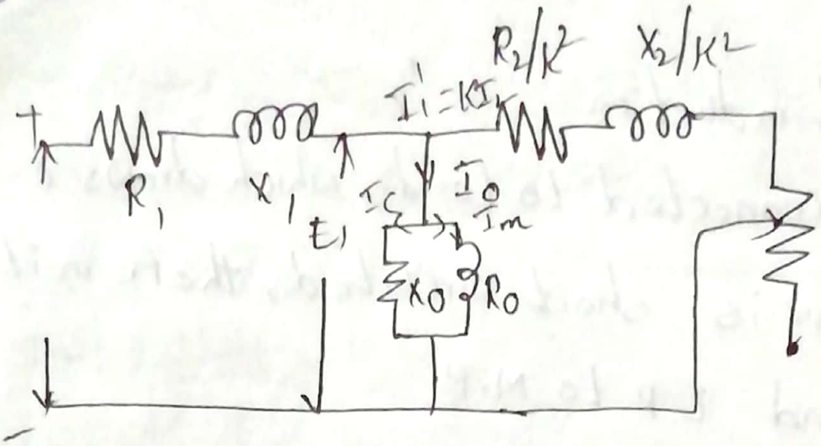
$$\boxed{\frac{E_2}{E_1} = \frac{K W_2 T_2}{K W_1 T_1}}$$

from eqn (2) rotor current under running condition

$$E_2 = 4.44 K W_2 T_2 f_r \phi_m = S E_{2s}$$

induced emf in the rotor ckt is Max at the star Value of Normal slip under loaded condition is nearly 5% and rotor induced emf is nearly 20% of the Max. Value.

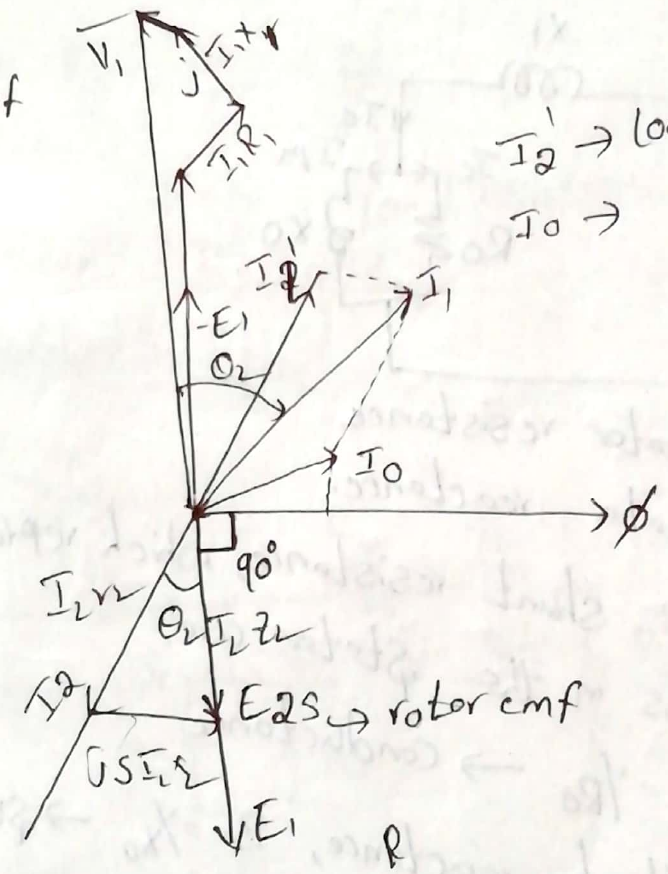
Equivalent ckt:

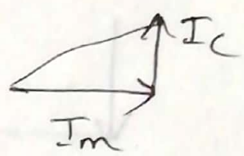


P.f → nature of Load

I_2' → load component

I_0 →





$G_0 = 1/R_0$ → conductance

$B_c = 1/X_0$ → susceptance

* Mutual induction

$f_r = sf$ → E.P to M.P

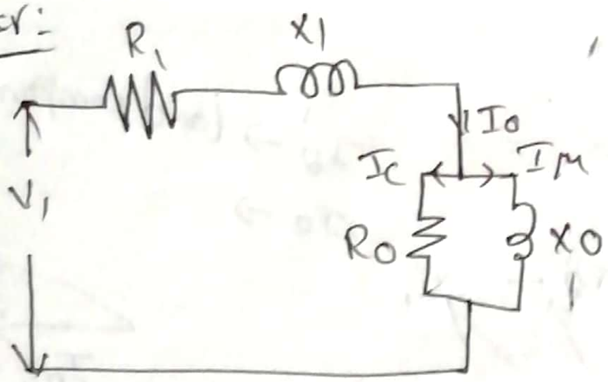
No-load current $= I_0$

$I_0 = I_c - jI_\mu$;

Stator, Rotor
↓
Mutual induction

* Rotor is connected to load, which draws E.P
* I.M rotor is short circuited, therfr in it is fr-sf and E.P to M.P

Stator:



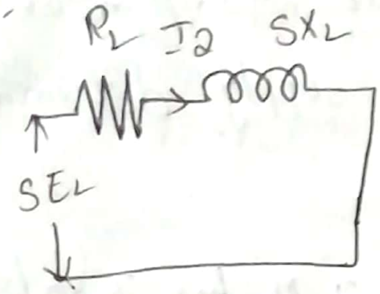
R_1 = stator resistance
X_1 = stator reactance.

* R_0 is the shunt resistance, which represents the iron loss in the stator core,
  $G_0 = 1/R_0$ → conductance
* X_0 → shunt reactance, $B_0 = 1/X_0$ → susceptance
* V_1 → line voltage to the stator,
  when the motor delivers no power o/p, the i/p is equal to the stator losses only

No load current = I_0

$I_0 = I_c - jI_M$

Rotor:



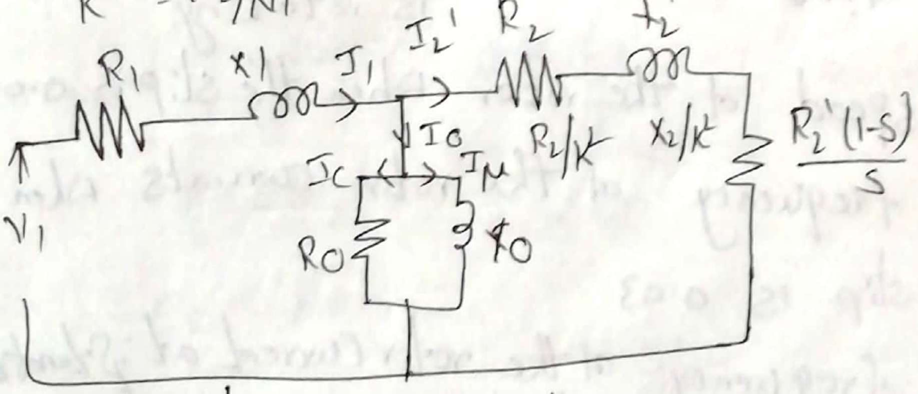
$$I_2 = \frac{SE_2}{\sqrt{R_2^2+(SX_2)^2}} \quad (\text{or}) \quad \frac{E_2}{\sqrt{(r_2/s)^2+X_2^2}}$$

$$R_2/s \Rightarrow \frac{R_2}{s} = \frac{R_2}{} + \frac{R_2(1-s)}{s}$$

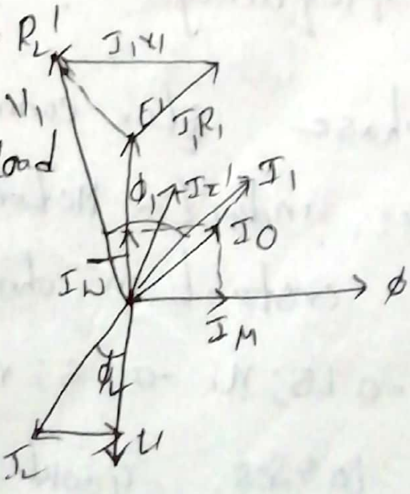
R. R Rotor ↓ Load resistance

∴ copper loss is $\frac{I_2^2 R_2(1-s)}{s}$

$K = N_2/N_1$



$$\left(\frac{N_1}{N_2}\right)^2 R_2 = R_2/K^2 = R_2'$$



$I_M \rightarrow$ 30% (or) 40% of full load current in the I.M

Torque Equation:-

The torque produced in the induction motor depends on the following factors

i, $T \propto \phi I_{2r} \cos\phi_{2r}$

$\phi \rightarrow$ flux responsible to produce induced emf

$I_{2r} \rightarrow$ rotor Running current

$\cos\phi_{2r} \rightarrow$ running pf of rotor

The flux ϕ produced by stator is proportional to E_1 i.e., stator voltage $\phi \propto E_1$

$$\frac{E_2}{E_1} = K \Rightarrow E_2 = K \cdot E_1$$

$$E_2 \propto E_1$$

$$E_2 \propto \phi$$

$$I_2 = \frac{E_2}{\sqrt{(r_2/s)^2 + X_2^2}} = \frac{SE_2}{\sqrt{R_2^2 + (SX_2)^2}}$$

$$\cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + (SX_2)^2}}$$

$$T \propto E_2 \cdot \frac{SE_2}{\sqrt{R^2 + (SX_2)^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + (SX_2)^2}} \Rightarrow T \propto \frac{S E_2^2 R_2}{R_2^2 + (SX_2)^2}$$

$$T = K \frac{SE_2^2 R_2}{R_2^2 + (SX_2)^2} \Rightarrow K = \frac{3}{2\pi n_s}$$

$$n_s = N_s/60$$

$$T = \frac{3}{2\pi n_s} \cdot \frac{S E_2 R_2}{R_2^2 + (S X_2)^2} \; N\text{-}m$$

Starting torque:

$$T_{st} = \frac{3}{2\pi n_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2} \; N\text{-}m$$

(S = 1) Standstill condition.

changing $R_2 \rightarrow T_{st}$ can be controlled

$$\frac{u}{v} = \frac{u'v + vu'}{v^2}$$

Maximum Torque:

$$dT/ds = 0$$

$$T = K \cdot \frac{S E_2^2 R_2}{R_2^2 + (S X_2)^2} = K \cdot \frac{S E_2^2 R_2}{R_2^2 + S^2 X_2^2}$$

$$\frac{dT}{ds} = \frac{(K S E_2^2 R_2) \, d/ds (R_2^2 + S^2 X_2^2) - (R_2^2 + S^2 X_2^2) \frac{d}{ds}(K S E_2^2 R_2)}{R_2^2 + S^2 X_2^2}$$

$$= (K S E_2^2 R_2) 2 S X_2^2 - R_2^2 + S^2 X_2^2 \cdot K E_2^2 R_2 = 0$$

$$2 K S^2 E_2^2 R_2 X_2^2 - (R_2^2 + S^2 X_2^2) K E_2^2 R_2 = 0$$

$$2 K S^2 E_2^2 R_2 X_2^2 = K E_2^2 R_2^3 + S^2 K R_2 E_2^2 X_2^2$$

$$K S^2 E_2^2 R_2 X_2^2 = K E_2^2 R_2^3$$

$$S^2 X_2^2 = R_2^2 \Rightarrow S^2 = \frac{R_2^2}{X_2^2} \Rightarrow \boxed{S = R_2 / X_2}$$

$S_m = R_2/X_2$

Magnitude of Max. Torque:-

$$S_m = R_2/X_2$$

$$T_m = \frac{K\, S_m E_2^2 R_2}{R_2^2 + (S_m X_2)^2}$$

$$= \frac{K \cdot R_2/X_2 \; E_2^2 \cdot R_2}{R_2^2 + \left(\frac{R_2}{X_2} \cdot X_2\right)^2}$$

$$= \frac{K R_2^2 E_2^2}{2 R_2^2 X_2} = \frac{K E_2^2}{2 X_2}$$

$$T_m = \frac{K E_2^2}{2 X_2} \quad N\text{-}m$$

Maximum Torque is not dependent on Rotor resistance R_2.

* A 3-ϕ, 400V, 50Hz, 4 Pole induction motor has star connected Stator winding. The rotor resistance and reactance are 0.1 ohms, 1 ohm respectively. The full load speed is 1440 r.p.m calculate torque developed on full load by the motor. Assume stator to rotor ratio as 2:1

... 4 pole 3-ϕ 50Hz stator connected ... rotor resistance and reactance per phase equal 0.01 ohm and 0.1 ohm respectively determine

i, starting torque

ii, slip at which max. torque will occur

iii, speed at which max torque will occur

iv, Max. Torque v, full load torque — 4%

Assume ratio of stator to rotor turns as 4

Torque slip characteristics:-

* No load to full load
  speed decreases hence slip increases.

The curve obtained by plotting torque produced against slip of I.M from S=1 to S=0 is called torque - slip characteristics.

$$T \propto \frac{SR_2}{R_2^2+(SX_2)^2}$$

i, low slip region.

$$T \propto \frac{SR_2}{R_2^2} \Rightarrow T \propto S/R_2 \rightarrow R_2 \text{ is constant}$$

$$T \propto S$$

$T \propto S \Rightarrow S=0;\ T=0;$ $\boxed{N=N_s}$

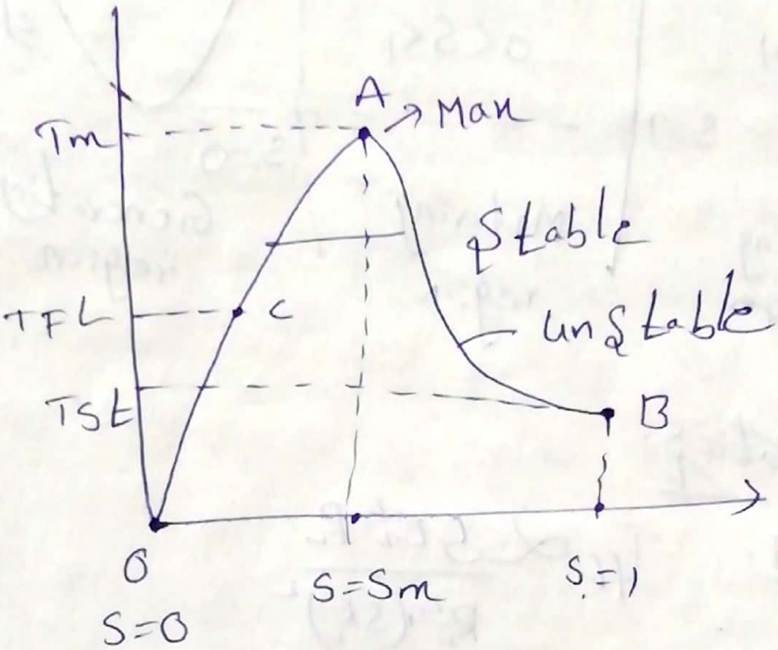
ii, High slip:

$$T \propto \frac{SR_2}{R_2^2+(SX_2)^2} \qquad T \propto \frac{SR_2}{R_2^2+S^2X_2^2}$$

$$T \propto 1/S$$

1α S, 1 α^2 / s
low high

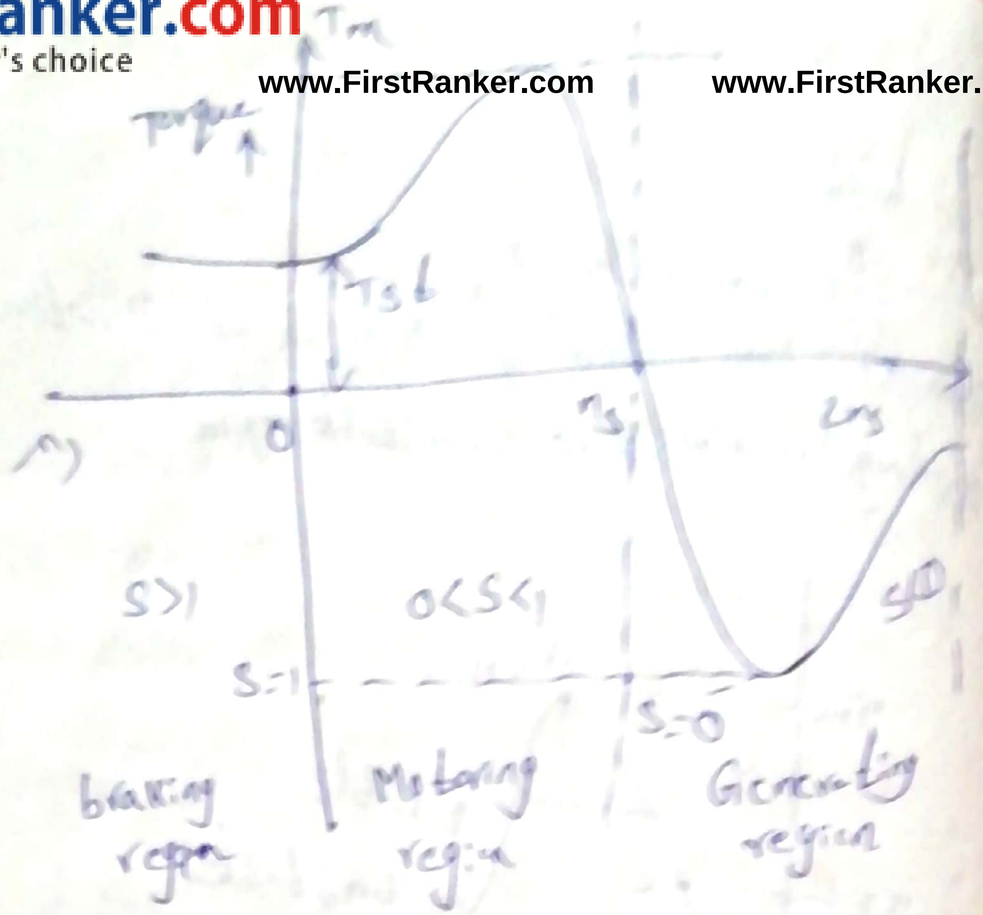
* Torque - slip characteristics
→ straight line called stable region of operation
→ Rectangular hyperbola — unstable region "



* in low slip region, as load increases, slip ↑ and torque also increases linearly.

S=Sm ; Max torque - Tm

* If load is increased beyond the limit, motor slip acts dominantly pushing motor into high slip region due to unstable condition i.e, Max torque which motor can produce is also called break down (or) pull out torque. s=0 to s=Sm → low slip stable.



Torque Ratios:

F.L & M.T: $T_{fl} \propto \dfrac{S E_2^2 R_2}{R_2^2 + (S X_2)^2}$

$T_{max} \propto \dfrac{E_2^2}{2X_2}$

$\dfrac{T_f}{T_m} = \dfrac{S E_2^2 R_2}{R_2^2+(SX_2)^2} \times \dfrac{2X_2}{E_2^2}$

$= \dfrac{2SR_2X_2}{R_2^2+S^2X_2^2}$

$R_2/X_2 = S_m$

$\dfrac{T_f}{T_m} = 2SR_2X$ $\Rightarrow \dfrac{2SR_2X_2}{X_2^2\left(\frac{R_2^2}{X_2^2}+S^2\right)}$

$$\frac{T_f}{T_m} = \frac{2S\,S_m}{S_m^2+S^2}$$

$$T \propto \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

$$\frac{T_{st}}{T_m} = \frac{E_2^2 R_2}{R_2^2 + X_2^2} \times \frac{2X_2}{E_2^2}$$

$$T \propto \frac{E_2^2}{2X_2}$$

$$\frac{T_{st}}{T_m} = \frac{2R_2 X_2}{R_2^2 + X_2^2}$$

$$\Rightarrow \frac{2R_2/X_2}{X_2^2\left(\frac{R_2^2}{X_2^2}+1\right)} = \frac{2S_m}{S_m^2+1}$$

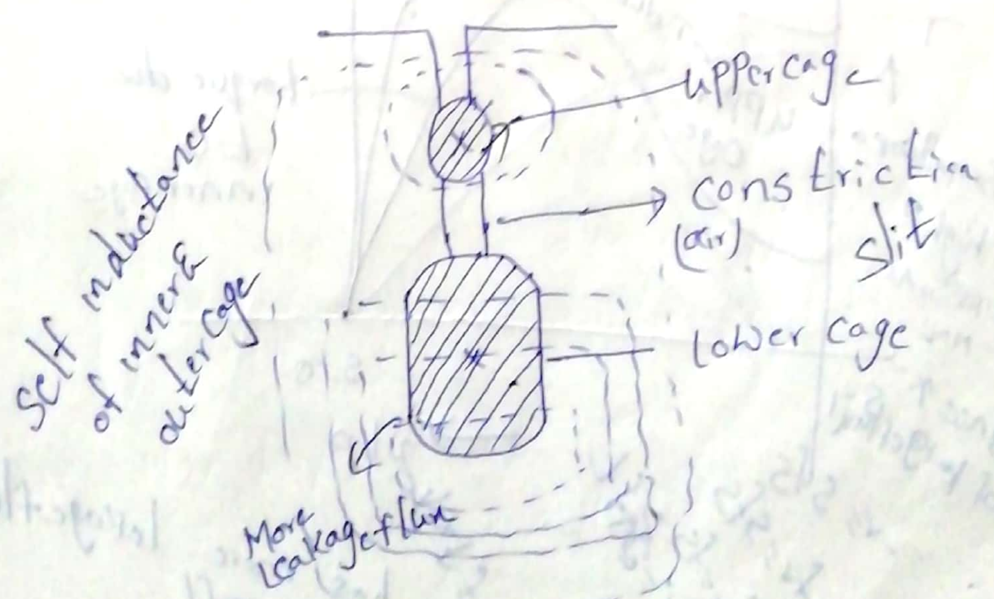
$T =$

Pb: A 24 pole, 50Hz, star connected induction motor has rotor resistance of 0.016 ohm per phase and rotor reactance of 0.265 ohm per phase at standstill. It is achieving its full load torque at a speed of 247 r.p.m calculate the ratio of

i, full load torque to max. torque

ii, starting torque to max. torque.

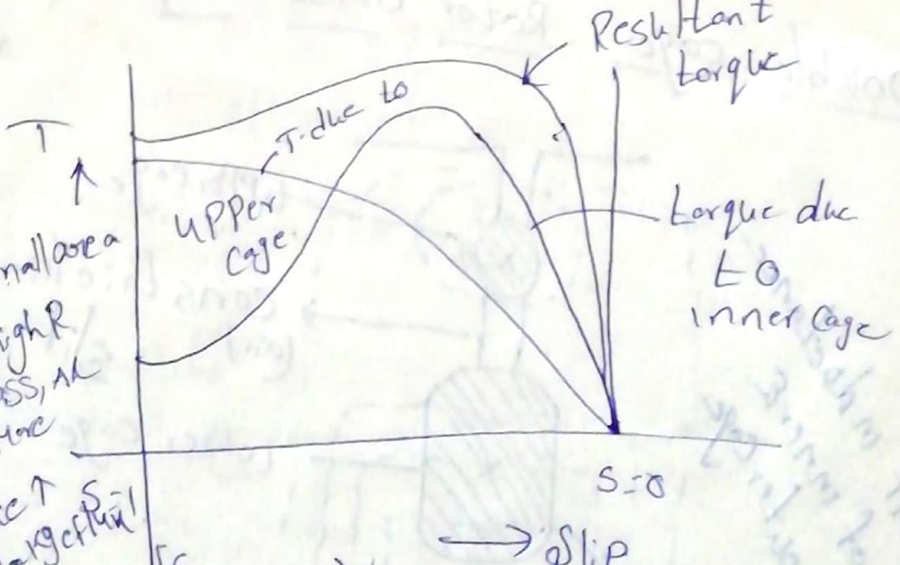
Double cage Rotor construction: $R = \rho l/A$



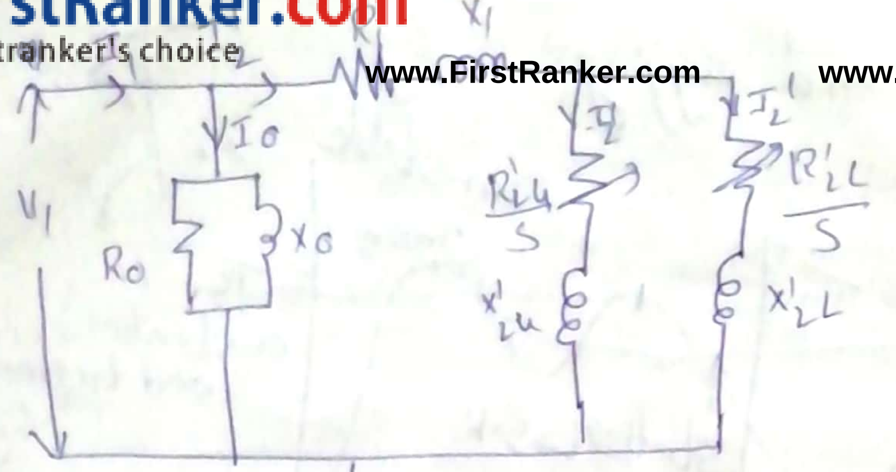
Better Performance than deep bar rotor

* The stator of double cage rotor induction motor is same as that of ordinary IM where as rotor consists of two cages (or) two layers of bars short circuited by end rings
* Since the upper cage is having smaller cross-sectional area than the lower cage.
* The upper cage is higher resistance than that of lower cage with equal cross-sectional areas of two cages the upper cage is made up of high 'R' Material like brass, Al, bronze etc.
* lower cage is made up of low 'R' material like copper.

$f_r = sf_s$

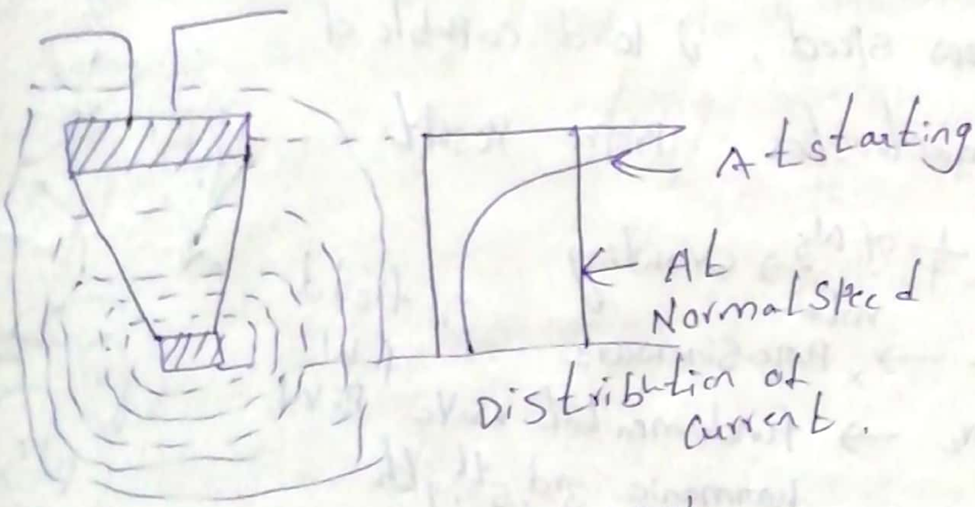


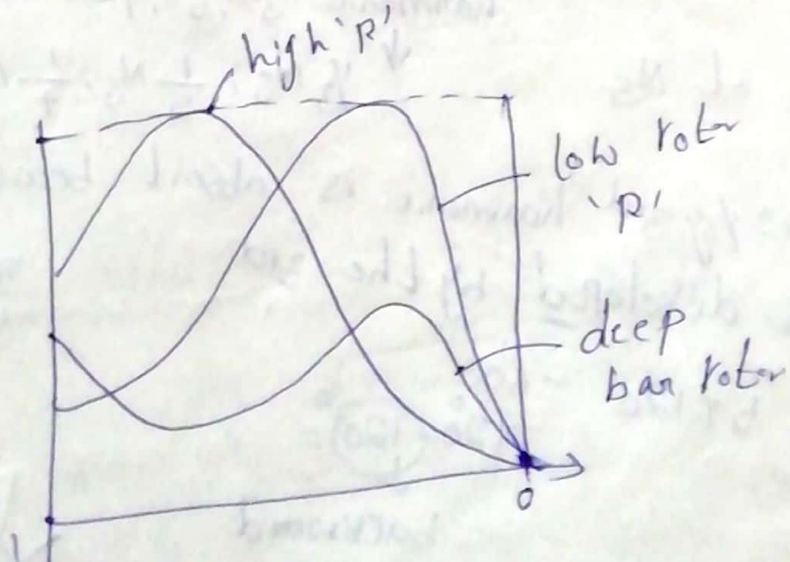
outer cage :- Small area high R brass, Al
Inner cage :- More
slot :- permeance ↑ of leakage flux
$f_r = sf_s$
$x_2 = s x_2$
inner cage has more leakage flux
$x = 2\pi f L$



double cage I.M

Deep bar construction:





* High starting Torque

Stator side:

* Supply frequency control to control N_s
  ↳ v/f control.
* supply voltage control
* controlling no. of stator poles to control N_s.
* Adding rheostats in stator circuit

Rotor side:

* Adding external -R' in the rotor circuit.
* cascade control
* injecting slip frequency voltage.

Supply frequency control: disAd:

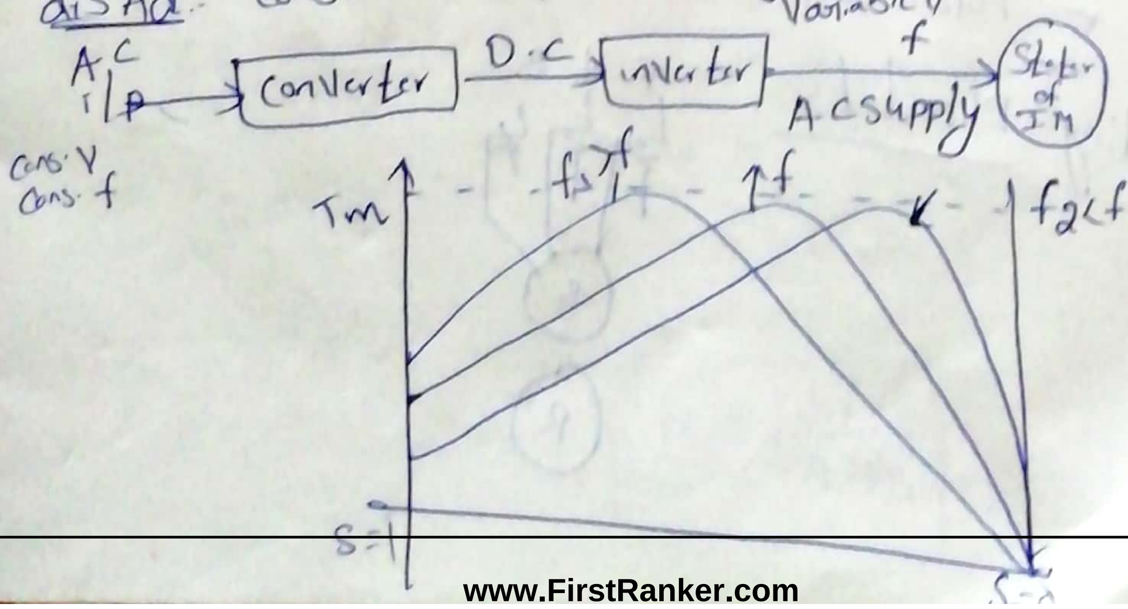
$$N_s = \frac{120f}{P}$$

$$E = 4.44 K_w T_{ph} \phi_m f$$

$$\phi_m = \frac{E}{4.44 K_w T_{ph} f} = \frac{1}{4.44 K_w T_{ph}} \cdot (V/f)$$

V/f → constant.

disAd: constant → V/f → costly.



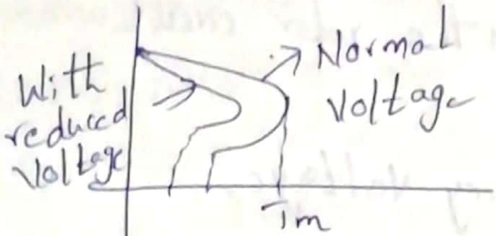
Supply voltage control

$$T \propto \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2}$$

$$E_2 \propto V$$

$$T \propto \frac{sV_2^2 R_2}{R_2^2 + (sX_2)^2} \Rightarrow \propto \frac{sV_1^2 R_2}{R_2^2}$$

$$T \propto sV^2 \rightarrow \text{low slip}$$



* Large change in voltage and small change in speed is required is the biggest disadvantage
* Due to ↑ I, the motor may get overheated

Controlling No. of Poles

'Pole changing' method.

* consequent poles method.
* Multiple stator winding method.
* Pole amplitude modulation method

Adding Rheostats

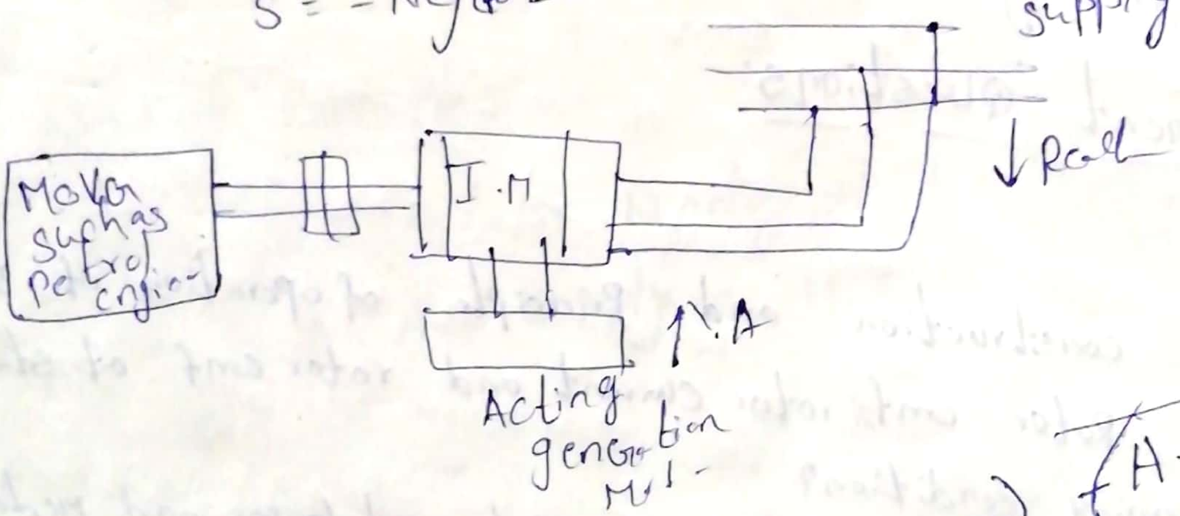
Induction Generator:-

* To run the induction Machine as a generator its slip must be less than zero i.e. Negative

$$S = \frac{N_s - N_r}{N_s} \times 100$$

$N_r \rightarrow$ above synchronous speed

S = "Negative"



$$\Rightarrow \; +(A - jB)$$

$$I_2 = \frac{SE_2}{R_2 + jSX_2} = \frac{E_2}{R_2/s + jX_2}$$

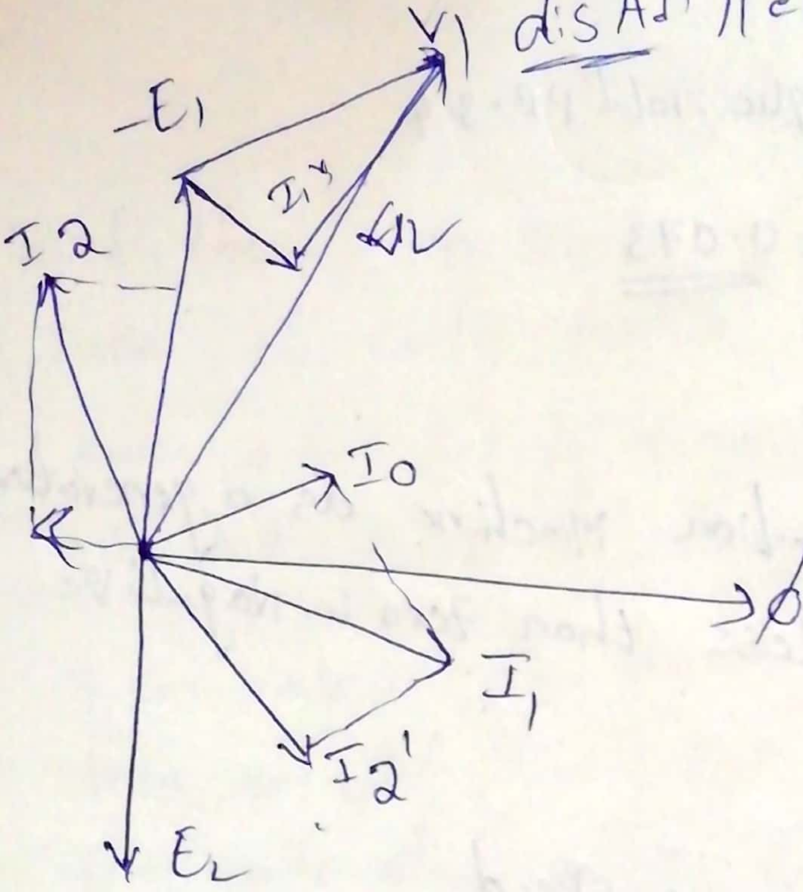
$$= \frac{E_2 \left[R_2/s - jX_2 \right]}{\left(R_2/s + jX_2\right)\left(R_2/s - jX_2\right)}$$

$$= \frac{E_2 \left(R_2/s \right)}{\frac{R_2^2}{s^2} + X_2^2} - j\frac{E_2 X_2}{R_2^2/s^2 + X_2^2}$$

Phasor:

Ad: for high speeds.
s.c → reduces to zero.

disAd: ‖el with syn. Machine



railway for braking purposes.

UNIT-III

Measuring Instruments

Measurement: It is the process of comparison between the unknown quantity to standard unit & knowing the result of unknown quantity in terms of standard unit.

Instrument: It is a device that allows us to make this comparison.

Classification of Instruments:

- Instrument
  - Based on Methodology
    - Absolute
    - Secondary
      - Based on operation
        - Analog
        - Digital
  - Based on Indication at the end of m'ment
    - Deflection
    - Null deflection
  - Based on o/p
    - Indicating
    - Integrating
    - Recording

Essential Requirements of Indicating Instruments

there are 3 Torques which are responsible for indicating effective reading

(i) Deflecting Torque
(ii) Controlling Torque
(iii) Damping Torque

(i) Deflecting Torque: Deflecting Torque is used to deflect the pointer away from zero position. it is produced by various effects. those are

→ Magnetic Effect
→ thermal "
→ Electrostatic "
→ Induction "
→ Hall Effect

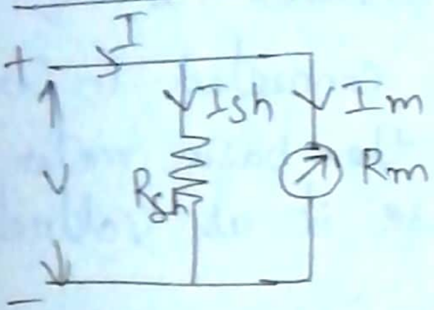
(ii) Controlling Torque: It is used to control the motion of the pointer. it is provided by → Spring control
→ Gravity control Methods.

(iii) Damping Torque: It is used to remove the oscillations of the pointer

Damping Torque is provided by 3 methods

1. Airfriction damping
2. fluid friction "
3. Eddy current "

Basic DC Ammeter:



voltage drop a/c Rsh = vol drop a/c Rm

$$\Rightarrow I_{sh} R_{sh} = I_m R_m$$

$$R_{sh} = \frac{I_m R_m}{I_{sh}} \rightarrow (1)$$

from fig, $I = I_m + I_{sh}$

$$\Rightarrow I_{sh} = I - I_m$$

$$(1) \Rightarrow \quad R_{sh} = \frac{I_m R_m}{I - I_m}$$

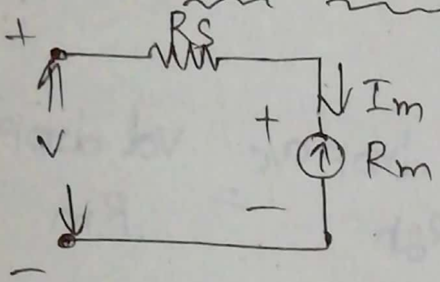
$$R_{sh} = \frac{R_m}{\left(\frac{I}{I_m}\right) - 1}$$

$$R_{sh} = \frac{R_m}{m-1}$$

$$\Rightarrow \boxed{m = \frac{R_m}{R_{sh}} + 1}$$

Thus to increase the range of ammeter 'm' times, R_{sh} req is $\frac{1}{m-1}$ times R_m. this is nothing but extension of ranges of an Ammeter.

Basic D.C. voltmeter:



The Resistance is req to be connected in Series with the basic meter to use it as voltmeter

from fig $V = I_m(R_m + R_s)$

$$\Rightarrow I_m R_s = V - I_m R_m$$

$$R_s = \frac{V}{I_m} - R_m$$

$$m = \text{multiplying factor} = \frac{V}{v}$$

$$m = \frac{I_m(R_m + R_s)}{I_m R_m}$$

$$m = 1 + \frac{R_s}{R_m}$$

hence multiplier Resistance

$$\boxed{R_s = (m-1) R_m}$$

Permanent Magnet Moving coil Instrument (PMMC):

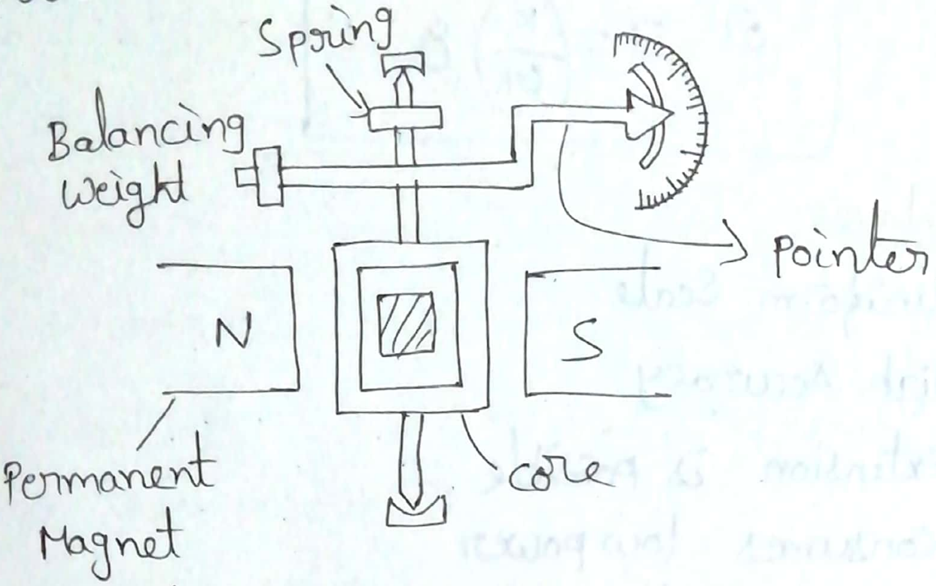


fig: Construction of PMMC

PMMC mainly works on the motoring principle. when a current carrying coil is placed in Mag field, it experiences a force & pointer moves. this basic principle is called D'Arsonval principle

Deflecting Torque is given by

$$T_d = NBAI$$

$$T_d = GI$$

Controlling Torque is provided by Spring

$$T_c \alpha \theta$$

$$T_c = K\theta$$

At steady state $T_d = T_c$

$$\theta = \left(\frac{G}{K}\right) I$$

$$\text{or} \quad I = \left(\frac{K}{G}\right)\theta$$

Advantages:
-> uniform Scale
-> High Accuracy
-> Extension is possible
-> consumes low power
-> High Sensitivity

Disadvantages:
-> Suitable for d.c m'ments only
-> Aging of permanent Magnets introduce Errors
-> High cost

* Moving Iron Instruments

these instruments are classified as

(i) MI Attraction type Inst

— the Basic working principle of these Inst is a soft iron piece if brought near the magnet gets attracted by Magnet.

(ii) MI Repulsion type Inst

Due to the repulsion of like

polarities, there is a force of repulsion b/w the 2 Vanes causing the movement of the Vane.

Torque Equation

Consider a small increment in current,

Mechanical work $= T_d \cdot d\theta$

In order to effect an increment dI in the current,

applied voltage $e = \frac{d(LI)}{dt}$

$= I\frac{dL}{dt} + L\frac{dI}{dt}$

elec. Energy $= eIdt = I^2 dL + ILdI$

change in stored Energy

$= \frac{1}{2}(L+dL)(I+dI)^2 - \frac{1}{2}LI^2$

$\therefore$ Energy Supplied = stored Energy + Mech work

$I^2 dL + ILdI = ILdI + \frac{1}{2}I^2 dL + T_d\, d\theta$

$\Rightarrow T_d \cdot d\theta = \frac{1}{2}I^2 dL$

$$\Rightarrow T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

$T_c = K\theta$

At steady state $T_c = T_d$

$$\theta = \frac{1}{2}\frac{I^2}{K}\frac{dL}{d\theta}$$

∴ MI Instrument gives Square law response

Adv:

→ used for both a.c. & d.c. measurements
→ friction Error is very less
→ wide range
→ good accuracy
→ it can be Extended
→ Rugged & Reliable.

Disadvantages:

→ Initially Cramped Scale
→ High power consumption
→ Errors due to Hysteresis, freq, stray Magnetic fields.

* Electrodynamometer type Instruments

It is a Transfer Instrument. it is one which is calibrated with a d.c. source & used without any modification for a.c. measurements.

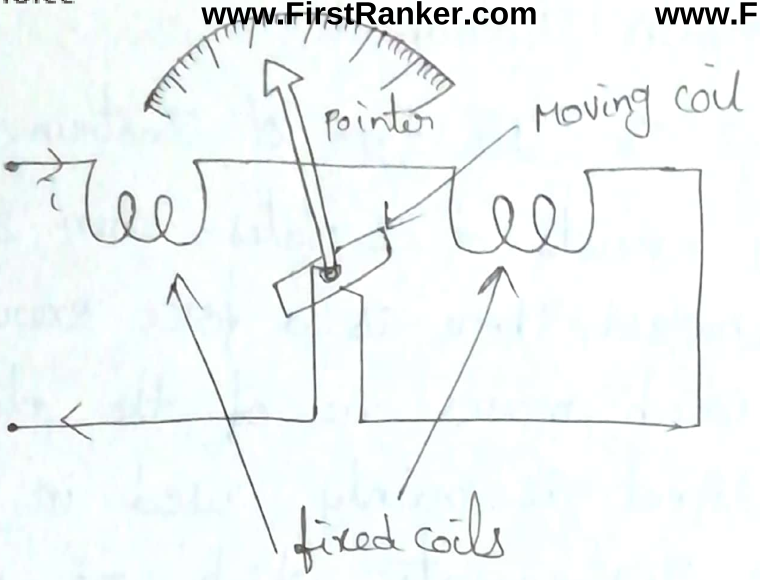
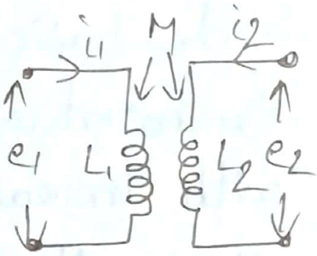


fig: Electrodynamometer type Instrument

Average deflecting Torque over 1 cycle is

$$\tau_d = \frac{1}{T}\int_0^T \tau_i \, dt$$



$$\tau_d = \frac{dM}{d\theta} \cdot \frac{1}{T}\int_0^T i_1 i_2 \cdot dt$$

Let $i_1 = I_{m1} \sin\omega t$

$i_2 = I_{m2} \sin(\omega t - \phi)$

$$\tau_d = \left(\frac{I_{m1} I_{m2}}{2}\right) \cos\phi \frac{dM}{d\theta}$$

$$= I_1 I_2 \cos\phi \frac{dM}{d\theta}$$

I_1, I_2 are RMS values of I_{m1} & I_{m2} resly

At steady state $\tau_c = \tau_d$ $[\tau_c = K\theta]$

$$\boxed{\theta = \frac{I_1 I_2}{K} \cos\phi \frac{dM}{d\theta}}$$

*. Electrostatic Instruments:

In this type of Instruments, it mainly consists of 2 plates. when 2 plates are charged, there is a force exerted b/w them, which moves one of the plates. this effect is mainly used in electro static Instruments which are normally voltmeters.

*. Errors and Compensation:

→ the basic PMMC Inst is Sensitive to temperature. coil resistance Increases with Increase in temperature.
Hence the temperature compensation is provided by Appropriate use of Series & shunt resistances of copper & manganin.

→ To reduce frictional errors in PMMC, Torque to weight ratio is made very high.

→ the proper use of material & preaging during manufacturing can reduce the errors due to weakening of control springs

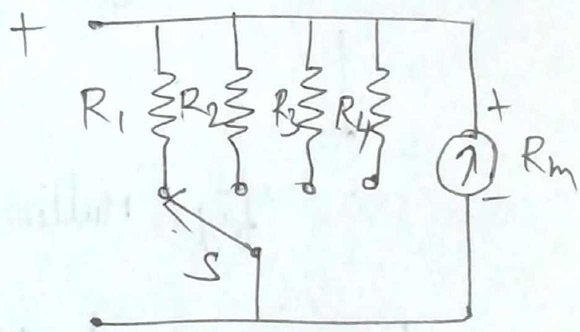
→ To reduce hysteresis error in [illegible], use smaller iron parts which can demagnetise quickly or to work with lower flux densities.

→ Resistivity of the metal parts used must be high to reduce Eddy currents in Electrodynamo Insts.

→ To reduce the effect of stray mag field, shields must be used for electro dynamo Insts.

* Extension of range using shunt Resistance

the range of the basic d.c. ammeter can be extended by using no. of shunts & selector switch.



All the shunts are very precise resistances & cost is high.

$$R_1 = \frac{R_m}{m_1 - 1}$$

$$R_2 = \frac{R_m}{m_2 - 1}$$

$$R_3 = \frac{R_m}{m_3 - 1}$$

$$R_4 = \frac{R_m}{m_4 - 1} - - - -$$

where m_1, m_2, m_3 --- are the shunt multiplying powers for the currents I_1, I_2, I_3 - - -

* Extension of range using Series Resistance

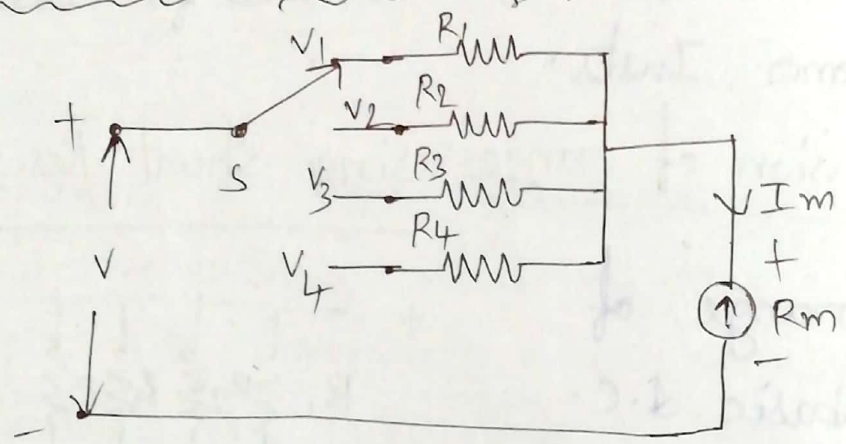


fig: Multirange voltmeter

the range of basic D.C. voltmeter can be extended by using Series Resistances. Such a meter is called multirange voltmeters.

$$R_1 = \frac{V_1}{I_m} - R_m$$

$$R_2 = \frac{V_2}{I_m} - R_m \quad - -$$

where Rm → meter Resistance

Sensitivity $S = \frac{1}{\text{full scale deflection current}}$

thus the high sensitivity voltmeter gives more accurate reading

* Instrument Transformers:

In heavy currents & high vol a.c. ckts, a specially constructed accurate ratio T/F are used. it is called as Instrument T/F. it is classified as (i) current Transformer
(ii) Potential Transformer

* Current Transformer:

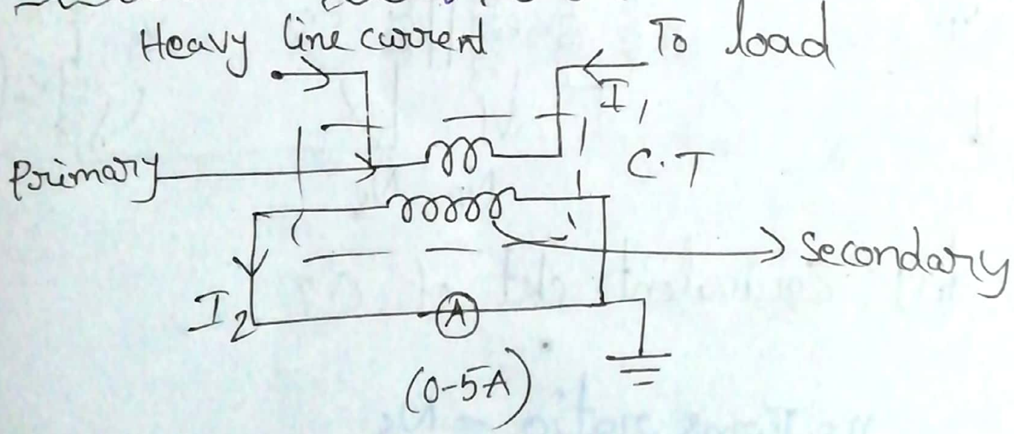


fig: Current Transformer

the large Alternating currents which cannot be sensed or passed through normal ammeters & current coils of wattmeters, Energy meters can easily be measured by use of C.T. along with normal low range Instruments.

working principle:

these T/F are basically step up T/F i.e, stepping up a voltage from primary to Secondary.

for a T/F $\frac{I_1}{I_2} = \frac{N_2}{N_1}$

theory of C.T:

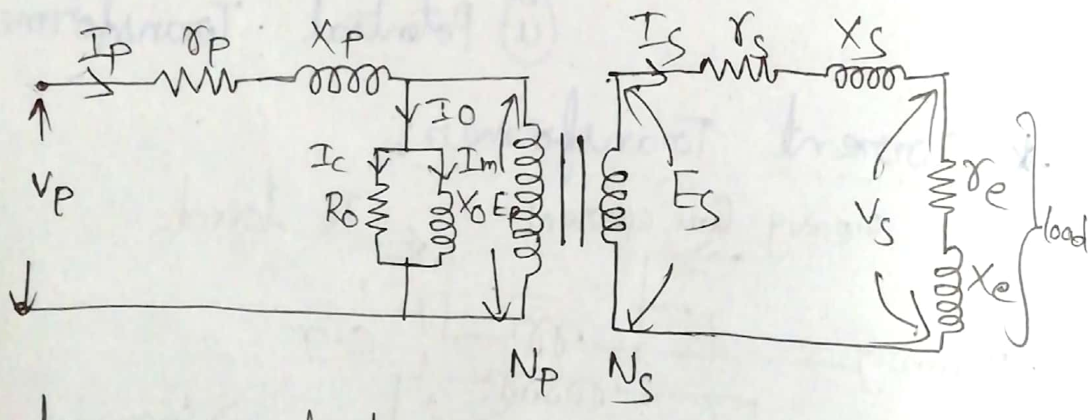


fig: Equivalent ckt of C.T.

n = Turns ratio $= \frac{N_s}{N_p}$

Actual ratio

$$R = \frac{\text{Mag of Actual primary Current}}{\text{Mag of Actual Secondary current}}$$

It is necessary that the T/F ratio must be exactly equal to turns ratio & phase of secondary terms must be displaced by exactly 180° from that of primary. 2 types of errors effects ch's of I.T.

1. Ratio Error:

$$\% \text{ Ratio Error} = \frac{\text{Nominal ratio} - \text{Actual ratio}}{\text{Actual ratio}} \times 100$$

$$= \frac{K_n - R}{R} \times 100$$

2. phase angle Error:

it is denoted by angle θ by which the phase diff b/w primary & secondary is different from 180°

$$\theta = \frac{180}{\pi}\left[\frac{I_m \cos\delta - I_c \sin\delta}{n I_s}\right]$$

Approximate results

δ is +ve & very small

$$\theta = \frac{180}{\pi}\left(\frac{I_m}{I_s}\right)$$

* Potential Transformers:

the basic principle of these T/F is Same as current transformers. the high alternating voltage are reduced in a fixed proportion for the measurement purpose with the help of potential Transformers.

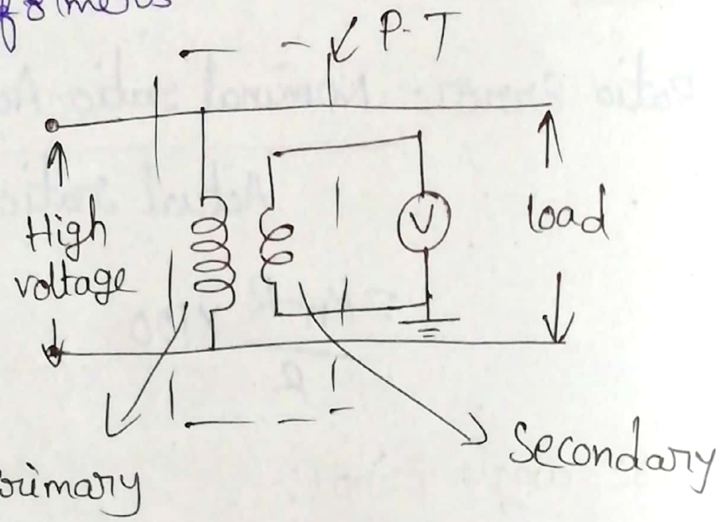
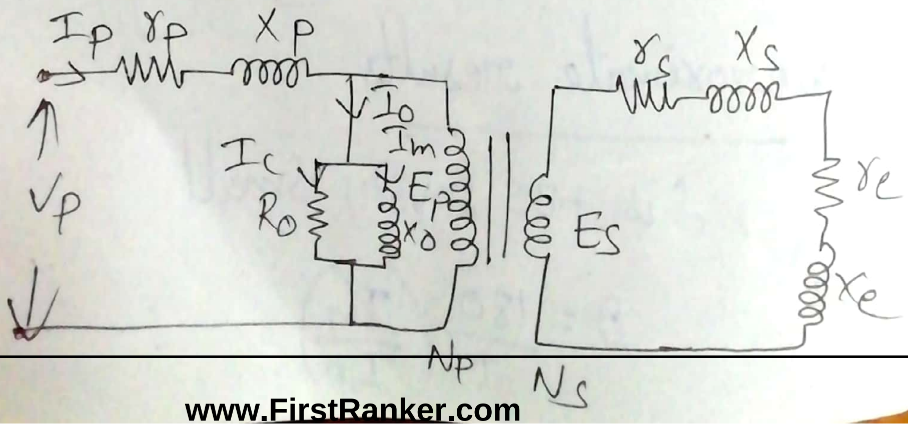


fig: Potential Transformer

As a normal T/F, its ratio

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

Equivalent circuit:



for P.T $n = \frac{N_p}{N_s} = \frac{E_p}{E_s}$.

it can be noted that the phase angle θ is treated positive when V_s reversed i.e, nV_s leads the primary winding voltage V_p. the θ is treated negative when nV_s lags the primary winding voltage V_p.

once R & θ are obtained then the errors in potential T/F are

$$\% \text{ ratio error} = \frac{K_n - R}{R} \times 100$$

$$\text{phase angle error} = \theta \text{ radians}$$

* Reduction of Errors in Instrument Transformer:

Design Considerations

the ratio & phase angle errors can be minimised by following methods

(i) Reducing the core loss & magnetising component of I_0

→ choosing low Inductance core

→ high permeability materials

→ using large cross section of core

(ii) Reduction of resistance & leakage reactance

The Resistance can be reduced by Increasing cross section of conductors & decreasing the length of the mean turn.

The leakage reactances depend on the leakage fluxes keeping flux density as high as practicable, the no. of turns req are less. hence leakage reactances get reduced.

(iii) Providing turns Compensation

for P.T

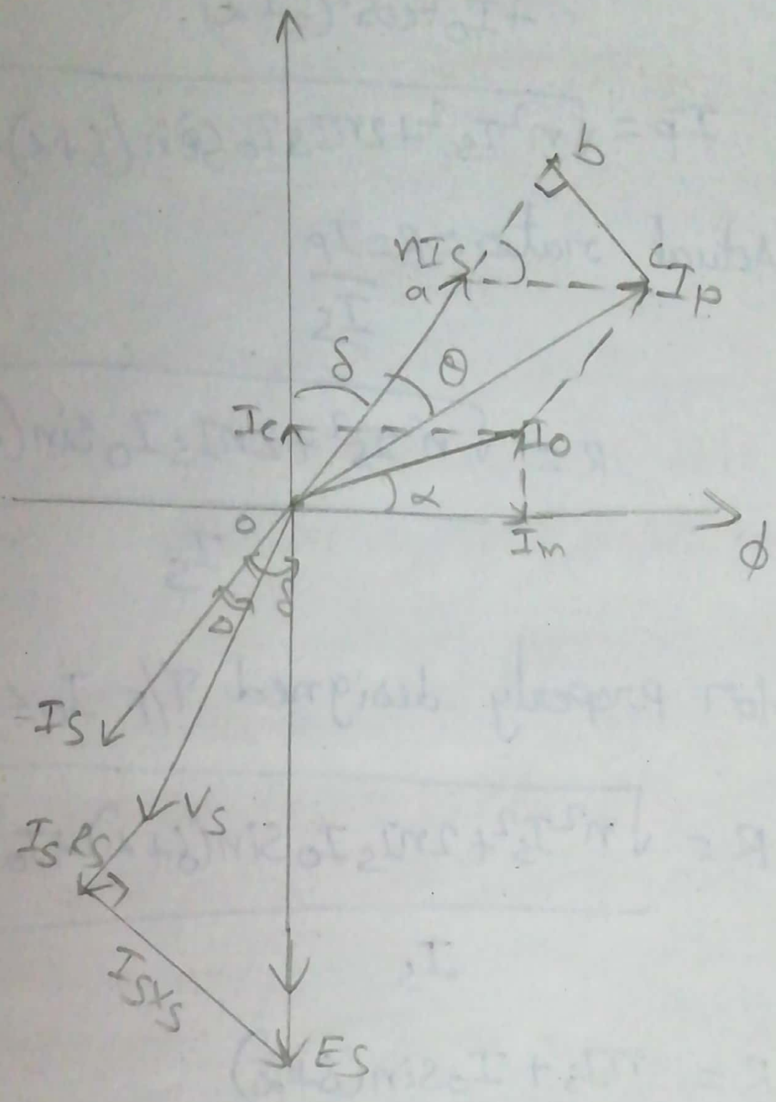
$$R = n + \frac{I_c r_p + I_m X_p}{V_s}$$

for C.T, approximate Exp for R

$$R = n + \frac{I_c}{I_s}$$

Thus Actual ratio becomes more than the nominal ratio. So turns ratio is reduced by reducing the secondary winding turns. this makes actual T/F ratio = nominal ratio. this is turns compensation for the current transformer.

Derivation of actual ratio



consider $\angle bac = 90-\delta-\alpha$, $ac = I_o$, $oa = nI_s$

$oc = I_p$

$bc = ac\sin(90-\delta-\alpha) = I_o\cos(\delta+\alpha)$

$ab = ac\cos(90-\delta-\alpha) = I_o\sin(\delta+\alpha)$

from $\angle obc$, $oc^2 = ob^2 + bc^2$

$= (oa+ab)^2 + bc^2$

$= [nI_s + I_o\sin(\delta+\alpha)]^2 +$

$[I_o\cos(\delta+\alpha)]^2$

$$I_p^2 = n^2 I_s^2 + 2nI_sI_0 \sin(\delta+\alpha) + I_0^2 \sin^2(\delta+\alpha) + I_0^2\cos^2(\delta+\alpha)$$

$$I_p = \sqrt{n^2 I_s^2 + 2nI_sI_0 \sin(\delta+\alpha) + I_0^2}$$

Actual ratio $= R = \frac{I_p}{I_s}$

$$R = \frac{\sqrt{n^2 I_s^2 + 2nI_sI_0 \sin(\delta+\alpha) + I_0^2}}{I_s}$$

for properly designed T/F $I_0 << nI_s$

$$R = \frac{\sqrt{n^2 I_s^2 + 2nI_sI_0 \sin(\delta+\alpha) + I_0^2 \sin^2(\delta+\alpha)}}{I_s}$$

$$R = \frac{nI_s + I_0 \sin(\delta+\alpha)}{I_s}$$

$$R = n + \frac{I_0}{I_s} \sin(\delta+\alpha)$$

$$R = n + \frac{I_0}{I_s}\left[\sin\delta \cos\alpha + \cos\delta \sin\alpha\right]$$

but $I_m = I_0 \cos\alpha$, $I_c = I_0 \sin\alpha$

$$R = n + \frac{I_m}{I_s}\sin\delta + \frac{I_c}{I_s}\cos\delta$$

Derivation of phase angle (θ) of T/F

θ is defined as the angle b/w reversed secondary current phasor i.e, reflected secondary current phasor & the primary current

$$\theta = nI_s \wedge I_p$$

from phasor, $\tan\theta = \frac{bc}{ob} = \frac{bc}{oa+ab}$

$$= \frac{I_0\cos(\delta+\alpha)}{nI_s + I_0\sin(\delta+\alpha)}$$

∴ θ is very small

$$\theta = \frac{I_0\cos(\delta+\alpha)}{nI_s + I_0\sin(\delta+\alpha)} \text{ radians}$$

but $I_0 << nI_s$

$$\theta = \frac{I_0\cos(\delta+\alpha)}{nI_s}$$

$$\theta = \frac{I_0[\cos\delta\cos\alpha - \sin\delta\sin\alpha]}{nI_s}$$

$$\theta = \frac{I_m\cos\delta - I_c\sin\delta}{nI_s} \text{ rad}$$

Converting to degrees

$$\theta = \frac{180}{\pi}\left(\frac{I_m\cos\delta - I_c\sin\delta}{nI_s}\right) \text{ deg}$$